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Thermal Study Of The Laminar Flow Of A Bingham Plastic In A Horizontal Circular Pipe Maintained At Uniform Temperature

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Viscoplastic fluids are widely used in industrial applications. They are characterized by a yield stress from which the fluid moves. The simplest representation of this type of fluids is the Bingham plastic. The purpose of this work is the numerical study by means of finite volume method, of heat transfer in forced convection of an incompressible viscoplastic fluid in a circular pipe maintained at uniform temperature. All the physical properties of the studied fluid are supposed to be constant. A subroutine based on the model of Papanastasiou was developed, in order to take into account the rheological behaviour of fluid of the Bingham type. The study is based on the influence of Reynolds number and the yield stress (Bingham number, $\mathcal{B}n$) on the coefficient of heat transfer represented by the Nusselt number. The comparison between the obtained results and those of Taegee Min et al. as well as those of Vradis et al. proves to be satisfactory.
Key words: Bingham fluid, yield stress, finite volume method, forced convection.

1. Introduction

A Bingham fluid is a substance which exhibits a yield stress that must be overcome before it will flow. It has a particular nonlinear behaviour law. Actually, when these materials flow in a pipe, there may be a central region which moves as a solid (plug flow) but near the wall the usual parabolic velocity profile of a newtonian fluid is observed.

Vradis et al. (1993) solved numerically the problem of simultaneous development of hydrodynamic and thermal fields in the entrance region of a circular pipe for a laminar flow of a Bingham fluid for which constant physical and rheological properties are assumed. They consider the flow with and without viscous dissipation and used a finite difference second-order accurate scheme in conjunction with a marching iterative solution technique. Min et al. (1997) studied numerically the hydrodynamically as well as the simultaneously developing laminar flows of a Bingham plastic in a circular pipe by using a four-step fractional method combined with an equal order bilinear finite element method. The latter found, for the simultaneously developing flow, that the heat transfer characteristics show the same trends as those predicted from the analytical method for the Greitz problem.

The purpose of the present work is to study the laminar forced convection flow of an incompressible Bingham fluid in a circular pipe maintained at uniform temperature by means of a numerical method based on the finite volume. All the physical and rheological properties of the fluid are supposed to be constant. Effects of the Reynolds number and the yield stress on the coefficient of heat transfer represented by the Nusselt number are presented and compared with previous studies.

2. The Governing Equations

Let's consider the laminar steady flow of an incompressible Bingham fluid inside a horizontal circular pipe of length L and radius r_w maintained at constant wall temperature T_w and let's suppose that all physical and rheological proprieties of the fluid are constant and uniform.

The non dimensionalized governing equations for the three dimensional flow of the studied fluid in cylindrical coordinates are given by:

$$\frac{1}{R}\frac{\partial(RV)}{\partial R}+\frac{1}{R}\frac{\partial W}{\partial \theta}+\frac{\partial U}{\partial X}=0 \tag{1}$$

$$\begin{aligned}\frac{1}{R}\frac{\partial(R\,V\,V)}{\partial R}+\frac{1}{R}\frac{\partial(W\,V)}{\partial \theta}+\frac{\partial(U\,V)}{\partial X}-\frac{W^2}{R}=-\frac{\partial P^*}{\partial R}+\frac{1}{Re}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(\eta_{eff}\,R\frac{\partial V}{\partial R}\right)+\right.\\ \left.\frac{1}{R^2}\frac{\partial}{\partial \theta}\left(\eta_{eff}\frac{\partial V}{\partial \theta}\right)+\frac{\partial}{\partial X}\left(\eta_{eff}\frac{\partial V}{\partial X}\right)\right]+\frac{1}{Re}\left[\frac{V}{R}\frac{\partial}{\partial R}(\eta_{eff})-\eta_{eff}\frac{V}{R^2}\right.\\ \left.-\frac{2}{R^2}\eta_{eff}\frac{\partial W}{\partial \theta}+\frac{\partial}{\partial X}(\eta_{eff})\frac{\partial U}{\partial R}+\frac{\partial}{\partial \theta}(\eta_{eff})\frac{\partial}{\partial R}\left(\frac{W}{R}\right)+R\frac{\partial}{\partial R}(\eta_{eff})\frac{\partial}{\partial R}\left(\frac{V}{R}\right)\right]\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{1}{R}\frac{\partial(R\,V\,W)}{\partial R}+\frac{1}{R}\frac{\partial(W\,W)}{\partial \theta}+\frac{\partial(U\,W)}{\partial X}+\frac{V\,W}{R}=-\frac{1}{R}\frac{\partial P^*}{\partial \theta}+\frac{1}{Re}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(\eta_{eff}\,R\frac{\partial W}{\partial R}\right)+\right.\\ \left.\frac{1}{R^2}\frac{\partial}{\partial \theta}\left(\eta_{eff}\frac{\partial W}{\partial \theta}\right)+\frac{\partial}{\partial X}\left(\eta_{eff}\frac{\partial W}{\partial X}\right)\right]+\frac{1}{Re}\left[\frac{2}{R^2}\eta_{eff}\frac{\partial V}{\partial \theta}-\eta_{eff}\frac{W}{R^2}\right.\\ \left.+\frac{1}{R}\frac{\partial}{\partial X}(\eta_{eff})\left(\frac{\partial U}{\partial \theta}\right)+\frac{1}{R^2}\frac{\partial}{\partial \theta}(\eta_{eff})\left(\frac{\partial W}{\partial \theta}+2\,V\right)+\frac{1}{R}\frac{\partial}{\partial R}(\eta_{eff})\left(\frac{\partial V}{\partial \theta}-W\right)\right]\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{1}{R}\frac{\partial(R\,V\,U)}{\partial R}+\frac{1}{R}\frac{\partial(W\,U)}{\partial \theta}+\frac{\partial(U\,U)}{\partial X}=-\frac{\partial P^*}{\partial X}+\frac{1}{Re}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(\eta_{eff}\,R\frac{\partial U}{\partial R}\right)+\frac{1}{R^2}\frac{\partial}{\partial \theta}\left(\eta_{eff}\frac{\partial U}{\partial \theta}\right)\right.\\ \left.+\frac{\partial}{\partial X}\left(\eta_{eff}\frac{\partial U}{\partial X}\right)\right]+\frac{1}{Re}\left[\frac{\partial}{\partial R}(\eta_{eff})\frac{\partial V}{\partial X}+\frac{1}{R}\frac{\partial}{\partial \theta}(\eta_{eff})\frac{\partial W}{\partial X}+\frac{\partial}{\partial X}(\eta_{eff})\frac{\partial U}{\partial X}\right]\end{aligned} \tag{4}$$

$$\frac{1}{R}\frac{\partial(R\ V\ \phi)}{\partial R}+\frac{1}{R}\frac{\partial(W\ \phi)}{\partial \theta}+\frac{\partial(U\ \phi)}{\partial X}=\frac{1}{Pr\ Re}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \phi}{\partial R}\right)+\frac{1}{R^2}\frac{\partial^2 \phi}{\partial \theta^2}+\frac{\partial^2 \phi}{\partial X^2}\right] \quad (5)$$

In order to avoid a numerical instability in the low shear rate region, Min et al. (1997) recommend to use the following constitutive equation proposed by Papanastasiou for which they advised to take m = 1000:

$$\eta_{eff}=1+\frac{Bn}{\dot{\gamma}}\left[1-\exp(-m\dot{\gamma})\right] \quad (6)$$

The resolution of equations (1-5) requires a set of boundary conditions. At the inlet, the flow is assumed to be uniform (V = W = 0, U = 1) as well as the temperature ($\phi = 1$). No-slip conditions are applied at the wall (V = W = U = 0) and a parietal constant temperature is assumed ($\phi = 0$). In addition, at the plane of symmetry ($\theta = 0$), we have: $\frac{\partial \phi}{\partial \theta}=\frac{\partial U}{\partial \theta}=\frac{\partial V}{\partial \theta}=W=0$.

The numerical technique used in the present study is the finite volume method proposed by Patankar (1980). The governing equations are put in the form of an algebraic equation which is solved using the SIMPLER algorithm. Since the rheological proprieties are independent of temperature, the hydrodynamic problem is independent of the thermal one.

3. Results And Discussion

3.1 Validation of the computer code

To validate our computing program, a comparison between the present numerical results and those obtained by Min et al. (1997) for a flow without viscous dissipation was done. The same hypothesis were taken for both studies except that Min et al. (1997) used the finite element method.

This comparison is showed on Figure 1 which presents the variation of the Nusselt number according to the Greitz number for Pr = 1, Bn = 1.99 and Re = 5, 25 and 50. Good agreements are observed between the two studies since that the maximum error does not exceed 3%.

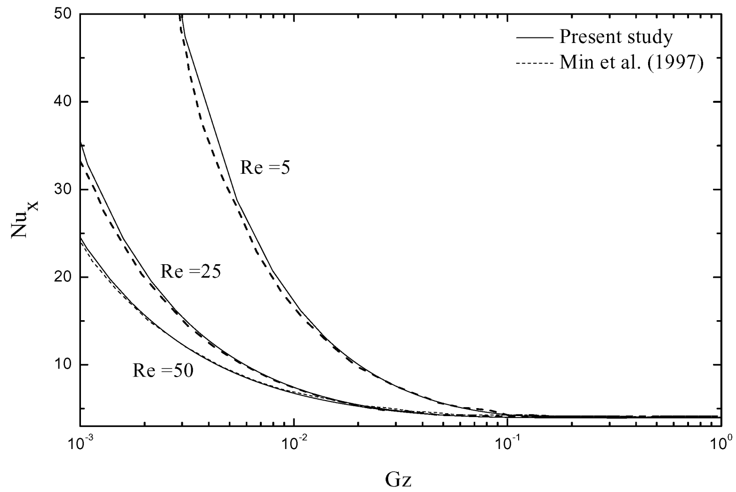

Figure 1: Comparison between the local Nusselt number resulting from the present study and that of Min et al. for various Re. Pr = 1, Bn = 1.99

3.2 Effect of the Reynolds number on heat transfer

Figure 2 shows the variation of the local Nusselt number according to the axial distance, for Pr = 1, Bn =1.99 and Re = 5, 25, 50, 75, 100 and 215. The curves present the same form: a pronounced decrease near the inlet then an asymptotic tendency to a same value of the Nusselt number independent of Re; this represents the fully developed conditions. On the other hand, the local Nusselt number as well as the thermal entrance are shown to be higher for higher values of Re. These results seem to be in agreement with those found by Min et al. (1997) and Vradis et al. (1993).

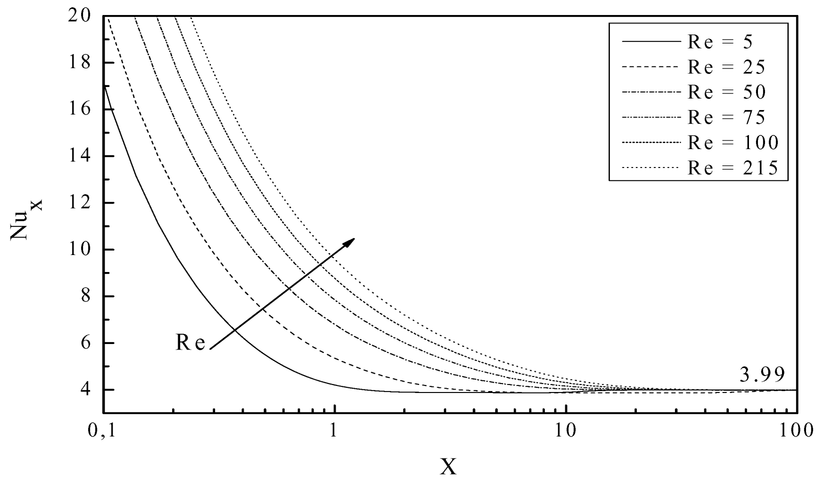

Figure 2 : Local Nusselt number with respect to the Reynolds number.
Pr = 1, Bn =1.99.

3.3 Effect of the Bingham number on heat transfer

Figure 3 represents the evolution of the local Nusselt number according to the axial distance, for Re = 25, Pr = 1 and Bn = 0, 1.99, 3 and 5.65. The curves show a light increase in the value of the Nusselt number for the higher value of Bn, located at the fully developed region. But this effect remains, even so, weak given that the viscous dissipation was not taken into consideration in the present study. These observation was also made by Min et al. (1997) and Vradis et al. (1993).

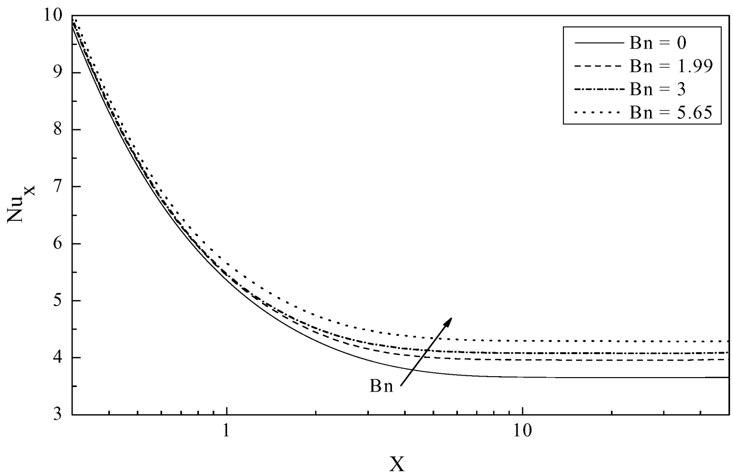

Figure 3 : Variation of the local Nusselt number according to the axial distance for various values of Bn. Re = 25, Pr =1.

3. Conclusion

The study of the laminar forced convection flow of an incompressible Bingham fluid in a circular pipe maintained at uniform temperature was investigated by means of a numerical method based on the finite volume. In order to avoid a numerical instability in the low shear rate region, the viscosity model of Papanastasiou was adopted to describe the behaviour of the Bingham fluid. The validity of the present computing program was confirmed by comparing the results with those of the literature.

The present results showed that the heat transfer characteristics, that is to say the Nusselt number and the thermal entrance, are strongly affected by the variation of the Reynolds number. On the other hand, the Bingham number presents a weak influence on the heat transfer characteristics in the entrance region because the viscous dissipation was not taken into consideration in the present study.

3. References

Vradis G.C., J. Dougher and S. Kumar, 1993, Entrance pipe flow and heat transfer for a Bingham plastic, Int. J. Heat Mass Transfer, 36, 3, pp. 543-552.

Min T., Hyoung Gwon Choi, Jung Yul Yoo and Haecheon Choi, 1997, Laminar convective heat transfer of a Bingham plastic in a circular pipe-II. Numerical approach-hydrodynamically developing flow and simultaneously developing flow, Int. J. Heat Mass Transfer. 40, 15, pp. 3689-3701.

Patankar S.V., 1980, Numerical heat transfer and fluid flow, McGraw, New York.

4. Nomenclature

Bn	Bingham number, τ_0 R / μ_p V_0
C_p	Specific heat at constant pressure.
D	Pipe diameter.
k	Thermal conductivity.
L	Length of the pipe.
m	Exponential growth parameter in equation (6).
Nu	Nusselt number, $(-2/\phi_m)(\partial\phi/\partial R)\big|_{R=1}$
p^*	Pressure.
P^*	Dimensionless pressure, $p^* / \rho V_0^2$.
Pr	Prandtl number, $\mu_p C_p / k$.
r	Radial coordinate.
r_w	Radius of the pipe.
R	Dimensionless radial coordinate, r / D.
Re	Reynolds number, $\rho V_0 D / \mu_p$;
T	Temperature.
T_0	Entrance temperature.
T_w	Wall temperature.
U	Dimensionless axial velocity, V_x / V_0.
V	Dimensionless radia velocity, V_r / V_0.
V_0	Average velocity.
W	Dimensionless azimutal velocity, V_θ / V_0.
x	Axial coordinate.
X	Dimensionless axial coordinate, x / D.

Greek symbols:

$\dot{\gamma}$	Rate of strain.
η	Effective viscosity of the Bingham fluid.
η_{eff}	Dimensionless effective viscosity, η / μ_p.
μ_p	Plastic viscosity.
ρ	Density of the fluid.
τ_0	Yield shear stress.
ϕ	Dimensionless temperature, $(T - T_w) / (T_0 - T_w)$
ϕ_m	Dimensionless bulk temperature, $(T_m - T_w) / (T_0 - T_w)$