Modelling a Vertical Thermosyphon Reboiler Operating Under Vacuum Using Data from Rigorous Experimental Studies

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Thermosyphon reboilers represent effectively a pumpless system, in which natural, gravity-assisted circulation takes place. Although these units are the most commonly used in the chemical industry, Arneth and Stichlmair (2001), their operation has only been considered within the context of wider experimental programmes with few studies conducted below atmospheric pressure. In addition, previous research carried out to determine the operating characteristics of thermosyphon reboilers decoupled the problems related to heat transfer into a tube side and a shell side, usually by means of an electrically heated single tube (uniform heating). Thus, it is of paramount importance to look at the coupled problem to obtain better estimates of the heat transfer coefficients for the condensing steam in the shell and the heated process fluid in the tubes. The work described in the present article is carried out in this context and provides a detailed description of a mathematical model developed to predict the steady-state performance of a vertical thermosyphon reboiler. A number of operating variables have been predicted. Analysis of these predictions and comparisons with the generated experimental data, reported by Alane and Heggs (2007), resulted in good agreement. The resultant model could be used for optimisation studies on existing thermosyphon reboilers and the design of new ones.

1. Introduction

Flow in the two-phase thermosyphon reboiler is induced by the buoyancy forces arising from the combined density gradients, as a result of presence of temperature gradients, and the body force that is proportional to density, Alane and Heggs (2007). This work was conducted using data collected from a full scale replica of an industrial sized natural circulation thermosyphon reboiler comprising 50 vertically-mounted 25 mm OD tubes of 3 m length, Alane and Heggs (2007). Each tube within the bundle of the reboiler may behave differently owing to the uneven heat transfer from the condensing steam and the mal-distribution of the process (water) fluid entering the tubes as well as the location of the tubes with respect to the three segmental baffles inside the shell and
the steam inlet. The length of the subcooled region is most likely to differ for each tube. For convenience, however, it is assumed in the present model that the heat load from the condensing steam and the flow of the process fluid are distributed uniformly across the 50 tubes of the reboiler. In Figure 1 is a schematic, which illustrates the configuration of the process streams around the vertical thermosyphon reboiler.

Figure 1: Configuration of Streams around the Thermosyphon Loop

2. Numerical Model

The mathematical model detailed herein was developed to describe the behaviour of the vertical thermosyphon reboiler system in the Morton Laboratory and is specific to its physical configuration. Nonetheless, the number of tubes and their dimensions as well as the material of construction can be changed. The calculations sequence behind the model is written in Visual Basic for Applications. The following assumptions are made:

- The steam condensation on the outside surface of the tubes can be assumed to follow a Nusselt type analysis, which neglects the interfacial shear stress,
- The tubes behave identically with a uniform distribution of flow and heating,
- The vapour and liquid are one dimensional, steady-state, Newtonian flows,
- Compressibility of the vapour is negligible,
- The pressure drop in the condensate film is negligible,
- The cold leg of the thermosyphon loop is adiabatic,
- No heat is transferred in the region $0 \leq x \leq \delta_{\text{plate}}$, the bottom tube sheet,
- Condensate on the shell side is well mixed and at a fixed temperature, $\delta_{\text{condensate}} \leq x \leq L_{\text{tube}}$
The model relies on existing correlations for the prediction of the profiles of heat transfer coefficients, temperature and pressure. These correlations were rigorously selected based on published studies conducted to assess their accuracy against experimental data. A stepwise calculation method was adopted to solve a number of integrals of non-linear nature, which prompted the use of iterations.

### 2.1 Solved Equations and Solution Method

The temperature profile of the process fluid in the subcooled region is represented by the following equation:

\[
(MC_p) \frac{dT_p}{dx} = U_o A_o (T_{sat} - T_p)
\]  

(1)

In Eq. (1), the LHS corresponds to the temperature profile along the tube and the RHS accounts for the rate of heat transfer from the condensing steam to the process fluid. The initial condition for Eq. (1) is at \( x = 0, \ T_p = T_{p,1} \). Three regions of interest are considered in the subcooled/sensible heating zone on the process side:

1. Condensate on the shell side, \( 0 \leq x \leq \delta_{condensate} \)
2. Up to the onset of nucleate boiling, when \( T_{w,i} = T_{p,sat} \), but \( T_p < T_{p,sat} \),
   \( \delta_{condensate} \leq x \leq \delta_{ONB} \)
3. Up to the position when \( T_p = T_{p,sat} \), \( \delta_{ONB} \leq x \leq L_{sub} \)

Above the subcooled/sensible heating zone is the two-phase boiling region, \( L_{sub} \leq x \leq L_{tube} \). Beyond this point, the process fluid temperature is equal to the saturation temperature so that \( T_p = T_{p,sat}(x, p) \). So, over the two-phase boiling region, \( L_{sub} \leq x \leq L_{tube} \), Eq. (1) now becomes:

\[
\dot{Q}(x) = M \left( \frac{dx}{dx} \cdot \frac{dT_p}{dx} + c_{pf} \frac{dT_p}{dx} \right) = U_o A_o \left[ T_{sat} - T_{p,sat}(x, p) \right]
\]  

(2)

Throughout the length of the tube the value of the overall heat transfer coefficient, the tube inside and outside wall temperatures and the saturation temperature of the process fluid vary with position. In addition, the inside and outside film heat transfer coefficients change with position along the tube, which cause the overall heat transfer coefficient to vary accordingly, so that: \( T_p(x), T_{wall,i}(x), T_{wall,o}(x), T_{p,sat}(x, p), U_o(x) \). The rate of heat transfer from the condensing steam (\( \delta_{condensate} \leq x \leq L_{tube} \)) to the process fluid inside the tube is described by the following equation:

\[
\dot{Q}(x) = \alpha_o A_o (T_{sat} - T_{wall,o}) = -\frac{2\pi \lambda_o L}{\ln \left( \frac{d_o}{d_i} \right)} (T_{wall,o} - T_{wall,i}) = \alpha_o' A_o (T_{wall,i} - T_p)
\]  

(3)

In light of the expressions in Eq. (1) to Eq. (3), it is clear that the present system is non-linear and would require an iterative approach to solve the equations. An initial guess of
8000 W m\(^{-2}\) K\(^{-1}\) is assumed for the condensing coefficient, while the temperature of the outside surface of the tube is taken as \(T_{\text{sat}} - T_{\text{wall,0}} = 10^\circ\text{C}\). These values proved to generate good convergence throughout the calculations. Based on Eq. (1), the temperature of the process fluid at the different increments up the tube (using a step size \(dz\)) is calculated by means of the following equations:

\[
\begin{cases}
\text{Condensate} & \delta_{\text{Condensate}} \leq x \leq L_{\text{sub}} & T_{p,j+1} = T_{p,j} + \frac{U_{\text{cp}}A}{MC_{\text{pf}}} (T_{\text{sat}} - T_{p,j}) dz \\
\text{L}_{\text{sub}} \leq x \leq \text{L}_{\text{tube}} & T_p = T_{p,\text{sat}}(x, p)
\end{cases}
\]  

(4)

The process side heat transfer coefficient, \(\alpha_c\), is a function of the type of flow inside the tube and varies between the subcooled single-phase region and the two-phase boiling zone. These calculations are continued until the process fluid temperature at the outlet of the tube is obtained. The resultant values of the temperature of the process fluid and the heat transfer coefficients are then utilised in Eq. (5) to estimate the corresponding profile of the surface temperature along the outside tube wall, \(T_{\text{wall,0}}\) starting from the top of the tube moving downwards and using the same step size (\(dz\)):

\[
T_{\text{wall,0},j} = \left(\frac{1}{\alpha_c + f_o}\right)^{-1} T_{\text{sat}} + \left(\frac{1}{\alpha_w + f_i d_o + d_i \alpha_p}\right)^{-1} T_{p,j} \\
\left(\frac{1}{\alpha_c + f_o}\right)^{-1} + \left(\frac{1}{\alpha_w + f_i d_o + d_i \alpha_p}\right)^{-1}
\]

(5)

The calculation of the outside wall temperature is continued all the way down to the inlet of the tubes, where it is compared to the initial guess (\(T_{\text{wall,0}} = T_{\text{sat}} - 10^\circ\text{C}\)). At this point, the entire iteration process is completed. The ensuing \(T_{\text{wall,0}}\) is used as a guess for the second iteration only if it failed to converge with the guessed value in a process that is repeated until convergence at a tolerance of 0.1 is achieved after \(n\) iterations.

2.2 Heat Transfer Calculations

2.2.1. Subcooled Liquid Heat Transfer Coefficient

In the subcooled region, the process liquid is heated by convective heat transfer. The heat transfer coefficient for this process can be predicted accurately by the Petukhov equation. In the subcooled section there can be some enhancement due to nucleate boiling. This is accounted for by the model.

2.2.2. Two-Phase Inside Heat Transfer Coefficient

The method for calculation of the inside heat transfer coefficient is that recommended by Worley et al. (1985), which was later finalised by Gungor and Winterton (1986). The overall boiling heat transfer coefficient, \(\alpha_b\), is calculated from the combination of pure liquid convective heat transfer, \(\alpha_{f0}\), and nucleate boiling heat transfer, \(\alpha_{nb}\), as in Eq. (6):

\[
\alpha_b = E(1 - x_g)\alpha_{f0} + S\alpha_{nb}
\]

(6)
2.2.3. Steam Condensation Heat Transfer Coefficient
Steam condensation on the outside surface of the tubes can be assumed to follow a Nusselt type analysis. Assuming that the momentum and energy transfer by advection in the condensate film are negligible due to the low velocities associated with the film, then it follows that the heat transfer in the film occurs only through conduction and the temperature profile is linear, so that the heat transfer flux is given by Fourier’s law.

3. Experimental Studies, Comparison and Discussion of Results
The present model has been utilised to simulate a number of process conditions so as to study the individual effects of process pressure and heat load on the operation of the vertical thermosyphon reboiler. The results were compared with experimental data and analysed in the following discussion. From the simulated data, it can be inferred that the length of the subcooled region decreased with increasing pressures. The length of the subcooled zone was calculated to be approximately 2.54 m (82.93% of tube length) at 0.25 bar and 131 kW, decreasing steadily to reach a length of 1.95 m (63.66% of tube length) at atmospheric pressure and 140 kW. The longer subcooled lengths, obtained at lower pressures, are justified by the relatively larger effect of the static head of the fluid in the loop which pushes the boiling point upwards in the heated tubes. This increase in the subcooled length is also partly due to the need for more latent heat to evaporate the process fluid as pressure is decreased. The extended subcooled section implies that less area of the tubes was available for boiling. The effect of pressure is reduced dramatically at the higher heat loads, as these variations were confined within less than 5%. The short subcooled region and the reduced variations in the subcooled length with pressure at high heat loads are justified by the complex hydrodynamics and heat transfer mechanisms that are favourable to boiling under these conditions.

To validate the results obtained from the mathematical model, the predicted values are compared with the experimental data for the energy balance, the exit vapour mass quality, the exit temperature from the tubes, the overall heat transfer coefficient and the pressure around the loop under different process conditions. The errors in the calculations originate from the combination of the experimental uncertainties inherent to the data, and the deviations and limits associated with the correlations used for the calculations of heat transfer and pressure. The predictions of the temperature profiles around the loop provide the best reliability, since the overall error is maintained at significantly low values (-0.34% – 2.25%) throughout the entire process conditions that were examined. Contrary to the rest of the variables, the errors associated with the predictions of the pressure from the model tend to increase substantially as the heat load is increased, which gives rise to the largest errors. The significant discrepancies associated with the predictions of the overall pressure profile are justified by the presence of the two-phase flow. This is due to the fact that existing correlations for the single-phase flow can predict pressure drops satisfactorily with low inaccuracies, while the subject of two-phase pressure drop predictions is a topic of interest and ongoing research to develop more reliable and accurate correlations. Based on the modelled values, no clear patterns could be established for the effect of process pressure on the calculated errors. However, these results indicate that the reliability of the values generated by the model is largely dependent on heat load to the system.
4. Conclusions

The present work has successfully modelled the steady-state thermosyphon reboiler loop using existing correlations for the calculations of heat transfer and pressure. This model considers the coupled problem related to heat transfer in the tube side and the shell side of the reboiler. A stepwise calculation method was adopted to solve a number of integrals of a non-linear nature, which prompted the use of iterations. The profiles of a number of operating variables associated with the performance of the steady-state operation have been predicted (temperature, pressure, heat flux, local and overall heat transfer coefficients). Comparisons of these predictions with the experimental data resulted in good agreement. Throughout the conditions considered for the purpose of this study, the predictions of the exit temperature at the outlet of the tube produced consistently the best accuracy relative to the experimental values. The accuracy of the other predicted variables varied with pressure and heat load.

5. Nomenclature

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Special</th>
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</thead>
<tbody>
<tr>
<td>$A$ [m²]</td>
<td>Surface area</td>
</tr>
<tr>
<td>$c_v$ [kJ/kg K]</td>
<td>Specific heat capacity</td>
</tr>
<tr>
<td>$d$ [m]</td>
<td>Diameter</td>
</tr>
<tr>
<td>$E$ [-]</td>
<td>Enhancement factor</td>
</tr>
<tr>
<td>$f$ [-]</td>
<td>Fouling factor</td>
</tr>
<tr>
<td>$h$ [kJ/kg]</td>
<td>Specific enthalpy</td>
</tr>
<tr>
<td>$L$ [m]</td>
<td>Length</td>
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<tr>
<td>$M$ [kg/s]</td>
<td>Mass flow</td>
</tr>
<tr>
<td>$Q$ [kW]</td>
<td>Heat load</td>
</tr>
<tr>
<td>$S$ [-]</td>
<td>Suppression factor</td>
</tr>
<tr>
<td>$T$ ['C]</td>
<td>Temperature</td>
</tr>
<tr>
<td>$U$ [kW m⁻²K⁻¹]</td>
<td>Overall heat transfer coefficient</td>
</tr>
<tr>
<td>$x$ [m]</td>
<td>Position along the tube</td>
</tr>
<tr>
<td>$x$ [-]</td>
<td>Vapour mass quality</td>
</tr>
<tr>
<td>$z$ [m]</td>
<td>Incremental length</td>
</tr>
<tr>
<td>$a$ [kW m⁻²K⁻¹]</td>
<td>Heat transfer coefficient</td>
</tr>
<tr>
<td>$\delta$ [m]</td>
<td>Thickness</td>
</tr>
<tr>
<td>$\lambda$ [kW m⁻¹K⁻¹]</td>
<td>Conduction heat transfer coefficient</td>
</tr>
</tbody>
</table>

Subscripts

- $c$: Condensate
- $f$: Fluid properties
- $g$: Gas properties
- $i$: Inlet or inside conditions
- $o$: Outlet or outside conditions
- $ONB$: Onset of nucleate boiling
- $p$: Process conditions
- $s$: Steam
- $sat$: Saturation conditions
- $sub$: Subcooled zone
- $w$: Conditions at the wall

6. Reference


