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Contribution to Analytical Calculation Methods for Prediction of Uniform Fluid Flow Dividing in Tubular Distributor

Vojtech Turek*, Josef Kohoutek, Zdenek Jegla, Petr Stehlik
Brno University of Technology, Faculty of Mechanical Engineering, Institute of Process and Environmental Engineering, Technicka 2, 616 69 Brno, Czech Republic
turek@upei.fme.vutbr.cz

Uniform flow distribution plays a very important role in many different cases such as heat exchangers, burners, piping systems, etc. This paper describes one of the available analytical methods for distributor design. Moreover, it proposes two derived design procedures for optimum design of a specific non-cylindrical distributor (incompressible and compressible flow are considered).

1. Introduction

Although the principles of uniform fluid distribution have been known for a long time, they are very often neglected by designers. Therefore, important parts of process equipment unnecessarily suffer from flow maldistribution.
Fluid discharge into branches of a manifold is accompanied by static pressure variance owing to wall friction and to change of fluid momentum. In a dividing manifold, friction makes the pressure fall while sudden changes in direction of portions of the stream make the pressure rise. Conversely, the direction changes cause the pressure to fall in a combining one. As a result, it is not possible to keep static pressure inside a distributor perfectly constant and thus discharge flow-rates vary even for identical ports.

2. Successive Branch-by-Branch Approach

Scheme of a distributor is in the figure 1. We assume constant cross-section along its length, and uniform, one-dimensional, isothermal and incompressible flow.
Fluid flows as indicated by the velocities. Let us now consider one branch and one subsequent section of the distributor. A portion of fluid is discharged through the branch due to surplus static pressure in the distributor. Velocity of the remaining fluid inevitably decreases and thus its momentum changes. This consequently causes increase in pressure in the downstream direction. Although it may seem so, the discharging fluid does not generally lose all its original (axial) velocity. The discharge angle θ is then greater than zero and thus we need to introduce the coefficient of static regain C_r. Bailey (1975) defined it as the ratio of the difference in static pressure between the flow upstream and downstream of the branch to the difference in dynamic pressure.

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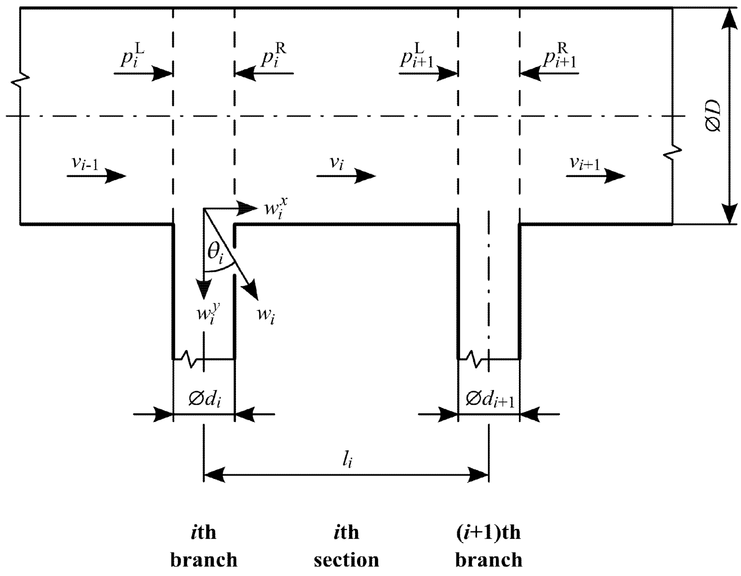

Fig. 1. Scheme of a distributor.

Considering the continuity equation, the amount of fluid discharging through the branch must correspond to the change in volumetric flow-rate between the section upstream and the section downstream of the branch. However, we need to introduce one more correction factor – the discharge coefficient C_d. The joint of the distributor and the branch is usually made in such a way that it is impossible for the streamlines to suddenly bend along the edge of the orifice. Therefore the stream is contracted due to axial momentum of fluid particles.

Fluid flowing through the section loses static pressure solely because of friction. Since we assume the cross-sectional area of each branch to be small compared to both the internal areas of the adjacent sections, the Darcy-Weisbach equation may be used without any additional corrections.

2.1 Governing Equations

We already know that the actual volumetric flow-rates through the branches are given by the variation of static pressure along the distributor. Hence, finding equations governing the static pressure means we can predict the discharge flow-rates.

Bailey (1975) assumed equidistant branches with identical cross-sections and, moreover, he neglected the effect of the gravitational field. He suggested the following three equations to be used, provided the necessary coefficients are known:

$$p_{i+1}^L - p_i^R = -f_i \frac{l}{D} \rho \frac{v_i^2}{2}, \tag{1}$$

$$p_{i+1}^R - p_{i+1}^L = \frac{C_{r,i+1}}{2} \rho \left(v_i^2 - v_{i+1}^2 \right), \tag{2}$$

$$D^2 \left(v_i - v_{i+1} \right) = d_{i+1}^2 C_{d,i+1} \sqrt{\frac{p_{i+1}^L + p_{i+1}^R}{\rho}}. \tag{3}$$

The discharge flow-rate is then given by the equation

$$Q_i^B = \frac{\pi d_i^2}{4} C_{d,i} \sqrt{\frac{p_i^L + p_i^R}{\rho}}. \tag{4}$$

Darcy friction factor for laminar flow depends only on Reynolds number:

$$f = 64/\mathrm{Re}, \tag{5}$$

whereas in case of turbulent flow it is a function of Reynolds number, absolute roughness of distributor surface and its hydraulic diameter. Then there are several ways of obtaining the value of f. We can either solve the (implicit) Colebrook-White equation

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \tag{6}$$

or use one of its explicit approximations (Churchill equation, Serghides equation, etc.). Considering the coefficients C_r and C_d, Bailey (1975) performed a series of experiments and found the following equations:

$$C_{r,i} = 0.780 + \phi_i \log_{10}\left(\frac{v_{i-1}}{v_{i-1} - v_i}\right), \quad \text{where} \quad \phi_i = 0.284 + 0.098\log_{10}\left(\frac{d_i}{D}\right), \quad \text{and} \tag{7}$$

$$C_{d,i} = 0.620 + 0.070\beta_i - 0.088\beta_i^2, \quad \text{where} \quad \beta_i = \log_{10}\left[\log_{10}\left(1 + \frac{p_i^L + p_i^R}{v_{i-1}^2}\right)\right]. \tag{8}$$

Acrivos, Babcock and Pigford (1959) did not neglect the effect of gravitational field and also derived the equations in dimensionless variables.
Although the successive branch-by-branch approach is not the only analytical one available, it can easily be used and algorithmized. There is also the differential approach which may in some cases be more suitable, but it requires solving differential equations (*cf.* Acrivos, Babcock and Pigford (1959) or Bajura R.A. and Jones E.H., 1976, Flow Distribution Manifolds, Journal of Fluids Engineering, vol. 98, 654-665).

2.2 Comparison with Experimental Data

Bailey (1975) and also Acrivos, Babcock and Pigford (1959) described experiments that had been performed to confirm theoretical results. In both cases the observed discharge flow-rates conformed to the flow-rates that were predicted.

3. Analysis of an Existing Dividing Flow Distributor

Analysis was carried out to find whether any significant improvement of performance of an existing distributor can be achieved without an excessive increase of production costs. The distributor is a part of a special U-tube heat exchanger and thus we require as uniform discharge flow-rates as possible. Air or water is used as a working fluid depending on the actual application of the heat exchanger. The exchanger itself is a component of a sludge incinerator and serves for both combustion air preheating and final flue gas cooling prior to a cleaning process.

3.1 Original Shape of the Distributor

Cross-section of the distributor is rectangular with constant width and height. A pair of tubes placed side by side forms a branch. Diameters of the tubes in equidistant branches are identical. The distributor is positioned horizontally and thus we do not have to consider the effect of the gravitational field.

3.2 Design Algorithms

Although an algorithm approximating compressible flow of air by an incompressible flow might be used, two separate algorithms were designed. Both algorithms make use of the successive branch-by-branch approach. To minimize the increase of production costs, only the linear decrease of height along the distributor was considered (however, discharge flow-rates can be calculated for any piecewise linear profile defined by heights in the middle of sections). Hence, also the change of pressure caused by the changing height has to be taken into account. Pressure drop due to hydraulic resistance of gradually contracting flow channel is neglected, since it is insignificant compared to other pressure changes. Mean velocities are no longer constant throughout sections. Moreover, values just upstream and just downstream of a branch are computed using the same flow channel height, i.e., the diameter of the branch is considered to be negligible. As an indicator of suitability of a certain distributor, non-uniformity of discharge flow-rates is employed (mass flow-rates are used instead of Q_i^B for compressible flow):

$$\delta = 1 - \min_i \{Q_i^B\} / \max_i \{Q_i^B\}. \tag{9}$$

In the algorithm assuming incompressibility, equation (2) can still be used with

$$C_{r,i} = 0{,}780 + \left[0{,}284 + 0{,}098 \log_{10}\left(\sqrt{\frac{\pi d_i^2}{2bh_i}}\right)\right] \log_{10}\left(\frac{v_i^L}{v_i^L - v_i^R}\right), \tag{10}$$

but equations (1) and (3) have to be modified to

$$p_{i+1}^L - p_i^R = \underbrace{-f_i \frac{l_i\left(b + h_i^M\right)}{2bh_i^M} \rho \frac{v_i^2}{2}}_{\text{friction}} + \underbrace{\frac{\rho}{2} v_i^2 \left(h_i^M\right)^2 \left(\frac{1}{h_i^2} - \frac{1}{h_{i+1}^2}\right)}_{\text{change of height}}, \tag{11}$$

$$bh_{i+1}\left(v_{i+1}^L - v_{i+1}^R\right) = \frac{\pi d_{i+1}^2}{2} C_{d,i+1} \sqrt{\frac{p_{i+1}^L + p_{i+1}^R}{\rho}}, \tag{12}$$

where v_i is the mean velocity in the middle of the ith section and

$$h_i = \left(h_{i-1}^M l_i + h_i^M l_{i-1}\right) / \left(l_{i-1} + l_i\right). \tag{13}$$

It is clear that velocities just upstream and just downstream of a branch have to be used when computing coefficients C_r and C_d (*cf.* equations (7) and (8)). To calculate the mean fluid velocities in the branches, equal static pressures are assumed at orifices:

$$p_i^B = \frac{p_i^L + p_i^R}{2} - \frac{\rho w_i^2}{2C_{d,i}^2} = p^B = \text{const.} \quad \forall i. \tag{14}$$

The second algorithm must regard the compressibility and thus it makes use of the momentum balance for compressible flow. Let us denote the density and the mean fluid velocity just upstream of a branch as ρ^L and v^L, and analogously just downstream of a branch as ρ^R and v^R. Let us also assume that we know all three quantities – pressure, density and mean velocity – just upstream of the current branch. To get the mean velocity just downstream of it, we need to solve the following implicit equation:

$$\frac{p_i^L}{\rho_i^L}\left[\left(\frac{Q_{m,i}v_i^L}{Q_{m,i-1}v_i^R}\right)^{\gamma-1}-1\right]= \\ =\underbrace{\left[0{,}780+\left(0{,}284+0{,}098\log_{10}\left(\sqrt{\frac{\pi d_i^2}{2bh_i}}\right)\right)\log_{10}\left(\frac{v_i^L}{v_i^L-v_i^R}\right)\right]}_{C_{r,i}}\frac{\gamma-1}{2\gamma}\left[\left(v_i^L\right)^2-\left(v_i^R\right)^2\right] \quad (15)$$

Any suitable numerical method can be used to find the solution, for instance the basic Newton's one. Density and pressure are then computed using these equations:

$$\rho_i^R=\rho_i^L\frac{Q_{m,i}v_i^L}{Q_{m,i-1}v_i^R}, \quad (16)$$

$$p_i^R=\rho_i^R\left[C_{r,i}\frac{\gamma-1}{2\gamma}\left[\left(v_i^L\right)^2-\left(v_i^R\right)^2\right]+\frac{p_i^L}{\rho_i^L}\right]. \quad (17)$$

Mean velocity just upstream of the subsequent branch is the solution of the equation

$$\frac{\left(v_{i+1}^L\right)^2}{2}-\frac{\left(v_i^R\right)^2}{2}=\frac{\gamma}{\gamma-1}\frac{p_i^R}{\rho_i^R}\left[1-\left(\frac{h_iv_i^R}{h_{i+1}v_{i+1}^L}\right)^{\gamma-1}\right]. \quad (18)$$

As before, we can use the Newton's method and then calculate density and pressure:

$$\rho_{i+1}^L=p_{i+1}^L\left[\frac{\gamma-1}{2\gamma}\left[\left(v_i^R\right)^2-\left(v_{i+1}^L\right)^2\right]+\frac{p_i^R}{\rho_i^R}\right]^{-1}, \quad (19)$$

$$p_{i+1}^L=\rho_i^R\frac{\gamma-1}{\gamma}\frac{h_iv_i^R}{h_{i+1}v_{i+1}^L}\left[\frac{\left(v_i^R\right)^2-\left(v_{i+1}^L\right)^2}{2}+\frac{\gamma}{\gamma-1}\frac{p_i^R}{\rho_i^R}\right]-f_i\frac{l_i\left(b+h_i^M\right)}{4bh_i^M}\rho_iv_i^2. \quad (20)$$

Equation (20), however, contains velocity v_i and density ρ_i (i.e., quantities in the middle of the ith section). Velocity v_i can be calculated similarly as above, but density ρ_i can only be estimated (pressure loss due to friction cannot be evaluated exactly and thus is neglected). Darcy friction factor f_i is also computed in the middle of the ith section. Mean fluid velocities in the branches are again calculated using the assumption of equal static pressures, but we have to replace ρ with $\left(\rho_i^L+\rho_i^R\right)/2$ in equation (14).

3.3 Optimized Shape

According to the above algorithms, non-uniformity of discharge flow-rates for the original shape is 12.2 % for water (incompressibility is assumed) and 12.9 % for air (compressibility is assumed). The optimized shape meeting all the restrictions we have set is the one with height starting from the original value and decreasing to the lowest feasible value possible at the end of the distributor. Now, non-uniformities are 7.7 % and 8.5 %, respectively. If we widened the distributor to 1.25 of its original width, non-uniformities would be even smaller: 5.5 % and 6.0 %, respectively. Additional improvement can be achieved by increasing the inlet height.

3.4 Future Work

Experimental verification of results obtained during the analysis of the existing distributor has to be done. Moreover, neither of the two algorithms that have been designed completely reflects the reality due to the actual heat exchanger construction (distributor – U-tubes – collector). Discharge flow-rates in the branches of the distributor are influenced by pressure fluctuations in the collector and thus both algorithms need to be modified to take the actual parallel flow into account.

4. Conclusion

The analytical successive branch-by-branch approach for discharge flow-rates computation was described. This approach assuming incompressible flow has been generalized to rectangular manifolds with variable cross-sections of both the main flow channel and the branches. Also, another algorithm for prediction of discharge flow-rates for compressible flow has been presented.

Notation

b Width of rectangular distributor
C_d Discharge coefficient
C_r Coefficient of static regain
d Diameter of branch
D Diameter of distributor
f Darcy friction factor
h Height of rectangular distributor at branch axis
h^M Height of rectangular distributor in the middle of section
l Length of section
p^B Static pressure at branch orifice
p^L Static pressure upstream of branch
p^R Static pressure downstream of branch
Q Volumetric flow-rate through section
Q^B Volumetric flow-rate through branch
Q_m Mass flow-rate through section
Q_m^B Mass flow-rate through branch
v Mean fluid velocity in section
w Mean fluid velocity in branch
w^x Mean fluid velocity in branch in the direction parallel to distributor axis
w^y Mean fluid velocity in branch in the direction perpendicular to distributor axis
γ Heat capacity ratio
δ Non-uniformity of discharge flow-rates
ε Absolute roughness of distributor surface
θ Discharge angle
ρ Fluid density in section
ρ^L Fluid density upstream of branch
ρ^R Fluid density downstream of branch
Subscript i denotes ith section or branch.

References

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