Availability-Based Optimization of Preventive Maintenance Schedule: A Parametric Study

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The current work proposes an approach based on equipment availability which is incorporated in a Benefit function, the latter being the net cost saved by applying a Preventive Maintenance (PM) vis-à-vis Corrective Maintenance (CM). The Benefit function is maximized to obtain the optimal maintenance schedule. The effect of variation of different PM parameters – namely, maintenance repair rate and cost, inspection rate and cost – on the optimal Benefit and PM schedule is also simulated. The results show that benefit increases with increase in repair and inspection rates, but decreases with increasing repair and inspection cost.

1. Introduction
Maintenance cost comprises nearly 20-30 % of the operating plant budget (Van Rijn, 1987); hence, optimization of maintenance actions is critical. Preventive Maintenance (PM) and/or Condition Based Predictive Maintenance (CBPM) strategies can augment equipment availability. In both instances the periodicity of maintenance interventions need to be optimized for greatest benefit. The present paper addresses the problem PM schedule optimization.

In PM, inspections may be Risk-based, Reliability or Availability-based. Optimizing maintenance schedule through the use of risk has been reviewed by Dey (2004) and Khan and Haddara, 2003). Comparison of different risks, i.e. like loss of human lives, environmental damage and economic loss, is a challenging task. The choice of the weighing factors of these risks is based on expert judgment, which renders the methodology sensitive to the experience of the analyst. Similarly, reliability function may also be used to optimize the PM schedule (Sachdeva et. al, 2008; Samrouta et. al., 2009; Ghosh and Roy, 2009).

There have been very few reported attempts at use of availability as a parameter to optimize PM (Wang, 2002; Naikan and Rao, 2005; Garg et. al, 2010; Khan and Haddara; 2008). Also, the applicability of these approaches to optimize the PM schedule is not often demonstrated explicitly. The present work proposes an approach which integrates the availability function for an equipment / system into a ‘Benefit’ function defined as the difference between net life cycle cost for using corrective maintenance (CM) and for using preventive maintenance (PM). The ‘benefit’ function is maximized to obtain the optimal PM schedule.
2. Theory

2.1 Corrective Maintenance Model

Steady-state availability \( A_{CM} \) under CM is given by:

\[
A = \frac{1}{\mu_{CM}} \left( \frac{1}{\mu_{CM} + \lambda_f} \right)
\]

(1)

Where,

\( \mu_{CM} \) = repair rate (repair/time),
\( \lambda_f \) = equipment failure rate (failures/time).

The total operative cost for CM model \((C_{CM})\) over the full life span of the equipment is:

\[
C_{CM} = (1 - A_{CM})C_L T_L + C_{CM} \lambda_f T_L
\]

(2)

Here, \( C_L \) = cost of loss of production/time; \( T_L \) = equipment life span; \( C_{CM} \) = cost per repair. The first term in eqn. (2) is the total cost of loss of production, while the second term is the total life span repair cost, \( \lambda_f T_L \) being the expected total number of repairs.

2.2 Preventive Maintenance Model

Fig. 1 depicts the possible states that a system can be in under a PM strategy, namely: operating (O), inspection (I), repair (R).

Figure 1. Markov Chain Diagram for PM model

Here, \( \mu_i \) = inspection rate, \( \mu_r \) = repair rate, \( \lambda_i \) = inspection rate, \( \lambda_f \) = failure rate. Shutdown is taken every \( t_i \) (=1/\( \mu_i \)) to inspect the equipment and is then sent to the repair state if a defect is detected; otherwise it is returned to operating state. If the equipment fails it is moved directly to repair state. The steady state availability is (Ebeling, 1997):

\[
A_{PM}(T) = \int_0^T R(t)dt / \left[ T + t_i + t_r \cdot [1-R(T)] \right]
\]

(3)

Here ‘\( T \)’ is the inspection interval. For a constant failure rate, \( R(T) = e^{-\lambda_f T} \), thus:

\[
A_{PM}(T) = (1 - e^{-\lambda_f T}) / \left[ \lambda_f \cdot [T + t_i + t_r \cdot (1 - e^{-\lambda_f T})] \right]
\]

(4)

As for the CM model above, \( C_{PM} \) for the total life span of equipment is defined by:

\[
C_{PM} = (1 - A_{PM})C_L T_L + [1 - R(T)] C_{PM} T_i / T + C_i T_i / T
\]

(5)

Here, \( A_{PM} \) = operational (steady-state) availability under PM; \( C_{PM} \) = per repair cost in PM mode; \( C_i \) = cost per inspection in PM mode. The first term in the eqn. (6) is the cost of loss of production; second term the repair cost, and the last term the inspection cost; \( T_i / T \) = total number of inspections. The net life cycle benefit for PM (with inspection interval of \( T \)) over CM using eqns. (3) and (6) is as follows:

\[
B(T) = (1 - A_{CM})C_L T_L + C_{PM} C_{CM} T_L - (1 - A_{PM})C_L T_L - [1 - R(T)] C_{PM} T_i / T - C_i T_i / T
\]

(6)
$B(T)_{total}$ is saved cost (or benefit) that results from employing PM mode as opposed to CM for the life span of the equipment. Thus, the benefit function per time $B(T)$ is:

$$B(T) = (1 - A_{CM})C_L + C_{rCM} \lambda_r - (1 - A_{PM})C_L - [1 - R(T)]C_{rPM} / T - (C_r / T)$$

(7)

3. Results and Discussion

3.1 Availability and Benefit Variation

Table 1 presents the representative parameter values used for estimation of the $A_{PM}(T)$ and $B(T)$ functions in terms of the PM interval $T$. The repair cost $C_{rPM}$ must be $< C_{rCM}$, without which PM can have no advantage over CM. Similarly, $\mu_{rPM} > \mu_{rCM}$, since failures detected by PM are expected to be sub-catastrophic. Finally $\mu_I$ is assumed to be 10 times more than $\mu_{rPM}$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repair rate in CM model ($\mu_{rCM}$)</td>
<td>0.05/d</td>
</tr>
<tr>
<td>Repair rate in PM model ($\mu_{rPM}$)</td>
<td>0.25/d</td>
</tr>
<tr>
<td>Inspection rate in PM model ($\mu_i$)</td>
<td>2.5/d</td>
</tr>
<tr>
<td>Cost per repair in CM model ($C_{rCM}$)</td>
<td>$4000</td>
</tr>
<tr>
<td>Cost per repair in PM model ($C_{rPM}$)</td>
<td>$800</td>
</tr>
<tr>
<td>Cost per inspection in PM model ($C_I$)</td>
<td>$600</td>
</tr>
<tr>
<td>Loss of production per day ($C_L$)</td>
<td>$600</td>
</tr>
<tr>
<td>Failure rate for both CM and PM ($\lambda_r$)</td>
<td>0.02/d</td>
</tr>
<tr>
<td>Life span ($T_L$)</td>
<td>3000 d</td>
</tr>
</tbody>
</table>

Using the above parameters the variation of $A_{PM}(T)$ availability with PM interval $T$ is plotted in fig. 2(a); the availability is maximum at $T = 5$ d. This optimal availability $A_{opt} = 0.81$ is greater than $A_{CM} = 0.61$ obtained using CM model (Eq. 2). Fig. 2(b) shows the variation of the benefit function, as in Eq. (7), with $T$. In contrast to fig 2a this curve has $T_{opt} = 13$ d, the corresponding, optimal benefit ($B_{opt} = $95/d. Therefore, a cost-based optimal schedule differs from that obtained by engineering considerations alone, i.e. using simple availability optimization by Eq 4.

![Figure 2: Variation of (a) Availability ($A_{PM}$) and (b) Benefit with PM interval ($T$)](image-url)
In the following sections, we present select results of sensitivity analyses on $B_{opt}$ and $T_{opt}$ for variation in the following PM parameters: repair and inspection costs, repair and inspection rates. For each simulated set only a single parameter is changed, while the others are held constant at values indicated in table 1, and $B_{opt}$ and $T_{opt}$ are obtained.

### 3.2 Variation of Optimal T and Benefit with Repair Cost

Fig. 3(a) shows the variation of $T_{opt}$ with $C_{rPM}$. Here, $C_{rPM}$ is varied as a function of $C_{rCM}$ such as $C_{rPM} = C_{rCM} / n , n = 1-3$, as $C_{rPM}$ must be $< C_{rCM}$. While $T_{opt}$ increases with $C_{rPM}$, the change is relatively small. Thus it may not be feasible to increase the $T_{opt}$ (that is, reduce the PM intervention frequency) just by spending more on repair. Fig. 3(b) shows the variation $B_{opt}$ with $C_{rPM}$; as is expected, $B_{opt}$ decreases as $C_{rPM}$ increases (see Eq. 7).

### 3.3 Variation of Optimal T and Benefit with Repair Rate

Fig. 4(a) shows the variation of $T_{opt}$ with $\mu_{rPM}$. Again, $\mu_{rPM}$ is varied in terms of $\mu_{rCM}$ as $\mu_{rPM} = \mu_{rCM} n , n = 1-3$, since $\mu_{rPM}$ must be $\geq \mu_{rCM}$. $T_{opt}$ decreases as the repair rate increases, i.e., inspections need be more frequent to obtain the optimal benefit. From fig. 4(b) we see that benefit increases as the $\mu_{rPM}$ increases, because the life-cycle downtime decreases, which in turn increases the optimal benefit. There is a critical value of $\mu_{rPM}$ above which optimal benefit is just positive. For the given set of parameters in table 1, the critical value of $\mu_{rPM} = 1.4 \mu_{rCM}$. Thus, a benefit is obtained by replacing CM with PM if the repair rate can be reduced by about 40% (using PM).

### 3.4 Variation of Optimal T and Benefit with Inspection Cost

Fig. 5(a) shows the variation of $T_{opt}$ with $C_I$. (As in table 1, $C_I = $ 500; thus, variation of inspection cost $C_I$ is desired as $C_I = 500 n , where, n = 1-3$). $T_{opt}$ increases with inspection; i.e., inspections need to be performed less frequently for optimum benefit. Fig. 5(b) shows that optimal benefit decreases as $C_I$ increases, as evident from Eq. (8).

### 3.5 Variation of Optimal T and Benefit with Inspection Rate

For PM to be effective, inspection rates will need to be higher than repair rates. Fig. 6(a) shows the variation of $T_{opt}$ with $\mu_I$, where, $\mu_I = n \mu_{rPM} , where, n = 1-3$. Fig. 6(a) shows that $T_{opt}$ decreases with increasing inspection rate; as inspection time decreases, one needs to do more frequent inspection for the optimal benefit. Fig. 6(b) shows that
optimal benefit decreases as $\mu_I$ increases. As the inspection time decreases, availability increases (Eq. 1), this lowers the loss of production, which in turn increases benefit.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure4.png}
\caption{$T_{opt}, B_{opt}$ vs. inspection cost $C_I$}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure5.png}
\caption{$T_{opt}, B_{opt}$ vs. inspection rate $\mu_I$}
\end{figure}

### 3.6 Effect of Failure Rate Behavior

While all the above results pertain to constant $\lambda_f$, here we depict the variation of benefit with $T$, for the case of linearly increasing failure rate: $\lambda_f=a+bt$, where $a=0.02/d$ and $b=0.002/d^2$, such that the starting value is the same for both cases ($=0.02/d$), as used earlier. Fig. 6 shows that $T_{PM, opt}$ for the linearly increasing failure rate is lesser. This implies that more frequent inspections are required for the linear failure rate to have the optimal benefit. This is because overall at any point of time the component is less reliable in case of a failure rate that increases with time. As a consequence, optimal benefit is also less than the constant failure rate case.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure6.png}
\caption{Benefit vs. PM Interval (T) for Constant and Increasing Failure Rates}
\end{figure}

### Conclusions

A new approach based on equipment availability incorporated in a Benefit function has been proposed for optimal Preventive Maintenance (PM) scheduling. ‘Benefit’ is defined as the net cost saved by applying a PM vis-à-vis Corrective Maintenance (CM). The benefit function is maximized to obtain the optimal maintenance schedule. For a equipment with constant failure rate, both the benefit and PM schedule increase with increase in both repair and inspection rates, but decrease with increasing repair and inspection cost. Also, if the equipment failure rate increases linearly over time, both the optimal benefit and PM schedule time are reduced in comparison with the former case.
References