Physical modelling of the drained flow on a suction box of a papermachine

Jean-Claude Roux*, Martine Rueff
Laboratory of Pulp and Paper Science and Graphic Arts, Grenoble Institute of Technology, Pagora, 461 rue de la Papeterie, BP 65, 38402 Saint-Martin d’Hères France
jean-claude.roux@grenoble-inp.fr

Suction boxes are used in the paper industry simultaneously to drain a pulp suspension and to form a fibre mat (or filter cake). This research address, in the formation unit of an industrial papermachine, the modelling of the fibre deposition assuming a filtration process and that of the flowing suspension drained through the building fibre mat and the wire on a suction box. From experimental data of the cumulative volume drained \( V \), per surface area, for two vacuum pressure \( \Delta P \) and 6 application times \( t \), an extension of the classical law \( (t/V) \) versus \( V \) is proposed, validated and used. This relation allows determining the average specific filtration resistance of the fibre mat over the box and the solid mass deposition before and over the suction box. The model obtained is as precise as 1% and can be used to limit and reduce the energy consumption of drainage assisted devices as suction boxes in the forming unit of an industrial papermachine.

1. Introduction

Before the paper manufacturing itself, the origin of a paper sheet is a pulp suspension composed of solid materials (cellulose fibres and mineral fillers) and a liquid phase: the water. The paper forming operation is the first process to eliminate water by drainage from the pulp suspension; it mainly proceeds through a filtration process. In fact, the filtration process occurs on a moving wire. In the relative referential attached to this wire, the process can be defined as a “dead-end filtration” where the pulp suspension flows perpendicularly through the building fibrous cake and the medium of filtration. Hence, the solid materials are deposited on the wire; the cellulose fibres entangle each other to form a filter cake that gives birth to a future sheet of paper.

Technically, the drainage process may operate through vacuum assisted devices named: suction boxes. Two acting variables are responsible for the solid/liquid separation: the suction pressure \( \Delta P \) applied through the thickness of both building fibre mat and wire and the duration time \( \Delta t \) applied. If the drainage flow versus the suction pressure and the application time was known, the mass of solids retained per unit area on the wire would be known. This would help us to quantify the flow drained on a suction box at a given vacuum. In the context of reducing the energy consumption in the paper industry, this research addresses an actual and serious problem.

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The second paragraph begins with a theoretical part in order to precise the hypothesis and to gather the main variables concerned by the analysis: the solid mass deposited on the wire per unit area named grammage \( w(t) \), the cumulative volume drained \( V(t) \) and the corresponding volumetric flow \( q(t) \). Then, the Darcy’s law is introduced and linked to the previous equations. The classical law in filtration science, \( t/V \) versus \( V \), is obtained. The third paragraph deals with materials and methods and proposes an extension of this classical law. In the fourth paragraph, the relation is validated and used to determine the average specific filtration resistance of the filter cake - the fibre mat - and the building grammage on the wire upstream and over the suction box.

2. Theoretical Part

2.1 Kinetics of the fibre mat (or filter cake)

The sheet forming process in the paper industry was precisely detailed by Meyer (1971). If a filtration process on a suction box is assumed with \( C \): the solid consistency of the pulp suspension, \( U \): the local drainage velocity of the filtrate, \( w \) the grammage, \( \eta \) the mass solid retention on the wire, then the kinetics of the solid deposition is given by:

\[
\frac{dw(t)}{dt} = \eta.C.U(t)
\]

(1)

If one assumes that the mass of solids on the wire is totally retained, taking into account a possible solid mass \( w_0 \), upstream of a suction box and the cumulative volume of filtrate \( V \) per unit area of wire, then equation (1) can be integrated and rewritten as:

\[
w(t) = w_0 + C.V(t)
\]

(2)

Hence, the solid mass deposition \( (w(t) - w_0) \) on the suction box can be quantified if the cumulative volume drained \( V(t) \) is known from measurements on line.

2.2 Introducing the Darcy’s law

Two porous media – the wire and the building filter cake - are submitted to a constant differential pressure, the suction pressure \( \Delta P \) applied by the suction box. The volume \( V \) filtrates according to a Darcy’s law and can be expressed (3) as given by Tien (2006). If the filtration resistance of the wire \( R_m \) is small compared to that of the filter cake where \( \alpha_{av} \) [m.kg\(^{-1}\)] is the average specific filtration resistance of the fibre mat and \( \mu \) [Pa.s] is the dynamic viscosity of the filtrate (that of the water), it comes:

\[
U(t) = \frac{\Delta P}{\mu \alpha_{av}.w(t)} \approx \frac{\Delta P}{\mu \alpha_{av}.w(t)}
\]

(3)

If equations (1) and (3) are combined with the hypothesis of a fully retained solid on the wire, then the kinetics of the mass deposition of the solid can be rewritten as follows:

\[
\frac{dw(t)}{dt} = C.\frac{\Delta P}{\mu \alpha_{av}.w(t)}
\]

(4)

After integration of equation (4) over the suction box, between time \( t_0 \) and time \( t \), and considering equation (2), a law well known in the filtration science is obtained:

\[
\frac{t - t_0}{V(t)} = \frac{w_0.\alpha_{av}}{\Delta P} + \frac{C.\alpha_{av}.w(t)}{2.\Delta P}
\]

(5)
If the first term is graphically shown versus the cumulative volume drained $V$, from the intercept, the initial mass $w_0$ already deposited is determined, then, from the slope, the average filtration resistance of the filter cake $\alpha_{av}$ is calculated.

3. Materials and Methods

3.1 Experimental trials
The data are taken from an industrial paper machine: a single wire type also named as a Fourdrinier papermachine. The wire speed is $V_m = 10 \text{ m.s}^{-1}$. The suction box analyzed is located in the beginning part of the forming unit, close to the headbox. The headbox is the hydraulic injector where the pulp suspension of 10-25 mm height is projected and delivered on the wire through its entire width 1-10 m.

The suction box is an assisted vacuum device used to separate fibres and water in the pulp suspension. The solid consistency of the pulp to be drained on the wire is $C = 10 \text{ kg.m}^{-3}$. In our investigations, 2 vacuum pressures are applied at 5 and 7 kPa. In the normal running conditions, the vacuum is applied with a maximum of 6 active slices of common width: $d = 6.9 \times 10^{-2} m$. The time duration ($t_k - t_0$) is chosen by letting active $k$ slices and by closing the other ($6 - k$) slices when $k$ is varying from 1 to 6.

3.2 Extension of the classical law in filtration science
The time duration and the cumulative flow drained per unit width of the wire over the $k$ active slices of the suction box can be expressed with discrete variables as follows:

$$t_k - t_0 = \frac{k.d}{V_m} \quad \text{and} \quad V(t_k) = \frac{q_{lk}}{V_m}$$

Replacing these variables in equation (5) leads to an extension of the continuous classical law $(t/V)$ versus $V$:

$$\frac{k.d}{q_{lk}} = \frac{w_0.\mu.\alpha_{av}}{\Delta P} + \frac{C.\mu.\alpha_{av}}{2.\Delta PV_m} q_{lk}$$

By dividing the two terms of equation (7) by $d$, the common slice width, another expression is obtained:

$$\frac{k}{q_{lk}} = \frac{w_0.\mu.\alpha_{av}}{d.\Delta P} + \frac{C.\mu.\alpha_{av}}{2.d.\Delta PV_m} q_{lk}$$

The equation (8) will be considered in the next paragraph for investigating the drainage flow and the fibre mass deposition on the wire over a suction box.

4. Results and Analysis
Two vacuum pressure and six active slices were investigated in our experimental trials on the industrial papermachine concerned. The results are given in the following table 1. In this table, the volumetric drained flow $q_{lk}$ per unit width of the wire is measured for an increasing number of active slices, from 1 to 6 active slices, where the constant suction pressure $\Delta P$ is applied.

4.1 Experimental results
A constant suction pressure at $\Delta P = 5 \text{ kPa}$ applies in table 1 for line 1 to 3.
In Table 1, lines 4 to 6 apply to a constant suction pressure at \( \Delta P = 7 \) kPa.

**Table 1: Measured flow drained through \( k \) active slices of a suction box at a constant suction pressure at 5 kPa (lines 1 to 3) and at 7 kPa (lines 4 to 6).**

<table>
<thead>
<tr>
<th>( k )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^3.q_{lk}[m^2.s^{-1}] )</td>
<td>0.903</td>
<td>1.78</td>
<td>2.62</td>
<td>3.43</td>
<td>4.23</td>
<td>5.00</td>
</tr>
<tr>
<td>( k/q_{lk} ) [s.m^{-2}]</td>
<td>1107</td>
<td>1121</td>
<td>1146</td>
<td>1165</td>
<td>1181</td>
<td>1200</td>
</tr>
<tr>
<td>( k )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>( 10^3.q_{lk}[m^2.s^{-1}] )</td>
<td>0.965</td>
<td>1.90</td>
<td>2.78</td>
<td>3.65</td>
<td>4.50</td>
<td>5.32</td>
</tr>
<tr>
<td>( k/q_{lk} ) [s.m^{-2}]</td>
<td>1036</td>
<td>1053</td>
<td>1078</td>
<td>1096</td>
<td>1111</td>
<td>1129</td>
</tr>
</tbody>
</table>

As the number \( k \) of active slices is increasing, the corresponding drained flow per unit width of the wire also increases.

### 4.2 Validation of equation (8) and consequences

From the data obtained at the suction pressure of \( \Delta P = 5 \) kPa, the following numerical correlation is found using the international units given in Table 1:

\[
\frac{k}{q_{lk}} = 1084 + 23174q_{lk} \tag{9}
\]

A similar correlation is also obtained at a constant suction pressure of \( \Delta P = 7 \) kPa.

\[
\frac{k}{q_{lk}} = 1015 + 21582q_{lk} \tag{10}
\]

This means that the proposed analysis is validated for the two constant suction pressures investigated since an equation similar to (8) is found. By identification of equation (8) and numerical correlations (9) or (10), the slope and the intercept can be determined as follows:

\[
\begin{align*}
\text{intercept} &= \frac{w_0-\mu.\alpha_{av}}{d.\Delta P} \\
\text{slope} &= \frac{C.\mu.\alpha_{av}}{2.d.\Delta PV_m}
\end{align*} \tag{11}
\]

From equation (11), two unknown variables can be quantified: the initial mass \( w_0 \) of solid already deposited upstream of the suction box and the average specific filtration resistance \( \alpha_{av} \) of the filter cake (fibre mat) for the two suction pressures investigated.

\[
\begin{align*}
w_0 &= \frac{C}{2V_m} \cdot \frac{\text{intercept}}{\text{slope}} \\
\alpha_{av} &= \frac{\text{slope}}{2d.\Delta PV_m} \cdot \frac{C.\mu}{\text{intercept}} \tag{12}
\end{align*}
\]

These two variables allow quantifying the mass of the solid deposited over the suction box and the cumulative volume of the drained flow in case of 6 active slices, as shown in Table 2. If the calculated data of Table 2 are analyzed, during the experimental trials, the running conditions on the papermachine are unchanged; this can be proven by the same numerical value obtained for \( w_0 \), taken at 23.5 g.m\(^2\), in the following analysis.
Table 2: Data calculated on a suction box with two vacuum pressures investigated: the mass solid deposition and the drained flow per unit width of the wire.

<table>
<thead>
<tr>
<th>( \Delta P [\text{kPa}] )</th>
<th>( w_0 [\text{g.m}^{-2}] )</th>
<th>( w_6 [\text{g.m}^{-2}] )</th>
<th>( (t_6 - t_0) [\text{ms}] )</th>
<th>( V(t_6) [\text{mm}] )</th>
<th>( q_6 [\text{m}^2.\text{s}^{-1}] )</th>
<th>( \alpha_{av} [\text{m.kg}^{-1}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>23.4</td>
<td>28.4</td>
<td>41.4</td>
<td>0.50</td>
<td>5.00 ( 10^{-3} )</td>
<td>1.60 ( 10^{10} )</td>
</tr>
<tr>
<td>7</td>
<td>23.5</td>
<td>28.8</td>
<td>41.4</td>
<td>0.53</td>
<td>5.32 ( 10^{-3} )</td>
<td>2.09 ( 10^{10} )</td>
</tr>
</tbody>
</table>

As the machine speed is constant, when the six slices are active, the only variable studied is the suction pressure. In case of an increasing suction pressure by 40% from 5 to 7 kPa, the average value of the specific filtration resistance of the filter cake increases from 1.60 to 2.09 \( 10^{10} \). This modification is mainly due to the compressibility of the filter cake that is depending on the suction pressure applied. This fact can be quantified if one assumes a classical power law with two parameters \( n \) and \( a \) as:

\[
\alpha_{av}(\Delta P) = a(\Delta P)^n
\]

The exponent \( n \) is calculated according to the data of table 2 after elimination of \( a \):

\[
n = \frac{\ln[a_{av}(\Delta P')/a_{av}(\Delta P)]}{\ln[\Delta P'/\Delta P]} = \frac{\ln[2.09/1.60]}{\ln[7/5]} = 0.794
\]

What can be learnt from this previous knowledge? For a given wire speed and a given time application with 6 active slices, when the suction pressure (driven force) is increased by 40% from 5 to 7 kPa, the average specific filtration resistance is increased by nearly 30%, that means that the cumulative flow drained will result of both influences driven and resistance forces. The result is an increase of the cumulative flow drained by only 6%. As the flow drained can be interpreted through equation (2) as mass solid deposition, it means that the increase in mass solid deposition over the suction box will only be of the order of 6%. When \( k \) slices of the suction box are active, from equation (8) and the power law (13) of compressibility of the filter cake, the cumulative flow drained per unit width of the wire can be rewritten as follows:

\[
q_{lk} = \frac{\Delta P \cdot 2. k. d}{\mu \cdot a_{av}(\Delta P)(w_k + w_l)} = \frac{(\Delta P)^{1-n} \cdot 2. k. d}{\mu \cdot a_{av}(\Delta P)(w_k + w_l)}
\]

This expression can be simplified for predicting the cumulative flow drained per unit width of the wire over a suction box with \( k \) active slices at a suction pressure at \( \Delta P = 7 \) kPa knowing the conditions at \( \Delta P = 5 \) kPa, with equations (14) and (15), it comes:

\[
q_{lk}(\Delta P) \approx q_{lk}(\Delta P') \left( \frac{\Delta P'}{\Delta P} \right)^{0.206} = 1.072 q_{lk}(\Delta P)
\]

Table 3 Comparison between calculated and measured cumulative flow drained at 7 kPa.

<table>
<thead>
<tr>
<th>( k )</th>
<th>1</th>
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<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^3 q_k [\text{m}^2.\text{s}^{-1}] measured</td>
<td>0.965</td>
<td>1.90</td>
<td>2.78</td>
<td>3.65</td>
<td>4.50</td>
<td>5.32</td>
</tr>
<tr>
<td>10^3 q_k [\text{m}^2.\text{s}^{-1}] calculated</td>
<td>0.968</td>
<td>1.91</td>
<td>2.81</td>
<td>3.68</td>
<td>4.53</td>
<td>5.36</td>
</tr>
</tbody>
</table>

Since the mean relative error between measured and calculated values is less than 1%, the analysis performed can be used with confidence. This means that the modelling
proposed together with the assumptions chosen – filtration process, Darcy’s law, and power law for the compressible filter cake – are validated to predict the physical phenomena that occur over a suction box on an industrial papermachine.

Table 4 also gives the prediction of the solid mass deposition over \( k \) active slices at \( \Delta P' = 7 \text{ kPa} \) knowing the conditions at \( \Delta P = 5 \text{ kPa} \) and the following expression deduced from the simultaneous consideration of equations (2), (6) and (16):

\[
(w_k - w_0) = \frac{C}{V_m} q_{lk} (\Delta P') \approx (w_k - w_0) \left( \frac{\Delta P'}{\Delta P} \right)^{1-n}
\]

(17)

Table 4: Comparison between calculated and measured mass of the solid deposited on the wire in case of \( k \) active slices over a suction box.

<table>
<thead>
<tr>
<th>( k )</th>
<th>1</th>
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</tr>
</thead>
<tbody>
<tr>
<td>((w_k - w_0) [\text{g.m}^{-2}]) measured</td>
<td>0.965</td>
<td>1.90</td>
<td>2.78</td>
<td>3.65</td>
<td>4.50</td>
<td>5.32</td>
</tr>
<tr>
<td>((w_k - w_0) [\text{g.m}^{-2}]) calculated</td>
<td>0.965</td>
<td>1.93</td>
<td>2.79</td>
<td>3.64</td>
<td>4.50</td>
<td>5.36</td>
</tr>
</tbody>
</table>

5 Optimization of the suction pressure

In the paper industry, a technical problem is the choice of the adequate suction pressure \( \Delta P \) over a suction box in order to simultaneously drain and insure a given mass solid deposition. When the pulp consistency \( C \), the machine speed \( V_m \) and the initial mass of solid \( w_0 \) are all known, the choice of a solid mass deposition is equivalent to the choice of the required cumulative drained flow per unit width of the wire as it is proven by:

\[
q_{lk} = \frac{V_m}{C} (w_k - w_0)
\]

(18)

The cumulative grammage deposited after the \( k^{th} \) slice \( w_k \) is calculated accordingly.

The optimum suction pressure must fulfill the following expression deduced from (15):

\[
(\Delta P)^{1-n} = \frac{q_{lk} + \alpha a (w_k + w_0)}{2 k d}
\]

(19)

Let illustrate a numerical application with the previous data, if one wants to retain 5.2 \( \text{g.m}^{-2} \) of solid cake on the wire over the suction box with 6 slices, the optimized suction pressure is calculated by equation (19) and the result is: \( \Delta P = 6.3 \text{ kPa} \).

From experimental data taken on the suction box of a papermachine of single wire type, if the time application is changing by closing a variable number of active slices, then different cumulative flow drained are obtained. The extension of the filtration model proposed in this article helps to determine the adequate vacuum for reducing the energy consumption of vacuum assisted devices as suction boxes in the paper industry.

References


Tien C., 2006, Introduction to cake filtration – Analyses, experiments and applications, Elsevier, Amsterdam, the Netherlands.