CHEMICAL ENGINEERING TRANSACTIONS Volume 24, 2011
Editor Sauro Pierucci
 ISBN 978-88-95608-15-0 ISSN 1974-9791
DOI: 10.3303/CET1124042

Dynamic Characteristics of Counter-Current Flow Processes

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In industry counter-current flow processes are common. Although these processes have been widely studied in literature, relatively little has been published on their dynamic behaviour. Two very common counter-current flow processes are heat exchangers and distillation columns. Ma's study based on dynamic models of heat exchanger's dynamic behaviour [1] reports an internal resonance effect, also earlier reported by Profos in 1943 [4] and Friedly in 1972 [3]. Here the study is extended to lumped models, first for heat exchangers and thereafter for an extremely simplified distillation column. Not unexpectedly, the dynamic properties change gradually as the number of lumps increases towards the distributed systems and for high frequencies similar internal resonance effects evolve with the envelopes showing a very low-order behaviour, which though somewhat surprisingly is independent of the number of lumps. Finally we show that the eigenvalues of the normed system matrix lie on a circle in the complex plane.

Keywords: Modeling, distributed/lumped model, Resonance effect, Frequency analysis

1. Introduction

Many industrial processes are based on two phases exchanging material and heat. The two phases are passing each other either in co-current or counter-current fashion often arranged in stages in each of which one drives the system towards equilibrium. The counter-current scheme is more commonly used, so we focus on this pattern.
Although counter-current flow processes have been widely studied in literature, little of it reports on their fundamental dynamic behaviour. Commonly used dynamic models for heat exchangers are simple empirical first-order-plus-dead-time models. Exceptions are Profos (1943) [4] reporting the internal resonance effects, Friedly (1972) [3] who derived reduced-order models and X H Ma [1] who derived a new set of high-fidelity low-order models also confirming the internal resonance effect, which years later has been show to exist in an experimental study by Grimm [2].
In Ma's a distributed model for heat exchangers the temperatures on the inner and the outer tube are considered as continuous functions of time and spatial coordinates yielding a set of partial differential equations. This model shows the presence of the internal resonance effect in the high frequencies domain. She splits the transfer function into a resonance and a non resonance part assuming a linear underlaying behaviour. This procedure yields high-fidelity analytical low-order models being the envelopes of the oscillating transfer function. For distillation columns however, no such behaviour has been reported. Since standard tray columns are better described as counter-current staged processes, Here we repeat Ma's study with lumped models aiming at a

Please cite this article as: Puschke J. and Preisig H.A., 2011, Dynamic characteristics of counter-current flow processes, Chemical Engineering Transactions, 24, 247-252
DOI: 10.3303/CET1124042

comparison with distillation models. The model is constructed as a network of communicating capacities for each of which a mass and an energy balance is constructed. The energy balances are being transformed into the alternative state space of the intensive quantity temperature all of which forms a set of ordinary differential equations. It is expected that, as the number of lumps approaches infinity, the solution approximates the solution of the distributed model hoping that the resonance effect shows also for low-order models. In a second step this is applied to a kind of a mass transport network representing a crude model of a distillation column.

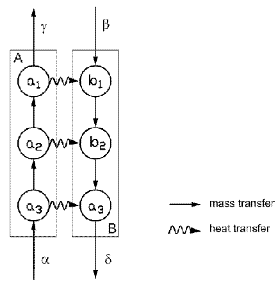

Figure 1: Model of the counter current double pipe heat exchanger with n equal three stages

2. Lumped Model of the Heat Exchanger

Ma’s double pipe heat exchanger is analyzed approximating it as series of heat-exchanging, paired lumps, on one side representing the hot stream and on the other the cold one. Since the result shall be compared with distillation models only the counter-current flow pattern is being considered. Ma’s work discusses different cases, which are based on different sets of assumptions. Here we focus on Ma's case I. We start with a 3-stage process as shown in Figure 1. The assumptions are:

- The total volume of stream A (consisting of all equal-sized lumps a_i) is the same as the total volume of stream B and both are constant: $V^A = V^B = V = const$
- The volume of each lump has the same size: $V_{a1} = \cdots = V_{an} = V_{b1} \cdots = V_{bn} = V/n$
- The heat transfer area O_i with $i = \{1,2,\cdots,n\}$ between two lumps with the same index i is constant and equal: $O_i = O_o/n$; $\forall i$, with O_o as the overall heat transfer area between stream A and stream B.
- Heat is only transferred between two lumps with the same index. No axial heat transfer.

2.1 State Space Model Equations for n Stages

The energy balances is drawn up for each lump and solved for the temperature T resulting in the state space representation: $\dot{x} = Ax + Bu$ with the two matrices A and B being

$$A = \begin{pmatrix} -\tau_A - d_A & \tau_A & & & d_A & & & \\ & \ddots & \ddots & & & \ddots & & \\ & & \ddots & \tau_A & & & \ddots & \\ & & & -\tau_A - d_A & & & & d_A \\ d_B & & & & -\tau_B - d_B & & & \\ & \ddots & & & \tau_B & \ddots & & \\ & & \ddots & & & \ddots & \ddots & \\ & & & d_B & & & \tau_B & -\tau_B - d_B \end{pmatrix} ; \; B = \begin{pmatrix} 0 & 0 \\ \vdots & \vdots \\ \tau_A & 0 \\ 0 & \tau_B \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix} \quad (3)$$

And the $y=Cx$ with the matrix $C=\begin{pmatrix}1 & \cdots & 0\\ 0 & \cdots & 1\end{pmatrix}$ with $\tau_m=\frac{\hat{V}^m}{V/n}$; $d_m=\frac{k_m O_i}{\rho_m c_{pm} V}$, $m\in\{A, B\}$. The quantities are: k_m :: heat transfer coefficient of stream *m*, O_i :: heat transfer area between two lumps, ρ_m :: density of the stream *m*, c_{pm} :: specific heat of stream *m*, V/n :: individual lump volume.

The state is $x=\left(T_{a1} \;\cdots\; T_{an} \;\; T_{b1} \;\cdots\; T_{bn}\right)^T$, the input is $u=\left(T_\alpha \;\; T_\beta\right)^T$ and the output is $y=\left(T_\gamma \;\; T_\delta\right)^T$.

2.2 Bode Plots

The dynamic behaviour of the models is depicted in Bode Plots of the model's transfer functions. The transfer functions are derived by transforming the state space model into the frequency domain solving for the output y = x in dependence of u. The transfer function matrix is then simply:

$$G=C(sI-A)^{-1}B \tag{4}$$

The transfer functions of input α to the output γ shows similar behavior as the one from the input β to the output δ. Only one of the two down stream responses, namely G_{11} from the input α to the output γ is shown in Figure 2. The same applies to the cross stream transfer functions, where only the transfer function G_{12} from the input α to the output δ is plotted. The behavior of all transfer functions approaches Ma's distributed model as the number of stages *n* approaches infinity which is also shown in Figure 2 as a reference.

Down Stream Response: The behavior of the transfer functions varies with the number of stages *n*. In the amplitude plot with an increasing number of stages *n* the slope of the amplitude decreases. As the number of stages approaches infinity the slope approaches zero. The latter implies that there exists only one gain. In the phase plot an increasing number of lumps increase the negative phase shift. For an infinite number of stages the phase lags go to minus infinity, which indicates the existence of a dead time. But there

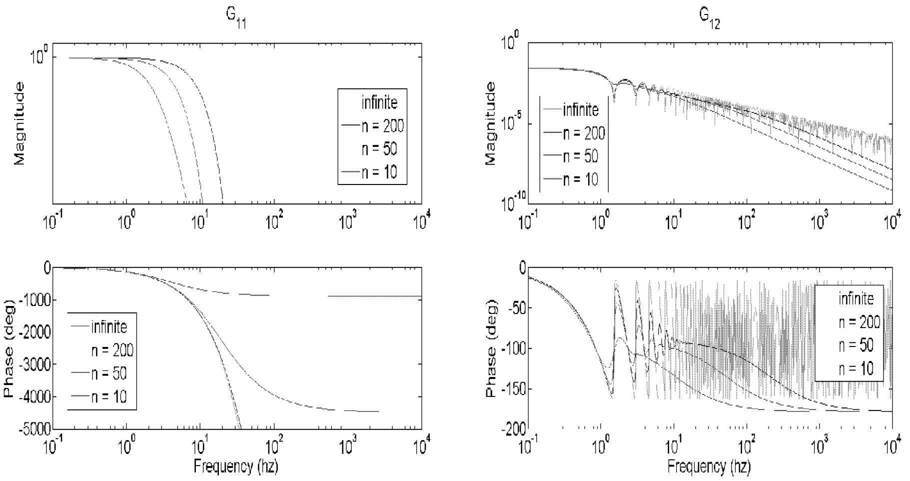

Figure 2: Bode plots of the down stream transfer function G_{11} (left) and of the cross stream transfer function G_{12} (right) with different numbers of stages n. The parameters are chosen to be $d_A=d_B=0.01$; $\tau_A=1$; $\tau_B=1$.

is not a resonance effect in the amplitude or the phase.

Cross Stream Response: The Bode plot shows the resonance effect in amplitude and phase. Furthermore one observes that the curves show a first corner at the frequency of ω=1Hz for the chosen set of parameters. Above this corner frequency the slope in the amplitude plot of the resonance part is in average minus one. And in the phase plot the resonance part average is -90 degree. The transfer functions with the number of stages being small than infinite show a decaying resonance part with increasing frequency, which finally disappears. The apparent length of the resonance part depends on the number of stages: With an increasing number of stages, the resonance part grows longer until the infinite case, where the resonance part does not decay anymore. Also the models with the number of stages being less than infinity, the final slope in the amplitude plot is -2 and the final phase lag is -180 degree. Hence this transfer functions show a second corner frequency under which the resonance part decays. Both corner frequencies depend on the number of stages.

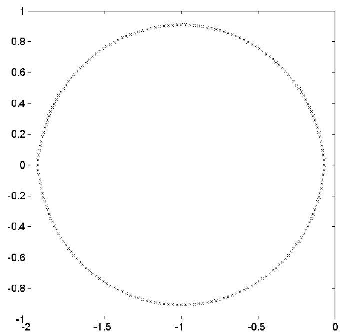

Figure 3: The normed eigenvalues of the system matrix A in the complex plane with n=100.

2.3 Detailed Analysis of the Cross Stream Response

To get more information about the second-order behavior of the cross stream response, one needs the pole excess of the transfer function. Due to the structure of the matrices B and C only four entries of the matrix $(Is\text{-}A)^{-1}$ are relevant for the transfer functions matrix and only two of these entries for the cross-stream transfer functions. The zeros of the transfer functions are the zeros of the adjoint matrix adj(Is-A). For the number of stages *n = 3 or 4* it is easy to show that the respective adjoint matrices have $2n$-2 zeros. Since the poles are the eigenvalues of A, their number is $2n$. So the pole excess is 2n-(2n-2)=2, which explains the observed second-order behavior. In addition, by closer examination of the poles, one finds that the normed eigenvalues of the matrix A form a circle with radius one and the center at (-1,0) as shown in Figure 3.

3. Lumped Model of a Distillation Column

Figure 4 depicts a distillation column and an abstraction there off which underlays the construction of the model equations [5].

3.1 The State Space Model

The mass balances drawn for each lump, assuming a linear transfer law making the mass transfer proportional to the composition differentiate and solved for the concentration c of the lumps yields again a linear state space model (A,B,C,D), with the state x , input u and the output y being

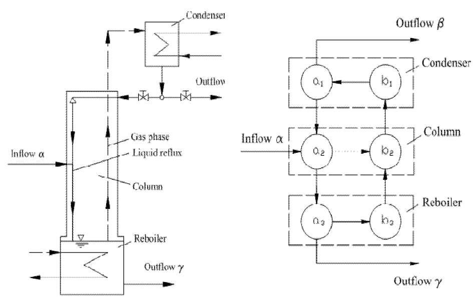

Figure 4: Process of the distillation column (left) and model of the distillation column (right)

$x = \begin{pmatrix} c_{a1} & \cdots & c_{an} & c_{b1} & \cdots & c_{bn} \end{pmatrix}^T$; $u = c_\alpha$ and $y = \begin{pmatrix} c_\beta & c_\gamma \end{pmatrix}^T$ with:

$$A = \begin{pmatrix}
-\tau_{a1}-\tau_\beta & & & & & \tau_{a1} & & & & \\
\tau_{a2} & -\tau_{a1}-d & & & & & d & & & \\
& \ddots & \ddots & & & & & \ddots & & \\
& & \tau_{an-1} & -\tau_{an-1}-d & & & & & d & \\
& & & -\tau_{an} & -\tau_{an}-\tau_\gamma & & & & & 0 \\
0 & & & & & -\tau_{b1} & & & & \\
& d & & & & & -\tau_{b2}-d & \tau_{b2} & & \\
& & \ddots & & & & & \ddots & \ddots & \\
& & & d & & & & & -\tau_{bn-1}-d & \tau_{bn-1} \\
& & & & \tau_{bn} & & & & & -\tau_{bn}
\end{pmatrix};$$

$$B = \begin{pmatrix} 0 & \cdots & 0 & \tau_\alpha & 0 & \cdots & \cdots & \cdots & 0 \end{pmatrix}^T ; \quad C = \begin{pmatrix} 1 & 0 & \cdots & & & \cdots & 0 \\ 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{pmatrix} ; \quad D = \begin{pmatrix} 0 & 0 \end{pmatrix}^T$$

with $\tau_m = \frac{\dot{V}^m}{V}$ and $\tau_{i2} = \tau_{i3} \ldots = \tau_{in-1}$, $i \in \{a, b\}$

These sparse system matrices for the distillation column show a similar structure as the system matrices of the heat exchanger.

3.2 Bode Plots

The transfer functions in the following Bode plots are obtained in the same way as this was done in the analysis of the heat exchanger.

The curves in the magnitude plot of the transfer function from the input α to the output β in the Bode plots of Figure 5 shows a resonance effect. This resonance part decays

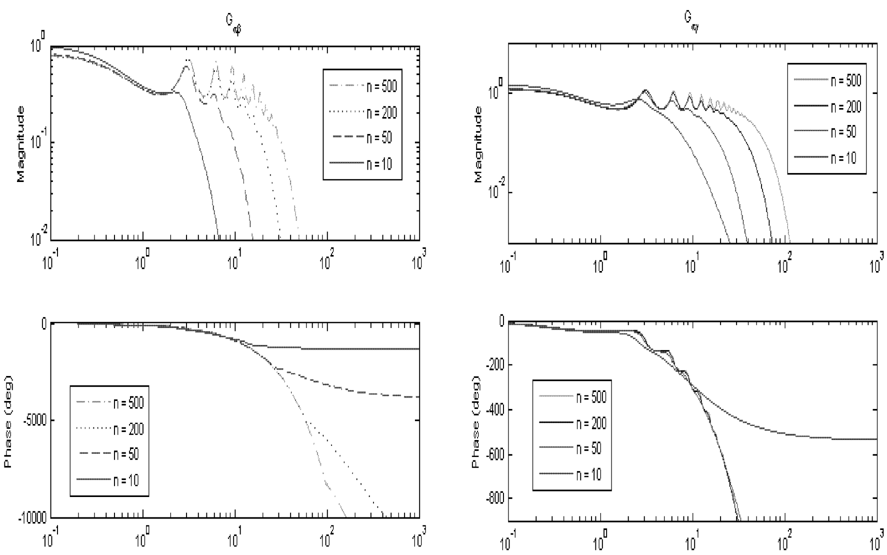

Figure 5: Bode plots of the transfer function $G_{\alpha\beta}$ (left) from the input α to the output β and of the transfer function $G_{\alpha\gamma}$ (right) from the input α to the output γ with different numbers of stages n.

before the curves reach a multiple corner frequency, which depends on the number of stages n. With a larger number of lumps the length of the resonance part is longer. If the number of stages go to infinity one could assume, that there is only a steady state gain with resonance, whereas in the phase shift plot the resonance part does not appear. The general behavior of the curves with the number of stages n going to infinity suggests the existence of a dead time.

The magnitude plot of the transfer function from the input α to the output γ (see Figure 5) shows a comparable response behavior as the transfer function from the input α to the output β. The differences are in the amplitude of the resonance part and the corner frequency. But the phase plot of the of the transfer function from the input α to the output γ is a resonance part.

By closer examination of the poles in the complex plane, one finds again that the standardized eigenvalues of A form a circle with radius one and the center at (-1,0) as Figure 3 shows.

4. Conclusion

The dynamic characteristics of two counter-current processes are compared: a single tube heat exchanger and a staged distillation column. For both simple linear transfer models are assumed yielding linear systems that are of very similar structure. If normed, both show the same behaviour with respect to the system eigenvalues: they lay on a shifted unit circle in the complex domain. Both systems show resonance effects for some parts. Heat exchangers show it for cross-stream transfer functions, but not for down-stream transfer functions, whilst in distillation one finds the resonance also in the down stream transfer function, at least in the amplitude. In both cases, the magnitude of the resonance effect is a function of the number of lumps or stages.

In case of the heat exchanger the pole excess is 2, but the second corner frequency approaches infinity as the number of lumps approaches infinity. Thus for the distributed system the pole excess is only 1. This behaviour is also detected in the phase plot with a max phase shift of -180 degrees for finite number of lumps and -90 degrees for the distributed system.

The cross transfer functions for the distillation column behaves like a dead time for high frequencies, though the position of the multiple zeros shifts to higher and higher frequencies as the number of stages increases.

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