# Combined Heat and Power Production Planning under Liberalized Market Conditions

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The need of efficient techniques for the modelling of combined heat and power (CHP) plants and for the optimization of their production has recently become imperative due to the liberalization of energy markets which compelled generating companies to compete on the energy markets. The goal of this paper is to present a general framework for modelling CHP plants which is able to deal with various configurations of technological equipment. The modelling framework also facilitates formulation of a mixed-integer linear programming (MILP) optimization problem dealing with plant operations scheduling, production planning and trading on spot and forward energy markets. The proposed framework has been implemented as a part of a decision support tool currently used by two district heating plants in the Central Europe.

## 1. Introduction

To be able to compete on deregulated energy markets, CHP plants need to achieve flexibility of their production by employing complex equipment configurations. As a result, the scheduling of their operations becomes more complicated problem due to the increasing number of degrees of freedom and sophisticated mathematical modelling and optimization techniques are needed to solve the problem of plant operations scheduling, production planning and trading on spot and forward energy markets.

This paper presents a general modelling framework for CHP plants able to deal with various configurations of technological equipment. The purpose of the framework is its utilization in the formulation of a MILP optimization problem.

This work is aimed on medium-sized or large CHP district heating plants employing thermodynamic cycle with installed capacity of boilers of 100 MWh and more (see Fig. 1 for example). Within the proposed framework, all components of the plants, i.e. turbines, boilers, heat exchangers, condensers, pressure reduction stations, water supply tanks etc., are modelled so that the over-all potential of the plant could be utilized.

The framework also allows for modelling the production of electricity in the form of power products which can then be traded on energy markets. The products are specified by four parameters – the time interval of demand (hour, day, week etc.), upper and lower bounds for power supply and the selection of hours within the interval in which the supply is demanded. The power supply in the (possibly not continuous) interval has to be constant. The example in Figure 2 shows the daily off-peak product with power supply demanded in hours 0:00 to 8:00 and 18:00 to 24:00 of each day.

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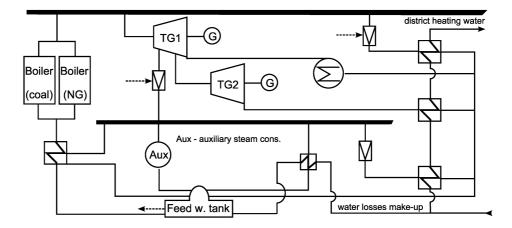


Figure: Example of a medium-size CHP plant.

#### 1.1 Literature Survey

The modelling of a CHP plant was addressed for instance by Chen et al. (2010), Thorin et al. (2005), Kong et al. (2005) and Lin et al. (2000). However, in these publications CHP plants are modelled by only several basic components (turbines, boilers and heat exchangers) for the purpose of electric utilities dispatch or process integration problems. In contrast, this paper deals with modelling complete thermodynamic cycles of CHP plants, i.e. including subsystems supplying boilers with water of specified parameters, water losses in district heating make-up and other parts of equipment essential to obtain effective and feasible everyday schedules of plant operations. Also, in the literature trading only on a spot market is considered or trading on markets is not considered at all. No publications proposing a general modelling framework of CHP plants or at least publications dealing with the modelling of a whole thermodynamic cycle for the purpose of operations scheduling were found.

#### 1.2 Problem Definition

To model the thermodynamic cycle of a CHP plant, its components are to be modelled first and then their interconnection into the cycle has to be implemented. In addition, electricity production in the form of power products has to be included.

Utilizing the model, a MILP optimization problem maximizing the objective function (1) is then formulated to obtain optimal operations schedule of every component, a plan of energy production and recommendations which power products are to be sold. Considering that a CHP plant is obligated to satisfy the heat demand, the revenues from heat are constant. The reason for including the heat revenues in the objective function is to allow easier interpretation of the objective value J as a profit of CHP plant.

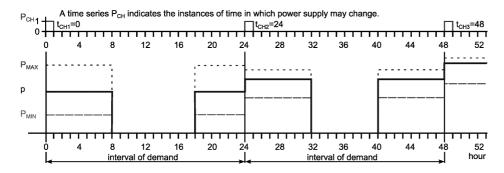


Figure 2: Example of a product definition – the daily off-peak.

## 2. Modelling Framework

In the following paragraphs a generalized component is introduced which may be utilized in the modelling of any typical component of a thermodynamic cycle. Its usage is then explained on the example of a turbine. Means for interconnection of the components into a complete thermodynamic cycle are then proposed. Component-specific constraints, such as ramping limits, are introduced next. Finally, the objective function (1) is concretized.

## 2.1 Generalized Component of the Thermodynamic Cycle

A component of thermodynamic cycle is generally a system with the set of inlet pipes A and the set of outlet pipes E. The state of the component in the time period t is specified by flows  $m_a(t)$  and  $m_e(t)$  of steam or water with enthalpies  $H_a$  and  $H_e$  through pipes  $a \in A$  and  $e \in E$  (Figure 3). Dynamics of all the components is neglected. The generalized component has to obey basic equations (1a) and (1b) formulating mass balance and energy balance respectively.

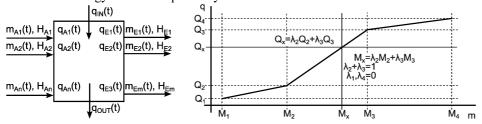


Figure 3: Generalized comp.

Figure 4. Example of a PWL function.

$$\sum_{a \in A} m_a(t) = \sum_{e \in E} m_e(t), \quad \forall t$$
 (2a)

$$q_{IN}(t) + \sum_{a \in A} m_a(t) H_a = q_{OUT}(t) + \sum_{e \in E} m_e(t) H_e$$

$$\Leftrightarrow q_{IN}(t) + \sum_{a \in A} q_a(t) = q_{OUT}(t) + \sum_{e \in E} q_e(t)$$

$$(2b)$$

The terms  $q_{IN}$  and  $q_{OUT}$  represent energy transferred into the component and out of the component in a different way than by the flows  $m_a$  and  $m_e$  respectively.

### 2.2 Modelling Non-Linear Characteristics

The enthalpies in (2b) are assumed to be independent on the flow. If this first approximation is found unsatisfactory a piecewise-liner (PWL) approximation (2c) is used instead of (2b).

$$q_{IN}(t) + \sum_{a \in A} h_a^{PWL} \left( m_a(t) \right) = q_{OUT}(t) + \sum_{e \in E} h_e^{PWL} \left( m_e(t) \right), \quad \forall t$$
 (2c)

It is possible to formulate a general PWL function  $q = h^{PWL}(m)$  as (3) using the type 2 special ordered set (SOS2), i.e. an ordered set of nonnegative variables  $\lambda_k$  of which at most two adjacent are allowed to be nonzero. The variables  $\lambda_k$  with  $k \in K$  define convex combination of points  $Q_k$  and  $M_k$  – the characteristic points of the PWL function with |K|-1 segments as illustrated in Figure 4.

$$m = \sum_{k \in K} \lambda_k M_k, \quad q = \sum_{k \in K} \lambda_k Q_k \tag{3a}$$

$$0 \le \lambda_k \le 1 \ \forall k, \quad \sum_{k \in K} \lambda_k = 1, \quad \left\{ \lambda_0, ..., \lambda_{|K|} \right\} \text{ is SOS2}$$
 (3b)

# 2.3 Utilizing Generalized Component in Model of Extraction-Condensing Turbine

Turbine TG has one inlet flow  $m_{AD}^{TG}$ , one outlet flow  $m_{EM}^{TG}$  and one extraction flow  $m_{EX}^{TG}$ . The mass balance (2a) is then formulated as (4a). Considering that emission enthalpy is not constant it may be convenient to formulate energy balance equation as (4b) and (3b), i.e. using PWL dependence of thermal power output  $q_{EM}^{TG}$  on the emission flow  $m_{EM}^{TG}$  with characteristic points  $\{M_{EM0}^{TG}, M_{EM1}^{TG}, M_{EM2}^{TG}\}$  and  $\{Q_{EM0}^{TG}, Q_{EM1}^{TG}, Q_{EM2}^{TG}\}$ .

$$m_{AD}^{TG}(t) = m_{EX}^{TG}(t) + m_{EM}^{TG}(t), \quad \forall t$$
 (4a)

$$m_{AD}^{TG}(t)H_{AD}^{TG} = q_{OUT}^{TG}(t) + m_{EX}^{TG}(t)H_{EX}^{TG} + \lambda_0^{TG}(t)Q_{EM0}^{TG} + \lambda_1^{TG}(t)Q_{EM1}^{TG} + \lambda_2^{TG}(t)Q_{EM2}^{TG}$$

$$m_{EM}^{TG}(t) = \lambda_0^{TG}(t)M_{EM0}^{TG} + \lambda_1^{TG}(t)M_{EM1}^{TG} + \lambda_2^{TG}(t)M_{EM2}^{TG}$$

$$\forall t \quad (4b)$$

# 2.4 Component-Specific Constraints

Aside from modelling mass and energy balances, operation of components may be additionally constrained by upper and lower bounds (5), ramping limits (RL, see Carrión et al., 2006 for formulation) and minimum up and down times (MUDT, see Hedman et al., 2009). Steam flow limits of a component may be formulated as (5) with a binary variable u(t) reflecting on/off state of the component in the time period t.

$$M_{MIN} u(t) \le m(t) \le M_{MAX} u(t), \quad \forall t \tag{5}$$

Also, it is convenient to define start-up and shutdown nonnegative continuous variables  $s_U(t)$  and  $s_D(t)$  taking implicitly values of one (zero) if the component is (is not) started up and shut down, respectively, in the period t. An effective formulation may be found in Hedman et al. (2009).

## 2.5 Interconnection of Components into Closed Thermodynamic Cycle

To create a full set of optimization constraints, the models of individual components have to be interconnected into the whole thermodynamic cycle. This can be implemented using a nodal component which is only a special case of the generalized component (2) with no energy lost or gained in the node. If the nodal component is employed to model a pipe junction or a steam header (as it usually is) the parameters of the steam flows are uniform, i.e. all the enthalpies are of the same value. Pipe junctions and steam headers are then modelled using equation (2a).

### 2.6 Model of Trading on Energy Markets

A power product can be modelled with a constraint (6a) where parameter  $P_{CH}(t)$  and instances of time  $t_{CH} \in \{t : P_{CH}(t) = 1\}$  are explained in Figure 2. The constraint (6a) is defined only for time instances specified by (6b).

$$-P_{MAX}(t)P_{CH}(t) \le p(t) - p(t-1) \le P_{MAX}(t)P_{CH}(t)$$
(6a)

$$t \in \{t_{CHi} + 1, ..., t_{CHi+1} - 1\} \cap \{t : P_{MAX}(t) > 0\}, P_{MAX}(t_{CHi}) > 0 \quad \forall i$$
 (6b)

#### 2.7 Objective Function

With the introduced models and constraints the objective function J (1) of the optimization problem can be reformulated as (7), where  $q_{REQ}$  is thermal energy supplied to the district with revenues  $R_Q$  per unit of energy,  $p^i$  is supplied electrical energy in the form of product i with revenues  $R_P^i$  per unit,  $q_{FUEL}$  is fuel consumption with costs  $C_{FUEL}$  per unit of fuel and  $C_{ALW}$  per  $CO_2$  allowance,  $C_{SU}^c$  and  $C_{SD}^c$  are costs per start-up and shutdown of a component and N is the set of components. The whole optimization problem is then formulated as follows:

$$\max_{\substack{q_{FUEL}, p^{p} \\ s_{U}, s_{D}}} J = \sum_{t \in H} \left( \overbrace{R_{Q}(t) q_{REQ}(t)}^{\text{revenues from heat}} + \sum_{i \in P} \overbrace{R_{P}^{i}(t) p^{i}(t)}^{\text{revenues from el. products}} \right) - \underbrace{\left( C_{FUEL}(t) + C_{ALW}(t) \right) q_{FUEL}(t)}_{\text{fuel costs}} - \underbrace{\sum_{c \in N} \left( C_{SU}^{c}(t) s_{U}^{c}(t) + C_{SD}^{c}(t) s_{D}^{c}(t) \right)}_{\text{start-up and shutdown costs}} \right)$$

$$(7)$$

subject to: (2) and (3) for all modelled components, (2a) for interconnection of the components, (5), (6), minimum up and down times and ramping limits constraints.

## 3. Implementation and Computational Results

The proposed model was implemented in Matlab. The optimization problem was formulated using optimization toolbox Yalmip (Löfberg, 2004) and solved using commercial solver Gurobi 4.0 on 8-core 2.5GHz Xeon, 16GB RAM computer.

Computational issues illustrated on various cases are presented in Tab. 1. Cases differ in horizon length, included constraints and in the set of tradable products. The set B includes only products with day interval of constant supply. The set A includes week

products in addition. All cases share the same real-world instance of a thermodynamic cycle comprising of 23 components.

Table 1: Comparison of computation time for various test-cases

Case	Products set	Horizon [h]	MUDT	RL	Computation time [s]
A1	A	168	yes	yes	2034
A2	A	168	no	yes	1455
A3	A	168	yes	no	1946
B1	В	168	yes	yes	131
B2	В	24	yes	yes	0,5

#### 4. Conclusions

A modelling framework for medium-sized and large CHP district heating plants was proposed and an optimization problem dealing with plant operations scheduling, production planning and trading on spot and forward energy markets was formulated. The main contribution of this paper lies in the generality of the proposed framework, facilitating many viable configurations of CHP plants to be modelled, and also in arbitrarily definable power products included in the optimization task, which is the feature that may improve ability of CHP plants to participate in forward energy markets. The proposed framework has been successfully implemented as a part of a decision support tool which has been in use by two district heating plants in the Central Europe region since the end of 2009.

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