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Subharmonic Oscillations , Relay On-Line Tuning and Process Identification

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In this paper a multiple relay arrangement for on-line process identification and controller tuning is proposed. The standard relay feedback method proposed by Astrom and Hagglund (1984) is very popular, because it is a simple and time efficient technique; moreover it is a closed-loop method and allows a tight control of the magnitude of the oscillations. Unfortunately only the critical point of the process (i.e. the process frequency response at the phase lag of $-\pi$) is detected and the acquired information may be insufficient for a correct identification of a large class of processes. Since in chemical processes the duration of each experiment may be critical, multiple identification sessions are undesirable and impracticable; therefore several alternative techniques have been proposed. However the information on the critical point of the process must be retained, often with the need of including some information at frequencies near the crossover point or in the low frequency range. Friman and Waller (1997) introduced a two-channel relay structure in order to extract more information about the process, e.g. to get a point in the third quadrant of Nyquist plane, Wang and Yang (2000) have provided the cascade relay for multipoint identification of the frequency response, Balestrino et al. (2006) have proposed a technique based on a standard relay with a variable hysteresis width, in order to originate relay transients. Several variants of the standard method have been presented in literature (see Wang et al., 2003): most of them consider various relay connections and give rise to very complex waveforms, usually difficult to analyse.

In this paper a master relay is configured as in the standard method of Astrom and Hagglund. Assume that the input of the master relay is periodic. The output of the master relay, u_m, is converted from real to binary format and, through a shift register or a chain of flip-flops, it gives rise to a sub-harmonic signal which is again converted from binary to real as the input to an auxiliary relay. Any sub-harmonic signal can be generated, but we restrict our attention to the simplest arrangement, where the sub-harmonic signal u_s (the output of the auxiliary relay) shows a frequency 2^{-m} of the master relay frequency. The input signal is the sum of the master and auxiliary relay outputs. The conditions assuring a limit cycle can be derived as in the classical approach of Tsypkin (1984). Eventually hysteresis can be added to the master relay, so that a strong robustness is assured with respect to output measurement noises.

The implementation of this technique is straightforward by using low cost electronics; examples of simulation tests using Matlab implementation illustrate the technique for some typical plants.

1. The Proposed Method

Relay feedback is a simple, powerful, and commonly used method of finding system parameters useful for designing and tuning standard proportional-integral-derivative (PID) controllers. Consider the standard relay feedback method proposed by Astrom and Hagglund (1984), as shown in Fig. 1, where the relay output amplitude is h. If the process has a phase lag of at least π radians most processes are brought into a condition of permanent oscillation. The resulting period T_c and amplitude E of the process output oscillation can be measured. The critical point in terms of the ultimate frequency ω_C and ultimate gain K_C can be determined by the describing function as:

$$\omega_C = \frac{2\pi}{T_C}; \qquad K_C = \frac{4h}{\pi\ E_C} \tag{1}$$

The above analysis originates from the standard theory where the amplitude ratio of the fundamental output from the nonlinearity and the input of the relay are considered. In the case of a relay nonlinearity the relay output is a square wave. Therefore (1) is an acceptable approximation if and only if the linear process G(s) behaves as a low-pass filter. Unfortunately only the critical point of the process is detected and the acquired information may be insufficient for a correct identification of a large class of processes. Several variants of the standard method have been presented in literature, e.g., Balestrino et al. (2006) have proposed a technique based on a standard relay with a variable hysteresis width, in order to originate relay transients, as shown in Fig.2. Using relay transients, instead of only stationary oscillations, more frequency points in a large neighbourhood of the critical frequency of the process transfer function can be estimate, but also this technique may require some care to be effective.

A different implementation may consider a sub-harmonic oscillator, as illustrated in Fig.3. The output of a standard relay, called master relay, u_m, is converted from real to binary format and, through a shift register or a chain of flip-flops, it gives rise to a sub-harmonic signal, which is returned from binary to real format and sent to an auxiliary relay. In a more general implementation, any sub-harmonic signal can be generated, but in this paper we restrict our attention to the simplest arrangement, where the sub-harmonic signal u_s (the output of the auxiliary relay) shows a frequency 2^{-m} of the master relay frequency. The sum of the master and auxiliary relay outputs (u) is the input signal of the process. The ratio of the amplitude of the auxiliary relay with respect to the amplitude of the master relay can be varied in a wide range, assuring a limit cycle with large frequency content. By adding a single sub-harmonic signal with frequency f/m to the output of the master relay with frequency f, the balance in the loop of the signals at frequencies f/m, 3f/m, f and 3f gives 4 complex equations allowing a direct evaluation up to 8 model parameters. The relay output u(t) consists of a periodic series of step changes with amplitude $\pm A_m \pm A_s$ and in most processes u(t) is a periodic function able to generate symmetric self-oscillations if the amplitude A_s is small and limited to the ratio $A_s / A_m \le 0.25$. For low-pass processes a positive amplitude ratio raises the limit cycle frequency above the value obtained by the approach of Astrom and Hagglund; the opposite effect is obtained by using a negative ratio. Asymmetric self-oscillations are usually generated if the ratio A_s / A_m increases.

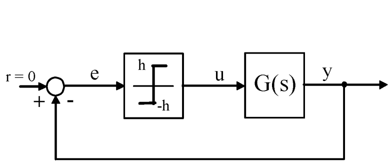

Fig. 1. Standard relay feedback system.

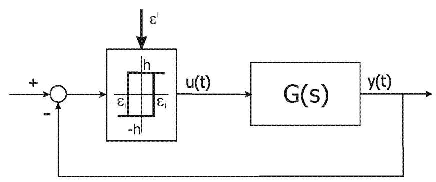

Fig 2. Modified relay feedback system.

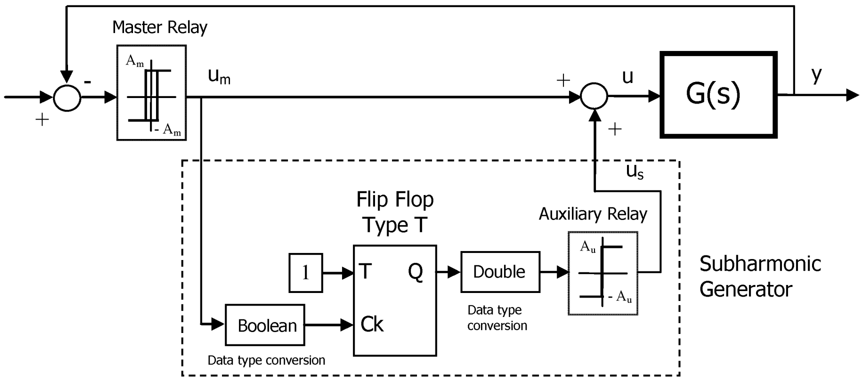

Fig. 3. Double channel relays and sub-harmonic oscillations with m = 1

After recording input and output signals, the process frequency response can be obtained using a standard Fourier analysis as the ratio:

$$G(j\omega_k) = \frac{\int_0^T y_{s(t)} e^{-j\omega_k t} dt}{\int_0^T u_{s(t)} e^{-j\omega_k t} dt} \tag{2}$$

where $y_{s(t)}$ and $u_{s(t)}$ are a period of the stationary oscillations of u(t) and y(t), respectively. The above relationship can be computed using the FFT algorithm. Since the method adopts spectrum analysis instead of the describing function, it will lead to accurate process frequency response estimation and it can identify multiple points on the frequency response from a single relay test. In order to extract the static gain, an asymmetric self-oscillation may be used. Because of the hysteresis in the master relay, the method is also robust in the case of output process signals corrupted by noise.

With respect to the parasitic relay methods proposed in Wang et al., (2003), the proposed implementation provides an oscillation with known sub-harmonics and such condition assures the occurrence of self oscillations that may be studied as in the classical approach of Tsypkin (1984).

3. An Analytic Approach

In this section the existence conditions of a limit cycle are established following the analytic approach of Tsypkin (1984). For sake of simplicity assume that the output $u_m(t)$ of the master relay is a symmetrical square wave with frequency f_m and amplitude A_m; moreover assume that the slave relay output is a symmetrical square wave $u_s(t)$ with frequency f_s and amplitude A_s. The output of the slave relay $u_s(t)$ is synchronized with the output $u_m(t)$ of the master relay: $u_m(0) = u_s(0)$, and the frequencies are such that: $f_m = N f_s$. Any square wave with unitary amplitude and pulsation ω may be described by its Fourier series:

$$\frac{4}{\pi}\sum_{k=0}^{\infty}\frac{1}{2k+1}\sin\left[\left(2k+1\right)\omega t\right] \tag{3}$$

If this signal is the input of a dynamic system described by the transfer function G(s) we can compute the steady state output by summing up the responses to all the harmonics.
Rewrite $G(j\omega)$ as $G[j(2k+1)\omega] = R_k(\omega) + j\, I_k(\omega)$ and define the auxiliary Tsypkin function:

$$T(j\omega) = \sum_{k=0}^{\infty}\left[\omega R_k(\omega) + j\frac{1}{2k+1}I_k(\omega)\right] \tag{4}$$

Note that this definition is slightly modified with respect to the original version in Tsypkin . Moreover define:

$$J_{m,s}(j\omega_s) = \frac{4}{\pi}A_m T(jN\omega_s) + \frac{4}{\pi}A_s T(j\omega_s) \tag{5}$$

Then the conditions for the existence of a limit cycle are:

$$\mathrm{Re}\left[J_{m,s}(j\omega_s)\right] < 0;\ \mathrm{Im}\left[J_{m,s}(j\omega_s)\right] = -h \tag{6}$$

The computation of $T(j\omega)$ is analytically possible and hence the computation of $J_{m,s}(j\omega_s)$. If we assume h = 0 and ω_s and N, so that $A_m T(jN\omega_s)$ and $A_s T(j\omega_s)$ are both in the same quadrant of the Nyquist plane, it is impossible to satisfy the conditions for the existence of the limit cycle. Of course if $A_s = 0$ we recover the classical conditions.
For low-pass smooth systems, analytical solution can be approximated by:

$$T(j\omega) = \sum_{k=0}^{\infty}\left[\omega R_k(\omega) + j\frac{1}{2k+1}I_k(\omega)\right] \cong \omega R_0(\omega) + jI_0(\omega) \tag{7}$$

as usually in the harmonic linearization approach, with N=2, we get:

$$\begin{aligned} &J_{m,s}(j\omega_s) = \frac{4}{\pi}A_m T(j2\omega_s) + \frac{4}{\pi}A_s T(j\omega_s) \cong \\ &\frac{8\omega_s}{\pi}A_m R(2\omega_s) + \frac{4\omega_s}{\pi}A_s R(\omega_s) + j\left[\frac{8}{\pi}A_m I(2\omega_s) + \frac{4}{\pi}A_s I(\omega_s)\right] \end{aligned} \tag{8}$$

If A_m and A_s are both positive or negative then a cycle limit is possible only if $G(j\omega_s)$ and $G(j2\omega_s)$ are in different quadrants of the Nyquist plane.
If $0<A_s<<A_m$ then $\omega_s < \omega_0 < 2\omega_s$ with $\angle G(j\omega_0) = -\pi$.

4. Analysis of Process Input and Output and Identification Tests

The performance of this new technique is illustrated for three typical chemical plants.
The auxiliary signals provide exciting inputs to processes under test: the signals considered for FFT are shown in Figs. 5, 7 and 9. The actual and the estimated frequency responses are shown in Figs. 6, 8 and 10. All relay tests has been carried out with $A_m = 1$, $A_u = 0.2$ and m =1, but different amplitudes have been condidered. Preliminary results show robustness with respect to output noisy signals, due to the selection of a suitable hysteresis in the master relay.

Example 1: $G(s) = \dfrac{10}{s(s^2+2s+2)}$

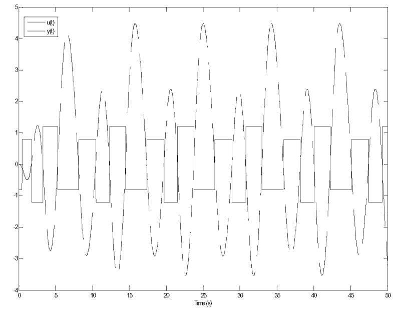

Fig. 4. Example 1: Limit cycle with $A_m = 1$, $A_u = 0.2$

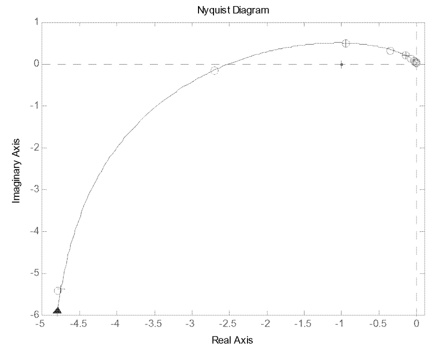

Fig. 5. Nyquist plots. (—) Actual, (o) Estimated.

Example 2: $G(s) = \dfrac{e^{-s}}{(2s+1)}$

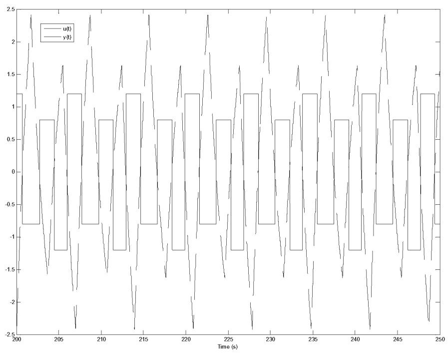

Fig. 6. Example 2: Limit cycle with $A_m = 1$, $A_u = 0.2$

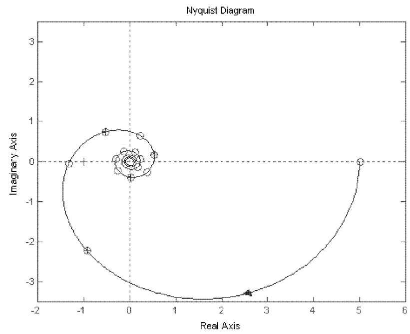

Fig. 7. Nyquist plots. (—) Actual, (o) Estimated.

Example 3: $G(s) = \dfrac{(1-s)\cdot e^{-s}}{(2s+1)^2}$

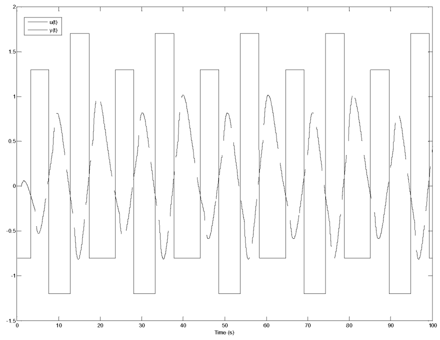

Fig. 8. Example 1: Limit cycle with $A_m = 1$, $A_u = 0.2$

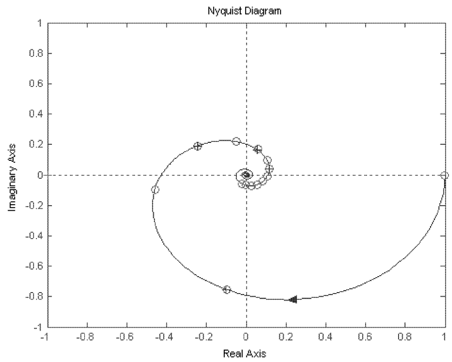

Fig. 9. Nyquist plots. (—) Actual, (o) Estimated.

4. Conclusions

A relay-based method for the estimation of the frequency response of linear process has been proposed. The method has several features:

- multiple points on frequency response can be identified from a single relay test;
- the hysteresis guarantees a high robustness with respect the noise;
- the method employs the FFT only once and the required computation burden is modest;
- it can be combined with some control tuning rules to form an auto-tuner for control systems;
- the implementation of this technique is straightforward by using low cost electronics.

5. References

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