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Optimal Design of Supply Chains by P-graph Framework under Uncertainties

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The utilization of renewable resources poses new challenges to process design. Consumption of renewable raw materials, or feedstock, often encounter limitations and uncertainties that are foreign to the design of processes based on fossil fuels whose availability is presumably uninterruptible. The algorithms and software of the graph-theoretic method based on process graphs (P-graphs) are elaborated for generating optimal and near-optimal supply networks for manufacturing a set of products at certain production levels with given reliabilities. Apparently, the sole currently available approach capable of rigorously and efficiently executing optimal process-network synthesis is that based on the P-graph framework. The approach leads to an algorithmically and mathematically proven solution for all steps involved. Such steps comprise superstructure generation, construction of the mathematical model, optimization, and the solution interpretation. The proposed method is illustrated with an example involving the utilization of renewable feedstock, namely agricultural products, whose uninterrupted availability tends to be uncertain.

1. Introduction

In recent years, there has been an increasing interest in developing processes based on renewable resources as feedstock. Various aspects of such processes, e.g., the process integration and supply chain design, have been intensively explored (Friedler 2009 and 2010, Lam et al. 2010a). Čuček et al. (2010) have assessed the production and supply of energy from biomass, and developed an integrated approach in synthesizing self-sufficient food-and-energy regional networks by utilizing multi-functional crops. Lam et al. (2010b) have studied the optimal synthesis of regional energy supply chains involving renewables based on the P-graph framework (Friedler et al. 1992, 1993, 1995). In the last decades, several robust and reliable process-network synthesis and optimization algorithms have been developed and implemented on the basis of the P-graph framework. The P-graph-based algorithms and the concomitant software are inordinately effective for enumerating feasible structural alternatives as well as determining the optimal or near-optimal *n*-best solutions. The original algorithms have been extended to consider the reliability of the availability of raw materials and to insure a predefined level of reliability for the overall process designed. The resulting

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algorithms are capable of synthesizing the optimal as well as near-optimal supply networks in the ranked order. The proposed method based on the extended algorithms is illustrated with an example involving the utilization of agricultural products.

2. Methodology

The optimal design of a supply chain under uncertainties attributable to the availability of the renewable resources as feedstock is carried out via the P-graph-based methodology. The mixed integer linear programming (MILP) problem, generated automatically via the algorithms of the methodology, serves as an input to subsequent optimization to determine the optimal supply chains in terms of multiple criteria (e.g., cost-optimal, performance optimal given a cost limit, etc.).

The problem definition contains the materials (raw materials, intermediates and products) and the operating units. The conventional directed/undirected graphs are inappropriate for process synthesis: The representation of the relationships between the operating units and the materials is ambiguous. In contrast, the process graph, or P-graph, is a directed bipartite graph that takes into account mathematically the combinatorial nature of process structures.

Let *M* be a finite set (material species), and let set $O \subseteq \wp(M) \times \wp(M)$ (operating units) with $M \cap O = \emptyset$, where $\wp(M)$ denotes the power set of *M*. Pair (*M*, *O*) is defined to be a P-graph with vertex set $M \cup O$ and arc set $A = A_1 \cup A_2$, where $A_1 = \{(x, Y) : Y = (\alpha, \beta) \in O, x \in \alpha\}$ and $A_2 = \{(Y, z) : Y = (\alpha, \beta) \in O, z \in \beta\}$. If $(\alpha, \beta) \in O$, then set α and set β are called the input-set and output-set of (α, β), respectively.

The P-graph-based methodology has been originally developed for synthesizing production systems, where operating units yield output materials by consuming input materials by assuming that their supply is uninterruptable. In graphical representation of P-graphs, each of the raw materials, products, and intermediate materials is denoted by a solid circle, while each operating unit is denoted by a horizontal bar.

In the P-graph-based methodology, algorithm MSG (Friedler et al. 1992) yields the maximal structure, i.e., the superstructure, for the PNS problem. The maximal structure is a rigorous superstructure, i.e., it is proven to contain at least one optimal structure as its sub-graph. This maximal structure serves as the input to the generation and solution of the mathematical model by algorithm ABB (Friedler et al., 1995). The solution-structure generator (algorithm SSG) determines exhaustively all the combinatorially feasible solution structures.

Algorithms MSG, SSG, and ABB are collectively executed by software PNS Studio (Varga et al., 2010; P-graph.com, 2010). For each problem, PNS Studio constructs a mathematical model via algorithm MSG, which is solved by an accelerated branch-and-bound algorithm, i.e., algorithm ABB. The model contains constraints for the operating units and the materials. Capacities of the operating units, consumption of the raw materials, and production of the final products are bounded. Moreover, mass conservation laws are defined for the intermediates and final products. The aim is either to identify the least expensive feasible structure or to maximize the profit. If the cost of an operation is defined by a stepwise linear function of its mass load, both the constraints and the objective function can be described by linear expressions. Alternatively, PNS Studio can exhaustively enumerate each combinatorially feasible structure via algorithm SSG.

In addition to costs and capacities, the reliability plays a significant role in selecting the process structure to be implemented under certain circumstances. As such, the reliability of availability (ROA) of renewable raw materials is considered in the process design in the current contribution. The value of ROA is assigned to each raw material as a function of its quantity. Moreover, the relationships are defined between the ROA values of different materials in the process structure.

When two input materials are fed to an operating unit, each of them is a raw material or produced independently from the other; subsequently, the ROA of each of its output materials is computed by multiplying the ROA's of its input materials. For example, if material m_3 is produced by a single operating unit from materials m_1 and m_2 with the ROA values of p_1 and p_2, respectively, the ROA of material m_3 is p_1p_2. The calculation can be generalized for n inputs, which are either raw materials or products. Thus, the ROA of any of the output materials of the operating unit is

$$p_{out} = p_{in,1} \cdot p_{in,2} \cdot \dots \cdot p_{in,n}. \quad (1)$$

In order to preserve the linearity of the mathematical model, the above expression is transformed as

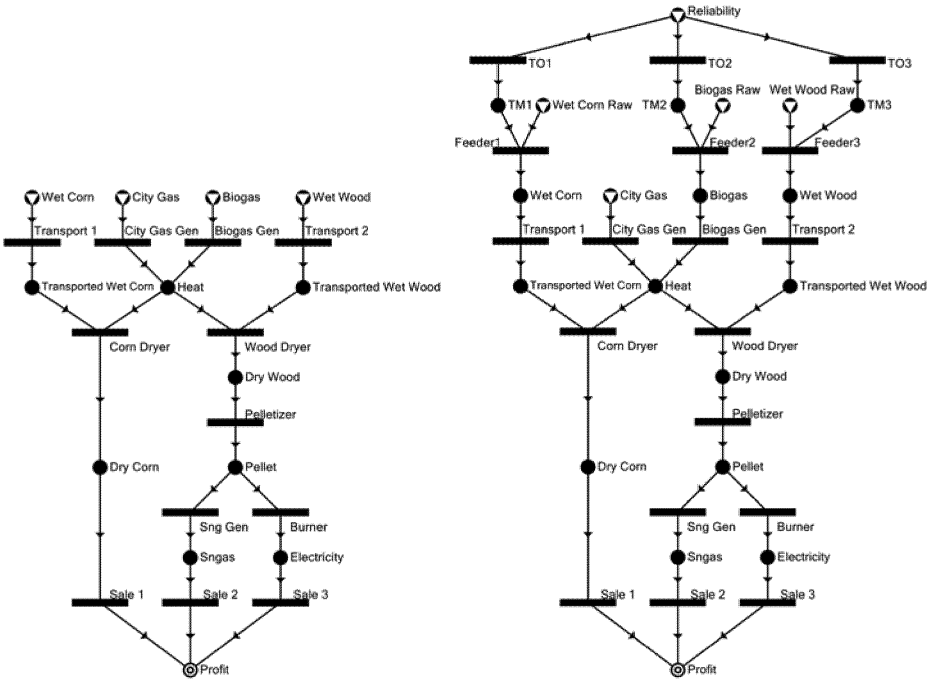
$$\log(p_{out}) = \log(p_{in,1}) + \log(p_{in,2}) + \dots + \log(p_{in,n}). \quad (2)$$

Note that this expression is linear and equivalent to equation (1) for positive values. Lower bounds for the reliability of the overall system, or for the ROA values of the products, can be similarly formulated and incorporated into the MILP model of the process-synthesis problem. Note that any process synthesis problem, which can be posed as a mixed integer linear programming problem solvable by a general MILP solver, gives rise to an optimal solution, but not the n-best solutions in ranked order yielded by algorithm ABB.

The P-graph representation of the reliability requires an initiating step prior to the construction of the mathematical model. The logarithm of the reliability of the overall system is regarded as a raw material, and a node representing it needs to be added to the P-graph of the model. This node is linked to each of the renewable raw materials by means of hypothetical operating units, which reduce the reliability of the system through the consumption of a raw material with uncertain availability.

3. Case Study

Corn and wood are typical agricultural or forest products of cultivated areas. This case study considers that dry corn, synthetic natural gas, and electricity can be produced from four raw materials including wet corn, wet wood, city gas, and biogas. Heat generated by burning city gas and biogas is deployed for drying the wet corn and wet wood. The dry corn is a potential final product, while the dry wood serves as an intermediate product to be pelletized and fed to the burner or utilized for the synthetic gas production. The values of ROA of the raw materials are given; while the city gas is always available, it is not the case for the wet corn, biogas and wet wood. Figure 1a portrays the initial structure described by P-graph, and Figure 1b represents the extended P-graph, where the reliability of the overall system is regarded as raw material; the hypothetical operating units (TO1, TO2, and TO3) with their corresponding materials (TM1, TM2, and TM3) are incorporated into the P-graph. Tables 1 and 2 list the parameters pertaining to the materials and the operating units.



(a) (b)

Figure 1: P-graph representations of: (a) the maximal structure of the case study and (b) its extension with reliability considerations

Figure 2 displays the three most reliable feasible structures for the case study with the reliability values of 89.3 %, 94 %, and 95 %, thus resulting in profits of 14719 €, 10172 €, and 4537 €.

Table 1: Properties of the raw materials for the case study

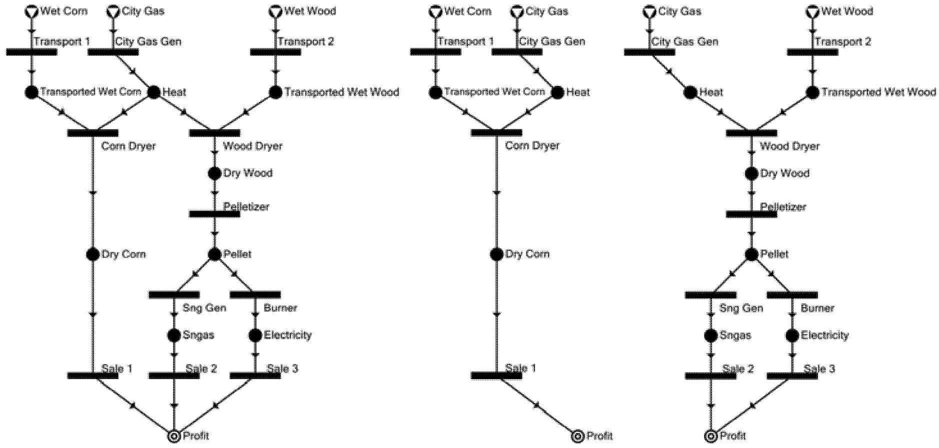
Materials	Type	Bound	Price	Reliability
Wet Corn	Raw material	2000 t/y	140 €/t	0.94
City Gas	Raw material	200000 m^3	0.5 €/m^3	1
Biogas	Raw material	260000 m^3	0.3 €/m^3	0.9
Wet Wood	Raw material	3000 t/y	80 €/t	0.95

Table 2: Inputs and outputs of the operating units for the case study

Unit name	Input(s)	Rate	Output	Rate	Capacity Bound	Inv. cost
Transport 1	Wet Corn	1 t	Transported Wet Corn	1 t	2000 t	1000 €/y
Transport 2	Wet Wood	1 t	Transported Wet Wood	1 t	3000 t	1000 €/y
City Gas Generator	City Gas	1 m^3	Heat	0.011 MWh	200000 m^3	10 €/y
Biogas Generator	Biogas	1 m^3	Heat	0.007 MWh	260000 m^3	10 €/y
Corn Dryer	Transported Wet Corn	1 t	Dry Corn	0.8 t	2000 t	5000 €/y
	Heat	0.35MWh				
Wood Dryer	Transported Wet Wood	1 t	Dry Wood	0.87 t	3000 t	5000 €/y
	Heat	0.15 MWh				
Pelletizer	Dry Wood	1 t	Pellet	1 t	3000 t	1000 €/y
Sng Generator	Pellet	1 t	Syn Gas	2.11 MWh	500 MWh	500 €/y
Burner	Pellet	1 t	Electricity	0.6 MWh	2000 MWh	500 €/y
Sale 1	Dry Corn	1 t	Profit	205 €/t	-	-
Sale 2	Syn Gas	1 MWh	Profit	55 €/MWh	-	-
Sale 3	Electricity	1 MWh	Profit	170 €/MWh	-	-

4. Conclusions

The P-graph-based methodology has been extended to take into account the overall reliability of raw-material availability in synthesizing the *n*-most profitable supply chain alternatives utilizing renewable resources. Exploiting the properties of the logarithm function, a linear mathematical model has been elaborated, which expresses the constraint on the overall reliability of a process system with uncertain feeds. The mathematical model has been formulated in the parlance of process-network synthesis as well by considering the reliability as a limited resource. The method is illustrated by a case study.



(a)	(b)	(c)
Reliability: 89.3 %	Reliability: 94 %	Reliability: 95 %
Profit: 14719 €	Profit: 10172 €	Profit: 4537 €

Figure 2: Three most reliable structures for the case study

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