Optimization of Process Plant Layout Using a Quadratic Assignment Problem Model

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The objective of this study is the optimization of the layout of a chemical process plant, minimizing the costs of piping, supports and pumping, while satisfying requirements of distance for safety, operation and maintenance.

The problem is formulated as a quadratic assignment problem, where the elements are allocated to a grid of predetermined positions. The objective function includes all costs, such as piping, pumping, installation and supports. The restrictions of a layout problem, such as minimum safety distances, are also included in the objective function as penalties.

The resulting quadratic assignment problem is solved using a simulated annealing method, due to the large size of the problem. The solution, although not proven optimal, is usually good and can be found in a short time.

The size and geometry of the grid should be carefully estimated for each case, because they can greatly influence the running time and the quality of the solutions. The proposed strategy was applied to a case study with 51 elements. The problem was solved in a small time, and the layout obtained was satisfactory.

1. Introduction

The main objective of a plant layout is to find the most economical spatial arrangement of process vessels and equipment and their connection by pipes, while satisfying requirements of construction, operation, maintenance and safety (Mecklenburgh, 1985). Therefore, the choice of the best layout is of great interest in the design of a chemical plant.

Most of the procedures for designing a plant layout come from practical experience (Anderson, 1982; Kern, 1977, 1978a; Mecklenburgh, 1985). A complete and final plant layout requires knowledge in the areas of chemical and mechanical engineering and considerations of geometrical design (Kern, 1978b).

An industrial process can be very complex and the optimization of the plant layout has a combinatorial nature that makes it very difficult to solve. The most difficult part is the determination of the best placement of the components, because of the nonconvexity of the feasible region (the positions of the vessels must not overlap). Therefore, mathematical models can be very helpful in the design of a process plant layout.

A number of different models and approaches have been proposed for the optimization of process plant layout. Some of them use continuous domain models, while others use discrete models or a grid of possible positions, concerning the way components could be located in the process plant. Models based on continuous domain

consider positions for the components that are represented by continuous variables, either as a mixed integer linear programming (Patsiatzis and Papageorgiou, 2003; Barbosa-Póvoa et al, 2002) or nonlinear optimization (Gunn and Al-Asadi, 1987). Models based on a grid of points consider positions in a discrete domain, where components could be assigned to (Georgiadis et al , 1999).

In this work, a quadratic assignment model based on a discrete domain approach is proposed. A grid of possible positions is defined, with a number of points greater than the number of components. The distance between neighboring possible positions is chosen so that it avoids overlapping of components. The objective function includes the costs of piping, pumping and supports, and a penalty cost for the requirements of minimum safety distances and restricted positions. Minimum distances can also be included in the definition of the distance between points in the grid. A simulated annealing algorithm is used for the minimization of the objective function.

2. Mathematical Modeling

2.1. Layout Problem Statement

The layout problem in this work has the objective of finding the best arrangement of process equipment so that the total cost is a minimum and the minimum distances between components are satisfied. Different pieces of equipment are not allowed to occupy the same space in the plant.

For a given number of process components, Nc, it is defined a three dimensional grid of points of possible positions. The total number of points in the grid, N, could be greater than or equal to the number of components.

The geometry of the grid of points and the distance between them are defined arbitrarily, but it can be done in a way so that two different components do not overlap and a minimum distance between components is satisfied. In this work, it was used a rectangular grid where all the points have the same distance d from their closest neighbor.

The layout problem is then formulated as quadratic assignment problem. The main strategy of the proposed approach is to transfer all the complexity of the layout problem into the definition of the matrices that appear in objective function, which includes real costs (piping, pumping, installation, supports) and penalty costs (minimum safety distances, restricted positions).

2.2. Quadratic Assignment Problem

The quadratic assignment problem can be stated as (Love et al, 1988):

$$\min z = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} a_{ijkl} \cdot x_{ik} \cdot x_{jl} \tag{1}$$

subject to:

$$\sum_{i=1}^{N} x_{ik} = 1 \qquad\qquad (k=1,\ldots,N) \tag{2}$$

$$\sum_{k=1}^{N} x_{ik} = 1 \qquad \left(i = 1, \ldots, N\right) \qquad (3)$$

$$x_{ik} \in \{0,1\}$$

where $x_{ik} = 1$ if component i is located at position k, and $x_{ik} = 0$ otherwise. Each component i then has its geometric center located at the point were it is assigned. The different applications of the QAP model can be found by the choice of the parameters a_{ijkl} and construction of the grid. This construction is a fundamental stage, because it already restricts the number of feasible solutions.

In some cases, there may be more positions than components or the components may require different amounts of space. The first problem is solved using dummy components in the model, the second by dividing larger components into smaller subcomponents. This kind of strategy for the optimization of process plant layout was used by Newell (1973), which used a two dimensional grid. In this work, to guarantee a larger flexibility, more positions than real components are used.

The quadratic assignment problem (QAP) is a generalization of the traveling salesman problem (TSP), and therefore is a NP-complete problem. The techniques that can be used to solve exactly the QAP to find the optimal solution include branch-and-bound and cutting plane methods, but they are limited to small problems ($N \sim 20$) in order to be solved in an acceptable time with current hardware. Therefore, heuristic methods are usually applied for large problems.

One heuristic method frequently used is to find some initial assignment, then try to improve it by pair interchanges, until no interchange can improve the objective function (Love et al, 1988). However, this method may only result in a local optimum. The simulated annealing algorithm (Aarts and Korst, 1989) is also a good way to solve the quadratic assignment problem, since it can avoid such local optimal solutions.

2.3. Matrices Definitions

The constants a_{ijkl} in Equation (1) include all costs and penalty functions used in the model formulation. In this work, they are given by:

$$a_{ijkl} = b_{ik} \cdot \delta_{ij} + \sum_{m=1}^{5} c_{ij}^{(m)} \cdot d_{kl}^{(m)} \qquad (4)$$

where $\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ if $i \neq j$.

There are two kinds of costs in Equation (4): (*i*) costs that depend only on the position of a component in the plant layout (b_{ik}), such as installation, supports and piping from fixed points in the plant (pipe racks), and (*ii*) costs that depend on the relative positions between components ($c_{ij} \cdot d_{kl}$), such as piping, pumping and penalty costs. Thus, the matrices in Equation (4) are defined as follows:

- Distance matrix $d_{kl}^{(1)}$: used to measure the Manhattan distance between points k and l.

- Cost matrix $c_{ij}^{(1)}$: unit cost of piping from component i to component j.

- Distance matrix $d_{kl}^{(2)}$: used to measure the vertical distance from point k to point l, if point k is below point l. It is used in the estimation of part of pumping costs.

- Cost matrix $c_{ij}^{(2)}$: pumping cost between components i and j proportional to height.

- Distance matrix $d_{kl}^{(3)}$: used in the penalty if some component cannot be assigned to a position k below another position l.

- Cost matrix $c_{ij}^{(3)}$: it results in a penalty M in the objective function if component i is placed at a position below component j, where M is a big number.

- Distance matrix $d_{kl}^{(4)}$: used for safety distances larger than d (point grid distance).

- Cost matrix $c_{ij}^{(4)}$: it results in a penalty in the objective function if components i and j are allowed to stay closer than or equal to some distance.

- Distance matrix $d_{kl}^{(5)}$: used for long components, requiring two positions in the grid.

- Cost matrix $c_{ij}^{(5)}$: it results in a penalty in the objective function if components i and j are not in adjacent positions in the grid.

- Cost matrix b_{ik}: it includes all costs that are only related to assigning a component i to a position k, such as costs of installation, supports, piping from or to the pipe rack.

3. Optimization Method

3.1. Size of the Problem

The layout problem formulated as a QAP, given by Equations (1) - (3), can be quite large. For example, for a cubic three dimensional grid with 10 points in each axis, the number of possible locations is 1000. Considering only 20 components to be located in these 1000 permissible positions, the number of ordered permutations is (1000!)/(980!), which is approximately 8.3 x 10^{59}. If ghost components are considered, so that the number of components to be placed is the same as the number of possible positions, the number of binary variables x_{ik} is 1000000. Therefore, the use of exact methods to solve this large QAP is not practical.

3.2. Heuristic Method

The optimization in this work was done using the simulated annealing method (Aarts and Korst, 1989). The concept of annealing is based on a strong analogy between the physical annealing process of solids and the problem of solving large combinatorial optimization problems. Using the Metropolis algorithm, it generates a sequence of solutions, where the transition from solution i to solution j is evaluated by the acceptance criterion given by (Aarts and Korst, 1989).

For the layout problem formulated as an assignment problem, a solution i is a given arrangement of components that satisfy the restrictions given by Equations (2) and (3). A new solution j is obtained from i by exchanging the positions of just two components, keeping the others in the same position. In order to avoid the interchange between two ghost components, which would change nothing in the layout, the first

component is chosen arbitrarily from the set of Nc real components and the second component is chosen arbitrarily from the set of N components (real + ghosts), excluding the one already selected as the first.

4. Case study

The case study tested in this work is based on the polyester process plant presented by Gunn and Al-Asadi (1987). Since that example considered a small production rate of 30 tons per day of polyethylene terepthalate, it is a batch process (Gunn and Al-Asadi, 1987).

For this work, it was considered that the plant had 50 major components. Since specific data for this example was not available, the sizes for the components were arbitrarily set. The values for the costs were estimated from literature, assuming some values for pipe specifications and supports (Peters and Timmerhaus, 1991). The unit costs for pumping were considered as the costs of energy to move a material from one piece of equipment to another, and they can be obtained from the energy balance. Some distances for minimum spacing of equipment were taken from process equipment spacing tables (Anderson, 1982), in order to give adequate safety, while others were arbitrarily fixed due to lack of specific information.

5. Results and Discussion

Cubic grids 5x5x5 (N=125), 6x6x6 (N=216) and 7x7x7 (N=343) were chosen to solve the problem, in order to offer a large flexibility to designate the components. It is important to point out that other kind of grids could be used (not cubic).

The distance between the grid points was arbitrarily set to 5m, which was large enough to avoid overlapping for all components, except one piece of equipment, which was divided in two parts, due to its larger size. The penalty value M was set to 100000.

The CPU time was around 10 min for the 5x5x5 grid, 15 min for the 6x6x6 grid and 20 min for the 7x7x7 grid. Grid 5x5x5 resulted in the lowest objective function values, without incurring in any penalty (no violation of layout restrictions).

The result found with the 6x6x6 and 7x7x7 grids had a higher objective function value, also without any penalty, which indicates that it was not really the optimal solution, since the 6x6x6 and 7x7x7 grids include the 5x5x5 grid as a subset.

As the simulated annealing is a heuristic method, there is no guarantee that the best solution found is the global optimal solution. However, the results were satisfactory, since the program obtained a reasonable layout in a low computational time and all the layout requirements were satisfied.

6. Conclusion

The objective of this work was successfully obtained with the adequate placing of the equipment for a case study with a large number of components. Equipment support, piping costs and pumping were minimized. The algorithm employed was very efficient. The requeried computing time was accetable, despite the great amount variables.

The main idea of the proposed approach is to define the layout problem as a quadratic assignment problem, so that most of the usual restrictions are transformed into

penalty functions. Then, a simulated annealing procedure is used to solve the resulting quadratic assignment problem. The proposed approach is easy to apply and can be used to find a reasonable layout in a short computational time, minimizing the cost while satisfying the requirements of space and safety distances. However, since it uses a heuristic procedure, it does not guarantee finding the global optimal solution.

The possible locations for the placement of all components are defined as a grid of points, which must be specified previously. The increase in the size of the grid offers a better flexibility to the method, although no advantage was observed in using larger grids for the particular case study tested in this work. Increasing the size of the grid results in a large increase in the number of possible locations and in the computational time, so the optimal solution also becomes more difficult to be found. A good strategy to solve the problem is to begin the resolution with a small grid of possible locations and then to increase the size until no better result is found, which was done in this work.

7. References

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