

# Optimal energy system design under uncertain parameters

Matthias Dubuis, François Maréchal<sup>1</sup>

Industrial Energy Systems Laboratory (LENI)

Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland

The aim of this paper is to discuss the integration of uncertainty analysis in a thermo-economic optimization method for process system design. Most of time energy systems are designed under constant parameters, whereas some of them are uncertain in real cases. The uncertainty may affect the design decisions, the objectives, although these may be compensated by control variables in some circumstances.

*Keywords:* energy system, uncertainty, design

## 1 Introduction

The concepts exposed in this paragraph consider specifically the problem of process design under uncertainty. The formalism has been mainly developed for mathematical programming formulation in [5], [6], [7] and [4].

As expressed in fig. 1, the main issue in energy system design is to determine the technologies allowing to provide one or several defined services from available resources. To achieve this, engineers have to deal with the context (cost, constraint,...) and the equipment (availability, efficiency,...) in order to take the most appropriate solution, which is characterized by its investment and operating condition.

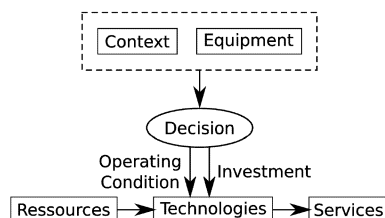


Figure 1: Goals and issues

<sup>1</sup>Corresponding author: francois.marechal@epfl.ch, phone: +41 21693 3516, fax: +41 21693 3502

Technologies are modeled as function of several variables. In a classical optimization problem formulation, they can be classified in several type:

- Decision variables  $X = \{Dv, Dd, O\}$ :
  - Design variables  $Dv = [dv_1, \dots, dv_{n_v}]$
  - Design decision  $Dd = [dd_1, \dots, dd_{n_d}]$
  - Operating variables  $O = [o_1, \dots, o_{n_o}]$
- Parameters, with some of them uncertain  $u = [u_1, \dots, u_{n_u}]$  with  $n_u$  the number of uncertain parameters

However, some elements of the context and the equipment are not clearly defined, due to lack of data. They are all the more uncertain since most of the installation have to last several years, predictive value being not accurate.

Then uncertainties on parameters will be considered in that paper. They might be due to several factors like measurement's imprecision, approximation of the model,... One way do deal with several values for the same variables is to carry out data reconciliation what allows to work with one value. Then the process can be designed taking only this value into account, what would be a loss of information. Another way to model the uncertainties is a probability density function with the assumption that the reconciliated value is its mean. The question become then how to include it in an optimization process.

## 2 Method

### 2.1 Model

The model considered here is a solid oxyd fuel cell (SOFC) coupled with a gas turbine as shown in fig. 2.

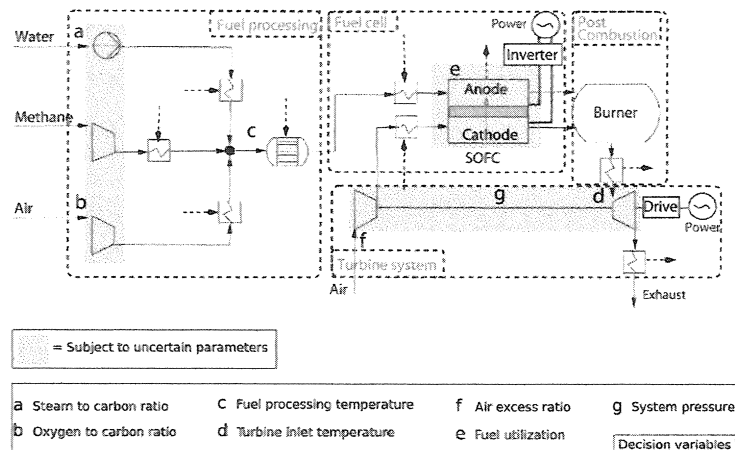


Figure 2: Model scheme

Each uncertain parameter is modeled by a normal law for a priori symmetric distribution and a beta law for others. Indeed, there's no reason why a specific cost should have been estimated higher than lower. On the other hand, a compressor efficiency will decrease if it is not used at its nominal point (whatever if it is at greater or lower capacity). So there is more chance that it will be lower than what is predicted.

## 2.2 Classical multi-objective optimization

The problem is first solved using a conventional approach based on a multi-objective optimization [8] considering the trade off between investment and efficiency. Details on the considered process design si given in [1]. An evolutionary algorithm is applied to solve the optimization problem and the uncertain parameters are fixed at their mean value  $u_i$ .

## 2.3 Monte-Carlo simulation

The first way of assessing the influence of the uncertain parameters is to calculate the performances of the points of the obtained Pareto curve using a Monte-Carlo simulation. This calculates for each set of decision variables in the pareto curve a cloud of performances that defines the influence of uncertainties of a given design. Fig. 3 shows the Pareto curve obtained for the system design. Desired range for objectives function varying uncertain parameter are artificially represented.

Total cost of the system can then be computed by combining investment and operating cost, allowing to deduce an optimal configuration from this analysis.

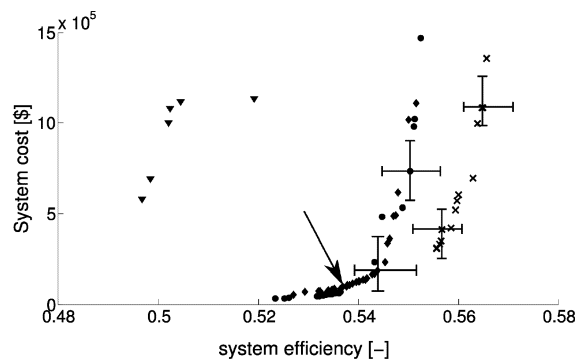


Figure 3: Representation of possible range of each parameters

In fig. 4, a Monte-Carlo with a Hammersley sampling on the uncertain parameters space has been performed at the point designed by an arrow in fig. 3. This shows that following the set of  $u$ , the objective functions can be better/worse than the best/worst solution of the pareto. This underline the strong influence of uncertainties.

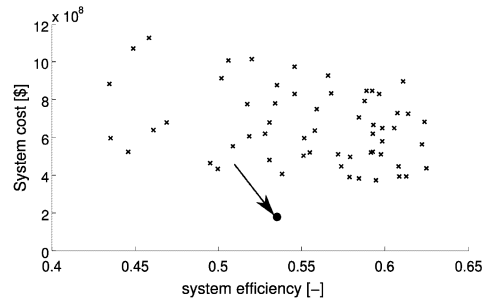


Figure 4: Monte-Carlo with Hammersley sampling around one point

Although the use of multi-objective optimization allows for handling uncertainties and reach optimal design, this approach has some default:

- This is just a partial way to include uncertainties in design. Indeed, by optimizing a system and then applying uncertainty to the obtained solutions, the variations of parameters are taken into account only retroactively. In other words, uncertainties are applied to a certain choice of solutions (the points of the pareto curve), but not in the process that generate these points. And therefore it is possible that best solutions have been eliminated from the optimum set.
- The use of evolutionary algorithm to solve the problem is already a time consuming task. Computing the clouds around each point is even worst. In order to reduce the computing time, the calculation of the multi-objective optimization has been parallelized. This is easy when using an evolutionary algorithm since the different points for the evaluation do not have any relation with the others. Instead of applying a pure Monte-Carlo simulation, sampling methods as proposed by Diwekar [3, 2] have been used.
- Complex model are strongly surjective, i.e. for different set of input variables, the model may return the same values of the objectives function. Then the probability to get a set of uncertain parameters is not the same than the one to get the corresponding solution. This is an advantage, because it means that variables can compensate each other, and then that if one uncertain parameter doesn't correspond to its desired value, others can be reevaluated to reach the predicted design.

## 2.4 Random Multi-Objective

Another way of solving the problem is to use a random optimization strategy. In this case, uncertain parameters are selected randomly and used as input in the black box model. Then, objectives functions are calculated and sent to the optimizer using evolutionary algorithm. Fig. 5 shows the comparison between a classical MOO as described in chapter 2.2 and the one with random parameters.

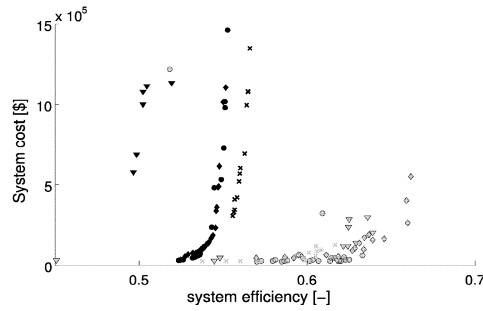


Figure 5: Comparison of "classical" (in black) and random (in grey) MOO

Two pareto curves are compared. The reference one is the one obtained by conventional optimization, the second one is the one calculated by the optimizer with random uncertain parameters values.

It appears that the "random" optimization gives better results (best point : 66%,  $5.5 \cdot 10^5$ €) than the normal one (best point : 57.5%,  $3.6 \cdot 10^7$ €), however both have been carried out with 10000 iterations at all. It is then clear that the "random" optimization has a less defined pareto curves dues to random nature of the parameters and would probably require more iteration before convergence. It shows the sensitivity of the model to these parameters, what explain the difference in the two curves of fig. 5

In the process engineering design them major outcome is the specification of the investment in terms of equipments and sizes. Fig. 6 shows decision variables differ if we consider the conventional optimization strategy or if we include uncertain parameters. What means that process design differ from one results to the other.

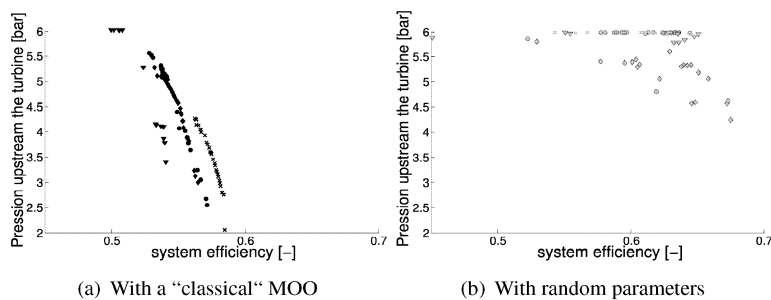


Figure 6: Pression upstream the turbine

The study of other decision variables shows that the operating set points are also different from one approach to the other. Then, the influence of the uncertain parameters could be compensated by adaptating the operating conditions of the real value of the parameter (e.g. fuel cell temperature). This means that multi-period approach will be needed in order to decide the best investment to be made (sufficient lage unit) to allow the

operating set points reaching the optimal performances.

### 3 Conclusion and further work

Since this is an application case, uncertain parameters in an SOFC have been selected and modeled. In real case, the more influential have to be determined and a statistic study has to be carried out in order to describe it the best way possible.

The challenges and issues about Monte-Carlo simulation have been exposed. The limitations of this method is mainly related to computing time and data storage. Moreover this approach allows only to include uncertainty in design procedure retroactively (after optimization). However, it shows clearly the influence of uncertain parameters on objectives functions. Then, random choice of the uncertain parameters has been included in an evolutionary algorithm. It demonstrated that both methods leads to different design and operating conditions.

A multi-period approach will then be applied in the optimization. This means that each set of uncertain parameters will represent a period, and its corresponding design. The maximum value for decision variables comparing all design will be kept, allowing to include every possible configuration in optimization.

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