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Simultaneous Batching and Scheduling in Multi-product Multi-stage Batch Plants through Mixed-Integer Linear Programming

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In this work, a novel sequence-based mixed-integer linear programming formulation for the simultaneous batching and scheduling in multi-product multi-stage batch plants is developed. The selection of batches, the allocation of batches to processing units and the sequencing of batches in each unit constitute the discrete decisions of our model. Batch processing times and sizes are variables. Batch size increment steps are included in an attempt to accommodate our model to real-life industrial practice.

1. Introduction

Multi-stage batch processes are commonly used for the production of high-value, low-volume products such as specialty chemicals and pharmaceuticals. In the last decade, a plethora of Mixed-Integer Programming (MIP) frameworks have been proposed for the scheduling of multi-product multi-stage batch plants in the Process Systems Engineering literature. Roughly speaking, multi-stage batch scheduling formulations can be classified into slot-based and sequence-based. In slot-based formulations, the scheduling horizon is divided into a set of different time slots for every equipment unit, wherein each batch is assigned to one time slot. In sequence-based formulations, sequencing binary variables are used, via big-M constraints, in order to identify the sequence of the batches. The advantage of this type of formulations is that sequence-dependent setup times can be modelled in a straightforward manner through the use of the sequencing binary variables. However, the higher the number of total batches, the bigger the size of the mathematical model. A comprehensive and neatly written review regarding the optimization methods for the short-term scheduling problem of batch processes can be found in Méndez et al. (2006).

Note that the main assumption in most of the previously proposed formulations is that the number of batches is predefined. In other words, the number of batches is fixed and it is not a decision variable. However, this assumption can be considered valid only in the case of fixed demand and identical (equal-capacity) parallel units. In industrial practice, this assumption is rarely met, since parallel units are often dissimilar in capacity in order to cover orders of different sizes or because some units have been installed later as part of a capacity expansion or retrofit project. Thus, a given order of a

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certain size (quantity) cannot necessarily be split optimally into different batch sizes outside the scheduling model, and the number and sizing of batches towards a product order must become part of the optimization in order to obtain true optimal solutions (Prasad and Maravelias, 2008).

At this point, we should emphasize that many of the state-of-the-art sequence-based formulations indirectly assume fixed processing times, for instance see Prasad and Maravelias (2008). In this type of approaches, the batch size is a variable however the batch processing time is a parameter. In industrial practice, this assumption is seldom met, since the processing time tends to depend on the batch quantity produced. Another important point to mention is that current MIP models treat the batch size as a continuous variable indirectly assuming insignificant increment runs. Nevertheless, industrial batch processing units do operate in specific batch-size increment steps. In production processes that batch-size increment steps are very small, the assumption of a batch size continuous variable can be valid. Finally, lag times (positive or negative) between consecutive stages usually exist and should be also appropriately modelled and included to the optimization procedure in order to avoid the generation of infeasible and/or suboptimal solutions.

2. Problem Statement

This study considers the batching and scheduling problem of multi-stage multi-product batch processes with the following features:

- A set of product orders i should be processed in a number of batches b by following a predefined sequence of processing stages s with, in general, unrelated processing units j working in parallel.
- Each product order i must follow a set of processing stages $s \in S_i$.
- Product order i can be processed in a specific subset of units $j \in J_i$. Similarly, processing stage s can be processed in a specific subset of units $j \in J_s$.
- Transition times between consecutive batches are expressed as the sum of two terms. One depends on both the unit and the product order being processed (π_{ij}) while the other also varies with the product order previously processed in that unit ($\gamma_{ii'j}$). Additionally, transferring lag times (λ_{is-1s}) may exist between consecutive processing stages.
- Model parameters like minimum and maximum product order demands (ζ_i^{min} and ζ_i^{max}), minimum and maximum unit capacities (β_i^{min} and β_i^{max}), batch size increment steps (α_{ij}), and processing rates (ρ_{ij}) are all deterministic.

The key decision variables are:

- The allocation of batches b of products i to units $j \in J_i$, Y_{ibj} ;
- The batch size and processing time of batches b of products i in units $j \in J_i$, Q_{ibj} and T_{ibj}, respectively;
- The sequence for any pair of batches in each unit, $X_{ibi'b'j}$;
- The completion time of batch b of product i at stage $s \in S_i$, C_{ibs}.

Alternative objective functions can be considered, such as the minimization of makespan, and the maximization of profit.

3. Mathematical Formulation

In this section, the proposed MIP scheduling model is stated and described in detail. Constraints have been grouped according to the type of decision (e.g., assignment, timing, and sequencing) they are imposed on.

3.1 Product-batch assignment to unit

Every product order batch goes through at most one unit $j \in (J_i \cap J_s)$ at each stage $s \in S_i$:

$$\sum_{j \in (J_i \cap J_s)} Y_{ibj} \leq 1 \quad \forall i, b \leq b_i^{max}, s \in S_i \tag{1}$$

3.2 Forbidden processing paths

In industrial applications, usually there does not exist connection between all processing units. These processing routes are called forbidden processing paths (J_j^{forb}) and should be appropriately incorporated into the optimization in order to avoid infeasible solution.

$$Y_{ibj} + Y_{ibj'} \leq 1 \quad \forall i, b \leq b_i^{max}, j, j' \in J_j^{forb}: j < j' \tag{2}$$

3.3 Product demand

The total production quantity of any product order *i* should be within the lower and the upper bound of its corresponding demand:

$$\zeta_i^{min} \leq \sum_{b \leq b_i^{max}} P_{ib} \leq \zeta_i^{max} \quad \forall i, b \leq b_i^{max}, s \in S_i \tag{3}$$

3.4 Batch sizing

Obviously, the batch size of a batch *b* of product order *i* should remain the same throughout all the processing stages:

$$\sum_{j \in (J_i \cap J_s)} Q_{ibj} = P_{ib} \quad \forall i, b \leq b_i^{max}, s \in S_i \tag{4}$$

Lower and upper bounds for the batch size of batch *b* of product order *i*, considering units capacities, are given by:

$$\beta_j^{min} Y_{ibj} \leq Q_{ibj} \leq \beta_j^{max} Y_{ibj} \quad \forall i, b \leq b_i^{max}, j \in J_i \tag{5}$$

3.5 Batch-size increments

The number of batch-size increments depends on the batch size increment step α_{ij} and is estimated by:

$$Z_{ibj} = \frac{Q_{ibj} - \beta_j^{min} Y_{ibj}}{\alpha_{ij}} \quad \forall i, b \leq b_i^{max}, j \in J_i \tag{6}$$

3.6 Batch processing time

The processing time for batch *b* of product order *i* that is assigned to unit $j \in J_i$ should be greater than a minimum batch processing time τ_{ij}^{min}. The additional batch processing time further depends on the batch-size increments steps, as given by:

$$T_{ibj} = \tau_{ij}^{min} Y_{ibj} + \frac{Z_{ibj} \alpha_{ij}}{\rho_{ij}} \quad \forall i, b \leq b_i^{max}, j \in J_i \tag{7}$$

3.7 Symmetry breaking constraints

For enhancing the solution process by eliminating equivalent symmetric solutions, the batch size of a smaller batch index towards a product order is forced to be greater than or equal to the batch size of a larger batch index:

$$\sum_{j \in (J_i \cap J_s)} Q_{ibj} \leq \sum_{j \in (J_i \cap J_s)} Q_{ib-1j} \quad \forall i, 1 < b \leq b_i^{max}, s \in S_i \tag{8}$$

3.8 Timing between consecutive stages

Transferring lag times between consecutive processing stages of every product i are explicitly considered. Therefore, the timing of a batch *b* of product order *i* is given by:

$$C_{ibs} - \sum_{j \in (J_i \cap J_s)} (\pi_{ij} Y_{ibj} + T_{ibj}) \geq C_{ibs-1} + \lambda_{is-1s} \quad \forall i, b \leq b_i^{max}, s \in S_i : s > 1 \tag{9}$$

3.9 Sequencing between batches in a processing unit

Our MIP model uses immediate precedence sequencing binary variables and sequencing constraints (for product-batches) similar to the ones presented by Kopanos et al. (2010).

3.10 Minimization of the makespan

The time point at which all product orders are accomplished corresponds to the makespan, and can be calculated by:

$$\text{makespan} \geq C_{ibs} \quad \forall i, b \leq b_i^{max}, s \in S_i^{last} \tag{10}$$

The makespan objective is closely related to the throughput objective. For instance, minimizing the makespan in a parallel-machine environment with sequence-dependent setup times forces the scheduler to balance the load over the various machines and to minimize the sum of all the setup times in the critical bottleneck path.

3.11 Maximization of profit

In the literature, demand is typically assumed fixed in most of the scheduling problems in multi-stage multi-product batch processes. Note that this condition implies that the objective function should either be time-related (e.g. minimization of makespan, earliness, etc.) for fixed product orders or minimization of production cost for orders with due dates; but it cannot be maximization of production or profit. Therefore, it is no surprise that most of earlier scheduling methods for multi-stage processes consider these two types of objectives only. In this study, the profit is equal to the total revenue minus total production costs. Total production costs consist of: (i) fixed costs, (ii) utilities costs (variable costs), and (iii) sequence-dependent changeovers costs.

$$\begin{aligned} \text{profit} = & \sum_i \sum_{b_i \leq b_i^{max}} \theta_i P_{ib} - \sum_i \sum_{b_i \leq b_i^{max}} \sum_{j \in J_i} \psi_{ij} Y_{ibj} - \sum_i \sum_{b_i \leq b_i^{max}} \sum_{j \in J_i} \xi_{ij} Q_{ibj} \\ & - \sum_i \sum_{b_i \leq b_i^{max}} \sum_{i' \neq i} \sum_{b_{i'} \leq b_{i'}^{max}} \sum_{j \in (J_i \cap J_{i'})} \varphi_{ii'j} X_{ibi'b'j} \end{aligned} \tag{11}$$

Parameter θ_i denotes the selling price of product order *i*, ψ_{ij} stands for the fixed production cost, ξ_{ij} corresponds to the utilities cost, and $\varphi_{ii'j}$ reflects the sequence-dependent changeover cost.

3.12 Remarks

Additional tightening constraints can be included to the mathematical formulation in order to make it more computationally efficient. For instance, in the case that makespan constitutes the optimization goal, lower bounds for the makespan can be calculated by:

$$\text{makespan} \geq \sum_{s'<S_i^{last}} \min_{j' \in J_{s'}} [\tau_{ij}^{min}] + \sum_i \sum_{b_i \leq b_i^{max}} (\delta_{ij} Y_{ibj} + T_{ibj}) + \sum_i \sum_{b_i \leq b_i^{max}} \sum_{i' \neq i} \sum_{b_{i'} \leq b_{i'}^{max}} \gamma_{ii'j} X_{ibi'b'j} \quad \forall j \in J_i, s \in S_i^{last} \tag{12}$$

4. Case studies

Two case studies have been solved in order to highlight the practical benefits of our mathematical formulation. All examples have been solved in a Dell Inspiron 1520 2.0 GHz with 2GB RAM using CPLEX 11 via a GAMS 22.8 interface (Brooke et al., 1998), under standard configurations.

Case Study I addresses the batching and scheduling problem of five product orders (A-E), which are produced in two stages. The optimal production schedule (Figure 1) results into 16.6 h of makespan and it was reached in 93 CPU s. It is worthy mentioning that a makespan of 37.25 hours is obtained if batching decisions are not optimized (i.e., $b_i^{max}=1$). Case Study II deals with the batching and scheduling problem of four two-stage product orders. The optimal production schedule results into a profit equal to 930.5 m.u. and it was reached in 124 CPU s.

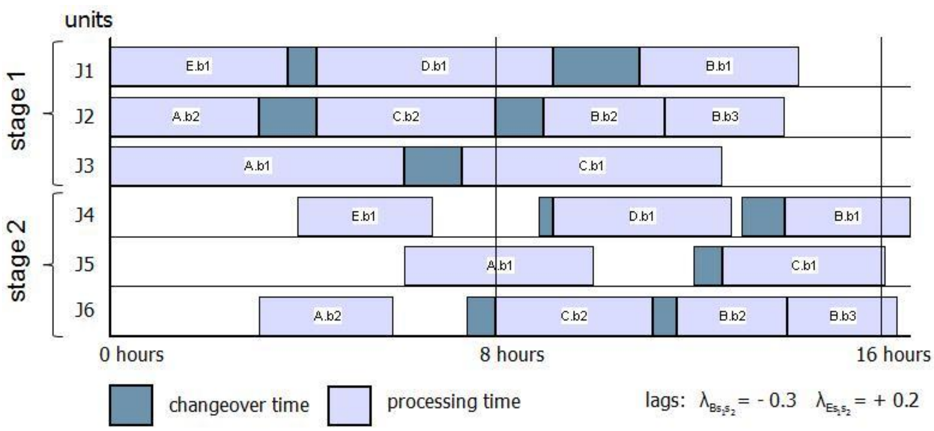

Figure 1: Case Study I: Optimal production schedule (makespan = 16.6 h).

5. Conclusions

In this work, a new batch-oriented MIP framework for the simultaneous batching and scheduling in multi-product multi-stage batch plants has been presented. Batch sizes and processing times are variables. A salient feature of the proposed MIP model is that it considers batch size increments in an attempt to simulate better real-life production processes. As a final point, simultaneous batching and scheduling problems are highly computationally demanding problems thus the development of elaborate solution methods for solving large-scale industrial problems reveals as a very promising and challenging future research task.

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