

NEW MEASUREMENT METHOD FOR LIQUID DISPERSION COEFFICIENTS IN FABRICS: FLASH MMT METHOD

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This work presents a new method called “flash MMT method” developed for the quantitative measurements of liquid dispersion coefficients in fabrics. Its principle is based on the combination of the well known flash method used for the measurement of thermal diffusivities of materials and the Moisture Management Tester (MMT) method used for qualitative characterization of liquid spreading in the fabrics.

It was shown that when the liquid injection at the top surface of a fabric follows a Dirac function, the flash MMT 2D model developed has an analytical solution where axial and lateral dispersion coefficients are involved as the unknown parameters. The latter were then deduced from the experimental measurements of water content on both top and bottom surfaces of the fabric. The agreement between the model prediction of water content and the experimental measurements is quite encouraging even though the liquid injection part of the method deserves to be improved.

1. INTRODUCTION

In the sportswear design, among the most influencing parameters of the fabric used, lateral (on both faces) and longitudinal dispersion coefficients of the liquid are of great importance. Their experimental measurement and theoretical prediction are therefore challenging.

Recently, a new method and instrument named Moisture Management Tester (MMT) has been developed and used to determine the liquid spreading and transfer rates of fabrics (Hu et al., 2005; Yao et al., 2006). Although the results obtained are quite interesting, it is however important to notice that only qualitative information can be determined with the MMT. The measurements do not contain the information which is able to quantify the dispersion coefficients. To overcome this problem, we combined the flash method with the MMT in what we called “Flash MMT” method.

The flash method was mainly developed for the measurement of heat dispersion coefficients of materials. It consists in imposing a pulse of heat during a very short time (flash) on one of the faces of a cylindrical sample of material. The temperatures of the material are then measured on the same face and on the opposite face as well. The heat dispersion coefficients are then deduced from the comparison of the measurements to the heat transfer model predictions (Parker et al., 1961; Lachi, 1991; Degiovanni et al., 1996; Demange et al., 1997).

In the present paper, the flash method is adapted to mass transport measurements in fabrics. Instead of a heat pulse in thermal measurements, a pulse of water is used. The MMT is then used for the measurements of water content on the two faces of the fabric. The mass dispersion coefficients are deduced from the comparison of the measurements of water content to the predictions of a transient 2D model (Quiniou, 2009).

2. PROCESS MODEL

We consider a cylindrical sample of a fabric with a thickness of L_z and a diameter of L_r . Water is injected on the upper surface of the fabric corresponding to the coordinate $z=L_z$ over a diameter R_{inj} less than $L_r/2$ as shown in figure 1. The model is developed assuming that the water injection is close to a Dirac function under isothermal

conditions. In this work, gravity effects are neglected since the value of the ratio between gravity and capillary forces, represented by the GS number (Li and Zhu, 2003), estimated in our experimental conditions, is around 0.1.

The transient 2D diffusion equation of water in the fabric is expressed as

$$\frac{\partial(\rho\varepsilon)}{\partial t} = \frac{\partial}{\partial r} \left(D_r \frac{1}{r} \left(\frac{\partial(r\rho\varepsilon)}{\partial r} \right) \right) + \frac{\partial}{\partial z} \left(D_z \left(\frac{\partial(\rho\varepsilon)}{\partial z} \right) \right) \quad (1)$$

where: $0 < t \leq t_f$, $0 < r < \frac{L_r}{2}$, $0 < z < L_z$.

The boundary conditions are defined assuming that there is no mass exchange with the surrounding. They are given as

$$\left. \frac{\partial \rho \varepsilon}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial \rho \varepsilon}{\partial z} \right|_{z=L_z} = 0, \quad \left. \frac{\partial \rho \varepsilon}{\partial r} \right|_{r=0} = 0, \quad \left. \frac{\partial \rho \varepsilon}{\partial r} \right|_{r=L_r} = 0$$

As in the thermal flash method, the initial condition must be represented by a Dirac function in order to ease the solution of the model equations using the variable separation method. Thus, at the initial time ($t=0$), the volume fraction of the liquid water in the fabric must be expressed by the product of an axial term and a lateral term as

$$\varepsilon(r, z, 0) = f_0(r) \cdot g_0(z) \quad \text{with} \quad f_0(r) = \begin{cases} 1 & \text{if } 0 \leq r \leq R_{inj} \\ 0 & \text{if } r > R_{inj} \end{cases} \quad \text{and} \quad g_0(z) = \begin{cases} \varepsilon_{inj} & \text{if } 0 \leq z \leq e \\ 0 & \text{if } z > e \end{cases}$$

This means that the water injected on the fabric surface during the experiment is initially located in the cylindrical zone defined by the radius of R_{inj} and the thickness e (see Fig. 1). In that zone, at the initial time, the liquid holdup is ε_{inj} and zero in the rest of the fabric.

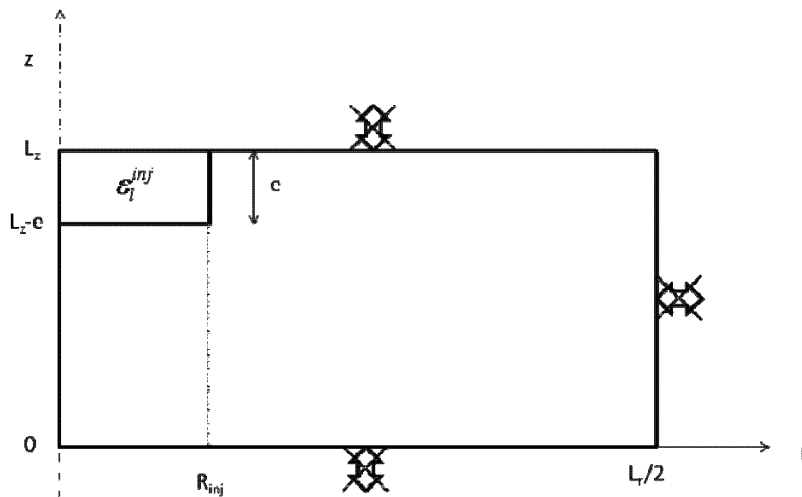


Figure 1: flash MMT geometry and initial conditions

According to the variable separation method, the solution is expressed as

$$\varepsilon(r, z, t) = A(z, t) \cdot R(r, t) \quad (2)$$

where the axial and lateral terms are given by the solution of the following partial differential equations :

$$\frac{\partial^2 A}{\partial z^2} = \frac{1}{D_L^z} \frac{\partial A}{\partial t} \quad (3)$$

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} = \frac{1}{D_L^r} \frac{\partial R}{\partial t} \quad (4)$$

The variable separation method is used again here to solve each of the two equations. The calculus details are developed in (Quiniou, 2009). The analytical solution of the model equation with associated initial and boundary conditions is expressed as

$$\varepsilon(r, z, t) = \left[A_0 + 2A_0 \sum_{k=1}^{+\infty} \cos\left(\frac{k\pi z}{L_z}\right) \exp\left(-D_l^z \frac{k^2 \pi^2}{L_z^2} t\right) \right] \times \left[B_0 + \sum_{k=1}^{+\infty} B_k J_0\left(\alpha_k \frac{2r}{L_r}\right) \exp\left(-D_l^r \frac{4\alpha_k^2}{L_r^2} t\right) \right] \quad (5)$$

where $A_0 = \frac{Me_0}{\rho\pi R_{inj}^2 L_z}$, $B_0 = \frac{4R_{inj}^2}{L_r^2}$, $B_k = \frac{4R_{inj}}{\alpha_k L_r} \frac{J_1(2\alpha_k R_{inj}/L_r)}{J_0^2(\alpha_k)}$, and α_k the k-th root of J_1 . Me_0 is the initial

mass of water in the fabrics, J_0 and J_1 are respectively the zeroth and first order Bessel function of first kind.

It can be seen that the solution involves two infinite series which have to converge in order to compute the value of liquid holdup at r , z and t . It is important to notice that the convergence rate of the series slows down when the time approaches the beginning of the experiment.

3. EXPERIMENTAL MEASUREMENTS

Here the Moisture Manager Test apparatus (Hu et al., 2005) is used to determine the liquid spreading and transfer rates of a fabric. Its schematic representation is given in figure 2, and a photograph is presented on figure 3. The principle of the method is based on the change of the electrical resistance of the fabric with its local water content. These two variables are linked through a proper calibration.

Six concentric rings (sensors) of different sizes are placed on both surfaces of the fabric. The distance between two consecutive rings is 5 mm except the first one which is at 1.5 mm from the centre. They allow us to measure the spreading and transfer of a 0.22 g drop of water during 2 minutes. More specifically they allow to determine the water content (WC) at different ring locations (local) and consequently on the overall (global) surface.

WC is the result of integration, over a ring, of the ratio between the weight of free water and the weight of dry fabric, given as

$$WC_i^{j,th} = \frac{2\pi}{S_i^j} \int_{r_i}^{r_{i+1}} \frac{\rho\varepsilon}{\rho_f \varepsilon_f} r dr \quad (6)$$

i (=1,2,3,4,5,6) refers to a ring and j (=top, bottom) to a surface. ε_f and ρ_f are the volume fraction and density of the fabric respectively, and S is the fabric surface bounded by a ring.

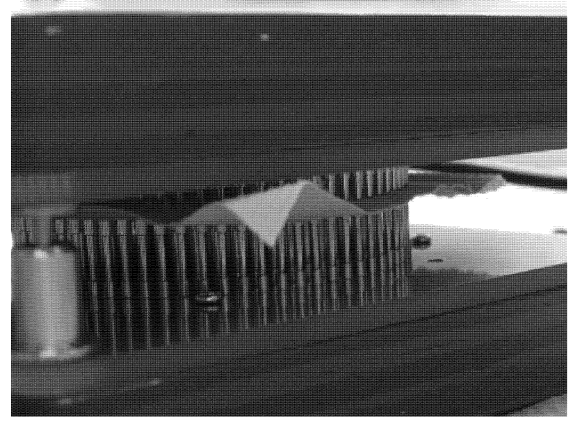
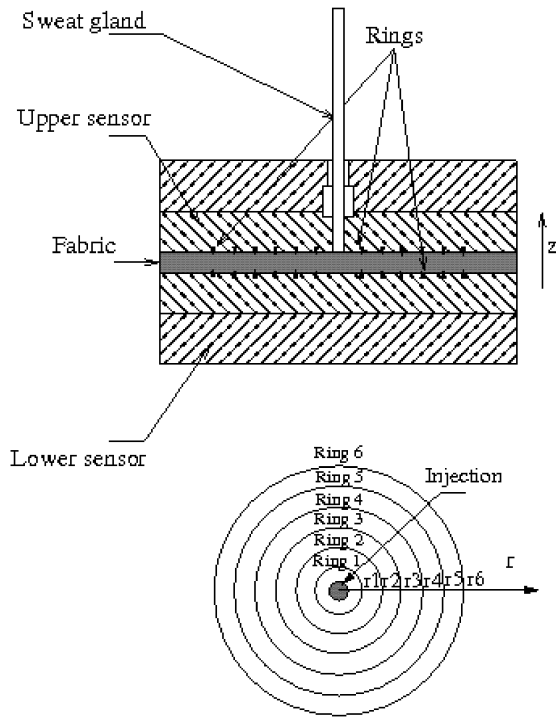


Figure 3: photograph of actual MMT device with a piece of fabric being tested

Figure 2: schematic view of MMT

4. PROCESS MODEL PARAMETERS IDENTIFICATION

The process model developed and solved analytically involves two unknown parameters, i.e. axial (D_z) and lateral (D_r) liquid dispersion coefficients. Their optimal values will be deduced from the comparison of the measured values of the water content using the MMT apparatus and those predicted using the process model. The identification procedure is as follows. For two (top and bottom) rings located on the same radius i , the ratio of predicted water contents is given as

$$\frac{WC_i^{bot,th}}{WC_i^{top,th}} = \frac{A(L_z, t)}{A(0, t)} = \frac{A_0 + \sum_{k=1}^{+\infty} A_k \cos(u_k L_z) \exp\left(-D_z \frac{k^2 \pi^2}{L_z^2} t\right)}{A_0 + \sum_{k=1}^{+\infty} A_k \exp\left(-D_z \frac{k^2 \pi^2}{L_z^2} t\right)} \quad (7)$$

It can be noticed that the only unknown parameter involved in the above ratio is the lateral dispersion coefficient D_z . It can be deduced from the minimization of the following objective function:

$$F(D_z) = \frac{1}{N_{mes}} \sum_{n=1}^{N_{mes}} \left[\frac{WC_i^{bot,exp}(t_n)}{WC_i^{top,exp}(t_n)} - \frac{WC_i^{bot,th}(t_n)}{WC_i^{top,th}(t_n)} \right]^2 \quad (8)$$

which is the sum of the least squares between the measured and predicted ratios of water contents using two rings located at the same radius.

On the other hand, for two neighboring rings located at the same (top or bottom) surface, the ratio of predicted water contents is expressed as

$$\frac{WC_{i_2}^{j,th}}{WC_{i_1}^{j,th}} = \frac{S_{i_1}^j \int_{i_2}^{i_2+1} \frac{\rho R(r,t)}{\rho_f \varepsilon_f} r dr}{S_{i_2}^j \int_{i_1}^{i_1+1} \frac{\rho R(r,t)}{\rho_f \varepsilon_f} r dr} = \frac{S_{i_1}^j \int_{i_2}^{i_2+1} \frac{\rho}{\rho_f \varepsilon_f} \left[B_0 + \sum_{k=1}^{\infty} B_k J_0(2\alpha_k r/L_r) \exp(-4D_r \alpha_k^2 t/L_r^2) \right] r dr}{S_{i_2}^j \int_{i_1}^{i_1+1} \frac{\rho}{\rho_f \varepsilon_f} \left[B_0 + \sum_{k=1}^{\infty} B_k J_0(2\alpha_k r/L_r) \exp(-4D_r \alpha_k^2 t/L_r^2) \right] r dr} \quad (9)$$

Here also, it can be noticed that the only unknown parameter involved in the above ratio is the axial dispersion coefficient D_r . It can be deduced from the minimization of the following objective function:

$$F(D_r) = \frac{1}{N_{mes}} \sum_{n=1}^{N_{mes}} \left[\frac{WC_{i_2}^{j,exp}(t_n)}{WC_{i_1}^{j,exp}(t_n)} - \frac{WC_{i_2}^{j,th}(t_n)}{WC_{i_1}^{j,th}(t_n)} \right]^2 \quad (10)$$

which is the sum of the least squares between the measured and predicted ratios of water contents using two neighboring rings located at the same surface.

It is required to adapt the MMT apparatus in order to get as close as possible to the main assumption made to build the process model, i.e. Dirac injection of water. However, in practical conditions, it is impossible to carry out a Dirac water injection. All what we can do is to change the MMT injection conditions in order to get as close as possible to a Dirac function. This could be realized by modifying the injection duration and flowrate. Unfortunately the MMT apparatus does not allow the modification of flowrate, only injection duration is able to be changed. The minimum (non-zero) injection duration allowed by the MMT is one second. But since the injection flowrate is 0.1g/s, only 0.1g of water would be injected in the fabric. In these conditions, only the first ring (top and bottom) will be reached by water. On the other hand, at least two rings should be reached by water in order to identify the lateral dispersion coefficient. Therefore the duration injection was fixed to three seconds.

5. RESULTS AND DISCUSSIONS

It is important to mention that the computation time needed to get water content at different rings is quite large. Instead, we used a CFD software, i.e. Comsol Multiphysics®, to simulate the flash MMT apparatus. The analytical solution developed above was mainly used to validate the numerical solution whose computation time is almost instantaneous.

The lateral and axial dispersion coefficients D_r and D_z are identified using the optimization code *fmincon* available within the Optimization Toolbox of Matlab® environment. This was made possible through a link between Matlab and Comsol Multiphysics. The latter was called within Matlab interface. The 95% confidence interval of each parameter was then computed in order to have an idea about the accuracy of its identification. The results are shown in Table 1 where the corresponding value of the objective function is reported.

Table 1: identification and confidence intervals for flash MMT experiments

Flash MMT	D_r	D_z	$F(\theta^{st})$
Value	2.4×10^{-7}	4.7×10^{-7}	5.5×10^3
95% Confidence interval (%)	4×10^{-2}	4.1	

Since the water is injected on the top surface and diffuses to the bottom surface of the fabric, the axial dispersion coefficient is two times greater than the lateral one. It should be noticed that this is true for the experimental protocol used. The same result is not guaranteed for a different experimental protocol. The computed confidence intervals show that the parameters were determined with a quite high accuracy.

Top of Figure 4 shows the time-varying of the overall water content obtained by adding the water content of each ring on both top and bottom fabric surfaces, whereas the two bottom graphs in figure 4 show the total water content on the top and bottom surfaces respectively. The total top and bottom water content are obtained by summing up the water content of the rings at the top and bottom fabric surface respectively.

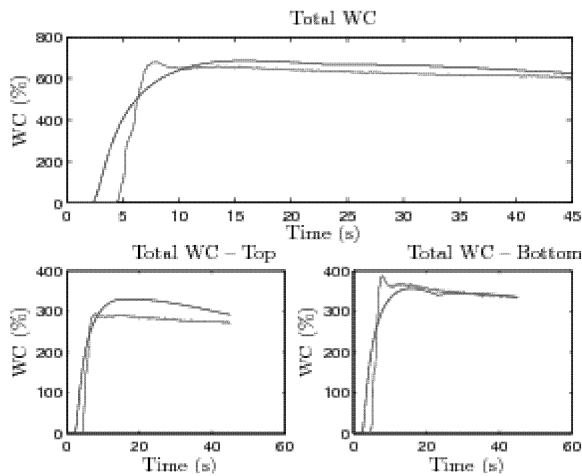


Figure 4: comparison between experimental and theoretical results

On figure 4, it can be seen that after a period of 10 s corresponding to the injection period (injection duration and fabric's response) the measured values of WC are in good agreement with those predicted by the model. This result is quite expected since the injection duration has not been accounted for in the model since it is considered as instantaneous. This is obviously one of the limits of the flash MMT method implemented in this work and deserves to be improved. Figure 4 shows that better results are obtained when total top and bottom WC are considered. However, when WCs of the first two rings are considered, the agreement between the measured and predicted values is quite poor particularly for the second ring (Figure 5).

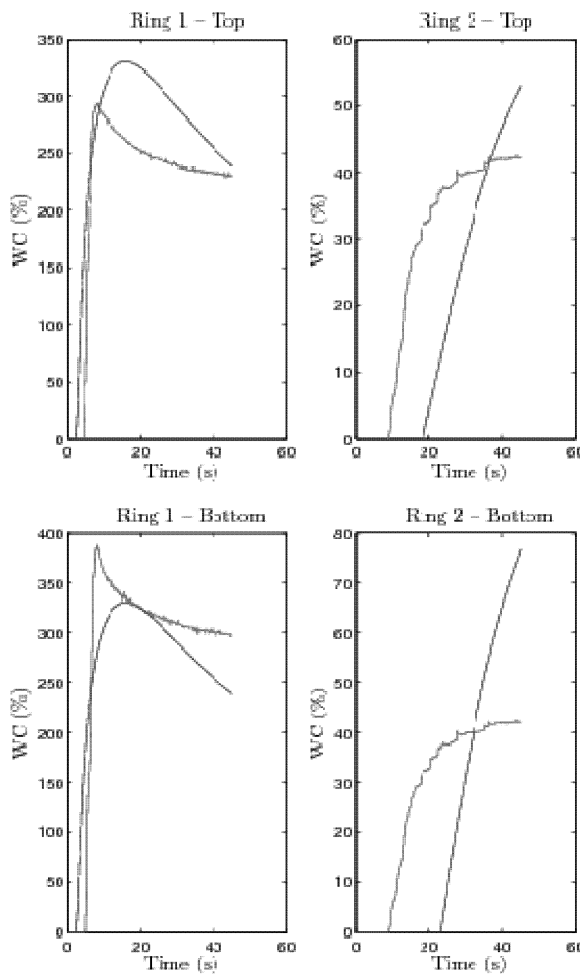


Figure 5: top and bottom results for rings 1 and 2

The main improvement that should be brought to the flash MMT method presented here is to design an injection system which will allow us to carry out a real flash water injection on the top surface of the fabric.

6. CONCLUSIONS

The flash MMT method presented in this work is based on the analogy with the flash method used in thermal engineering for the measurement of thermal diffusivities of materials. The 2D model developed simulates a water injection in the centre of the top surface of a fabric. The computation of the analytical solution of the resulting model is only possible if the initial condition is separable into a product of an axial term and a lateral term. This necessary condition for separation is that the water injection must be described by a Dirac function.

Since the computation time of the analytical solution is quite large particularly in the neighborhood of the initial time, the CFD software Comsol Multiphysics was used to compute the numerical solution which needs only few seconds. The analytical solution was then used only to validate the numerical solution.

The unknown model parameters, *i.e.* axial and lateral dispersion coefficients, were then identified from the available experimental measurements and their accuracy was quantified through confidence intervals.

The model predictions of WC were then compared to the experimental measurements at different levels: overall WC in the fabric, total WC at the top and bottom fabric surfaces and the WC at the two first rings on both top

and bottom surfaces. Although the results are quite interesting and encouraging some improvements are still to be brought to the experimental measurement setup, mainly at the water injection level.

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