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A MATHEMATICAL MODEL FOR WATER REMOVAL IN THE PRESS SECTION OF A PAPER MANUFACTURE INDUSTRY

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A production optimization problem concerned with the water removal in the press section in a paper machine is considered in the present paper. The proposed model seeks to determine the planning of production of paper in order to minimize a cost function that consists of replacement of the felts in the press section, cost of energy to operate the press and cost of energy in the drying section. The proposed model corresponds to a mixed-integer nonlinear programming (MINLP) where the most important decisions in the paper machine are: a) the sequence of paper to produce, or when to produce the paper, b) the need to exchange the felts, and c) when to exchange the felts. Numerical examples are presented to illustrate the performance of the model. This work was developed considering a real case with data from a Brazilian industrial paper plant. However, the optimized sequence calculated by the model was not implemented in practice. These results are just compared with the real sequence used in practice.

Keywords: Optimization, Schedule, GAMS, MINLP.

1. INTRODUCTION

With increased interest in optimizing parts of paper machines in recent years, there has been much work addressing to solve those problems. Most of the works that has been reported have focused in optimizing different parts in a paper producing mill, for example in paper-converting, as in Westerlund *et al* (1980). Also, several authors have studied water removal in the press section, and we can find references using as a basis pressing defined for one nip of vertical flow (Wahlstrom, 1960). Different models and mathematical models for that part of paper machine are reported in Kerekes (1991). The process for the manufacture of paper is common to any plant, initially preparing the mass of the paper, following the formation sector, going to the press section for water removal the web of paper, and finally going to the drying sector for drying the web with hot air. In a paper machine, specifically in the press section, we have two important parts, the felts whose function is to carry the wet web through the press nip (Farouk, 1991) and the nip which is the zone of contact between two rolls, where the water is transferred from the web to the felt. In the press section, which is the object of study of this work, the average life of a felt is different, depending on the type of the felt, but in general it is around 35-45 days.
We consider in this work the press section, which is an important part of the machine, affecting the properties of the paper, as well as having an impact on the final cost of manufacture. Low efficiency of this section causes difficulties like reduction of the tensile strength, increase in the steam consumption in the drying section of the machine, and in many cases, the reduction of the productivity due to reduction of the speed of the machine. It is more economical to squeeze water from the web in the press section than it is to evaporate water in the drying section (Reese, 2006). A reduction of 1% of humidity in the leaf results in a reduction of steam consumption in the order of 4.5%. The web of paper in the wire section is within 99.5% humidity approximately. In the press section we can remove water of the web economically, approximately 20% of water, and in the drying section we can remove 45%. Recently, there has been significant improvement in the operation of presses meeting requirements such as economy in the operational process, increasing the water removal in this section, and at the same time keeping or improving the characteristics of the web of paper. The work described in this paper has the novelty of minimizing the replacement of the felts, by using an optimal sequence of paper production, in order to improve the water removal in press section. The main goal of this work is to optimize the change of felts in a paper machine,

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while determining the optimal sequence of the production for the different types of paper, the felts to be replaced, and the optimal costs of production. The model for optimizing the press section has as an objective to obtain better sequence of production of the reels, aiming at the increasing the water removal for the press section. Comparison of the model with actual industrial values is presented.

2. PRESS SECTION AND FELTS

Motivated by the large replacement of the felts in the press section in the paper industry, this problem has received increased attention in the last few years. The press section of a paper machine is responsible for the removal of approximately 18 to 20% of water in the web of paper, which represents for 12% of the total cost of water removal in the machine. The maximum amount of water that can be removed by pressing the paper before it enters the drying section represents the largest potential savings since this section is responsible for drying and for 78% of the cost for water removal in the web of the paper. Felts are responsible for leading the web, removing the maximum amount of water, smoothing the surface, and removing small marks left in the formation section. The felts are different in size, cost and physical characteristics. The lifetime of the felts depends on the amount of water they can remove during their useful life. Commonly the water removal from pressing is 20 times cheaper than the one from drying (Roux, 2001).

3. PROBLEM STATEMENT

The objective is to minimize the total cost which involves the cost of new felts, the cost of energy in the pressing section, and the cost of steam in the drying section. We assume that we are given a fixed time horizon, which is the sum of the processing times of all reels, with different types and quantities of paper that need to be processed in the same paper machine, and new felts to be used at the start of the production. The decision variables are the processing order of the reels and when to change the felts.

4. DECISION VARIABLES

First of all we divide the programming for "reels in the end of the process", independently of the time of processing each one, which may be different from reel to reel. Initially the amount of reels to be produced is defined. Each set of reels contains a specific type of paper. The total production time T is given by:

$$T = \sum_{i=1}^{N} T_i \tag{01}$$

in which T_i is the processing time for each set of reels. These values are considered given in this model.

In this work the total time did not regard shutdowns due to unexpected events, such as rupture of the paper. A variable for the reels is then defined as follows:

$$x_{ik} = \begin{cases} 1 & \text{for reel } i \text{ processed in time interval } k \\ 0 & \text{otherwise} \end{cases}$$

where each time interval does not necessarily have the same time length, since it depends on the processing time of each reel. The following constraints then hold: each reel (or set of reels) can only be processed in one interval; each interval can only process one reel (or set of reels). These assignment constraints are given by:

$$\sum_{k=1}^{N} x_{ik} = 1 \qquad i = 1, \ldots, N \tag{02}$$

$$\sum_{i=1}^{N} x_{ik} = 1 \qquad k = 1, \ldots, N \tag{03}$$

Therefore, the time for each interval is given by:

$$t_k = \sum_{i=1}^{N} T_i \cdot x_{ik} \tag{04}$$

A binary variable y_{jk} is defined to represent the potential replacement of a felt in position *j* (press *j*) at the beginning of interval *k*. The binary variable for replacement of felts is defined as follows:

$$y_{jk} = \begin{cases} 1 & \text{if the felt in position } j \text{ was changed at the begining of time interval } k \\ 0 & \text{otherwise} \end{cases}$$

In this work, the paper machine has four press positions, and so four felts. Considering that at the beginning of the production all felts are new, at the beginning of the first time interval, $k = 1$, we have:

$$y_{j1} = 1 \qquad j = 1,2,3,4 \tag{05}$$

In the process, water is removed from the paper in the press section and in the drying section, according to *Figure 1.*

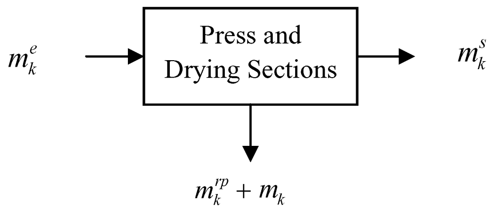

Figure 1: Water balance.

The mass balance of water in time interval *k* is given by:

$$m_k + m_k^{rp} = m_k^e - m_k^s \tag{06}$$

$$m_k^e = \sum_{i=1}^{N} \left[U_i^e \cdot A_i\right] \cdot x_{ik} \tag{07}$$

$$m_k^s = \sum_{i=1}^{N} \left[U_i^s \cdot A_i\right] \cdot x_{ik} \tag{08}$$

$$A_i = \left(v_p \cdot T_i\right) \cdot L_p \tag{09}$$

In this water balance, m_k is the mass of evaporated water in time interval k, m_k^{rp} is the amount of water removed from pressing, m_k^e is the amount of water in the entrance of the press section, and m_k^s is the amount of water at the end of the drying section. The values of the area of the processed reels (A_i) and the humidity of the paper by unit of area at the entrance and exit of the process (U_i^e and U_i^s) are considered as given data in this model.

5. EQUATIONS FOR THE DEWATERING IN THE PRESSING SECTION

In this section we present the mathematical model for dewatering in the press section. The mass of water that is removed by pressing during a certain interval t_k of time is:

$$m_k^{rp} = m_k^e - m_k^{sp} = \int_0^{t_k} \left(\dot{m}_k^e - \dot{m}_k^{sp}\right) \cdot dt \tag{10}$$

The water outflow in the entrance of the pressing process is given by:

$$\dot{m}_k^e = v_p \cdot L_p \cdot e_p \cdot \varepsilon_{H2O}^e \cdot \rho_{H2O} \tag{11}$$

As in the entrance of the pressing, for one given reel, these values remain constant, we then have:

$$m_k^e = \int_0^{t_k} \left(v_p \cdot L_p \cdot e_p \cdot \varepsilon_{H2O}^e \cdot \rho_{H2O}\right) \cdot dt = v_p \cdot L_p \cdot e_p \cdot \varepsilon_{H2O}^e \cdot \rho_{H2O} \cdot t_k \tag{12}$$

The water outflow inside the paper that leaves the press section in the interval k is given by:

$$\dot{m}_k^{sp} = v_p \cdot L_p \cdot e_p \cdot \varepsilon_{H2O}^{s} \cdot \rho_{H2O} \qquad (13)$$

The general balance of the volumetric fraction (cellulose, water and air) in each reel is given by:

$$\varepsilon_{\text{cel}} + \varepsilon_{\text{H2O}} + \varepsilon_{\text{air}} = 1 \qquad (14)$$

The porosity of the paper is the sum of the volumetric fractions of water and air:

$$\phi_p = \varepsilon_{\text{H2O}} + \varepsilon_{\text{air}} \qquad (15)$$

These values change with the pressing. Hence it is necessary to relate the volumetric fraction with the characteristics of the press and the felt used. On the other hand, it is considered that all of the reel properties (thickness, width) have the same values before and after the pressing (but not during the pressing; the thickness of paper change with the pressing, but it later returns to the previous value).
For each felt in each press, the general balance of volumetric fractions (fiber, water and air) is given by:

$$\xi_{\text{fiber}} + \xi_{\text{H2O}} + \xi_{\text{air}} = 1 \qquad (16)$$

The porosity of the felt is the sum of the volumetric fractions of water and air:

$$\varphi = \xi_{\text{H2O}} + \xi_{\text{air}} \qquad (17)$$

It was found in this work that, from experimental analysis of industrial cases, the porosity of the felts decreased with the running time. The values of porosity were adequately fitted by an expression as follows, for each felt j:

$$\varphi_j = \varphi_{0,j} \cdot \exp(-\alpha_j \cdot t) \qquad (18)$$

The mass fiber outflow of felt in each press machine j is given by:

$$\dot{m}_{\text{fiber}}^{j} = v_f \cdot L_f \cdot e_f \cdot \xi_{\text{fiber}} \cdot \rho_{\text{fiber}} \qquad (19)$$

Each felt has its proper characteristics, but for each felt the outflow fiber mass that enters in the point of contact with the paper (nip) is equal to the outflow fiber mass that leaves it, since it is a closed circuit. Moreover, the total fiber mass in each felt is the same at the beginning to the end of its useful life. In this way we have:

$$\xi_{\text{fiber}} \cdot e_f = \text{constant} \qquad (20)$$

Moreover, the relation between volumetric fraction of fiber and porosity in the felt is given by:

$$\xi_{\text{fiber},j} = 1 - \varphi_j = 1 - \varphi_{0,j} \cdot \exp(-\alpha_j \cdot t) \qquad (21)$$

The thickness of each felt them varies with the time as follows:

$$e_{f,j} = e_{f,j}^{0} \cdot \frac{(1 - \varphi_{0,j})}{(1 - \varphi_{0,j} \cdot \exp(-\alpha_j \cdot t))} \qquad (22)$$

In order to find the relation between the amount of water in the felt and the amount of water in the paper, a water balance is done, so that the sum of the water outflows (in the paper and in the felt) that enters the nip is equal to the sum of the water outflows that leaves it.
Additionally, using the condition of equilibrium (with an equilibrium constant, κ), relating the ratios of volumetric fraction of water/porosity, in the paper and in the felt, we have that:

$$\frac{\xi_{\text{H2O}}}{\xi_{\text{H2O}} + \xi_{\text{air}}} = \kappa \cdot \frac{\varepsilon_{\text{H2O}}}{\varepsilon_{\text{H2O}} + \varepsilon_{\text{air}}} \qquad (23)$$

and using the removal of water from the felt by the vacuum system of the pressing machine (assuming that the vacuum removes a constant fraction of water, $1-\beta$, from the total porosity of the felt), we arrive at:

$$\varepsilon_{\text{H2O}}^{j} = \varepsilon_{\text{H2O}}^{j-1} \cdot \frac{1}{1 + \Lambda_{jk}} \qquad j = 1,2,3,4 \qquad (24)$$

The extraction factor Λ_{jk} relates parameters of the felt and the processed reel:

$$\Lambda_{jk} = \frac{v_f \cdot e_{f,j} \cdot \varphi_j}{v_p \cdot e_p \cdot \phi_p} \cdot \kappa \cdot (1 - \beta) \qquad (25)$$

Using Equations (18) and (22), it follows that:

$$\Lambda_{jk} = \Lambda_{jk}^{0} \cdot \frac{(1 - \varphi_{0,j}) \cdot \exp(-\alpha_j \cdot t)}{[1 - \varphi_{0,j} \cdot \exp(-\alpha_j \cdot t)]} \qquad (26)$$

$$\Lambda_{jk}^{0} = \sum_{i=1}^{N} \lambda_{ij} \cdot x_{ik} \tag{27}$$

$$\lambda_{ij} = \frac{v_f \cdot e_{f,j}^{0} \cdot \varphi_{0,j}}{v_p \cdot e_p \cdot \phi_p} \cdot \kappa \cdot (1-\beta) \tag{28}$$

in which Λ_{jk}^{0} is the extraction factor of a new felt j for reel i processed at interval k, λ_{ij} is the extraction factor of a new felt j for each reel i, α_j is the coefficient of reduction of the useful life of the felt, $\xi_{0,j}$ is the volumetric fraction of fiber in a new felt, and $\varphi_{0,j}$ is the porosity of a new felt j ($\varphi_{0,j} = 1 - \xi_{0,j}$).

Observe that λ_{ij} is a constant that depends on each felt j and each processed reel i, grouping together properties of each paper reel i (e_p, ϕ_p), each new felt j ($e_{f,j}^{0}$, $\varphi_{0,j}$) and both (κ), as well as operating conditions of the pressing machine (v_f, v_p, β). However, it is considered that both felt and paper had the same velocity, $v_f = v_p$, since felt and paper are pressed together in the nip, without slipping.

The parameters λ_{ij} have to be determined experimentally from the ratio of exit and inlet volumetric fraction of water at each press machine, for example from the initial operation of the pressing (observe that for $t = 0$ we have that $\Lambda_{jk} = \Lambda_{jk}^{0}$). It is not necessary to determine the other parameters individually (κ, β, etc), since all of them are grouped into one parameter (λ_{ij}) for each reel i and each felt j.

In this work it is considered the case of four pressing machines, so that, for $k = 1, \ldots, N$:

$$\varepsilon_{\mathrm{H2O}}^{j} = \varepsilon_{\mathrm{H2O}}^{j-1} \cdot \frac{1}{1+\Lambda_{jk}} \qquad j = 1,2,3,4 \tag{29}$$

Therefore:

$$\varepsilon_{\mathrm{H2O}}^{s} = \varepsilon_{\mathrm{H2O}}^{e} \cdot \frac{1}{1+\Lambda_{1k}} \cdot \frac{1}{1+\Lambda_{2k}} \cdot \frac{1}{1+\Lambda_{3k}} \cdot \frac{1}{1+\Lambda_{4k}} \tag{30}$$

From Equations (11) and (13), and using (30), we then have:

$$\dot{m}_k^{sp} = \dot{m}_k^{e} \cdot \prod_{j=1}^{4} \frac{1}{[1+\Lambda_{jk}]} \tag{31}$$

In order to apply Equation (10) it is necessary to perform a mathematical integration over Equation (31). Since the time of processing a reel is much smaller than the time of the useful life of a felt, a numerical integration in each time interval can be done using the average point in the interval:

$$\int_a^b f(t) \cdot dt \cong f\left(\frac{a+b}{2}\right) \cdot (b-a) \tag{32}$$

In this way, from (12), (31) and (32), the equation for the water mass that leaves the sector of pressing in the time t_k in interval k is given by:

$$m_k^{sp} = m_k^{e} \cdot \prod_{j=1}^{4} \frac{1}{[1+\overline{\Lambda}_{jk}]} \tag{33}$$

where the extraction factor $\overline{\Lambda}_{jk}$ depends on the time of the felt, as follows:

$$\overline{\Lambda}_{jk} = \Lambda_{jk}^{0} \cdot \frac{(1-\varphi_{0,j}) \cdot \exp(-\alpha_j \cdot \overline{\tau}_{jk})}{[1-\varphi_{0,j} \cdot \exp(-\alpha_j \cdot \overline{\tau}_{jk})]} \tag{34}$$

$$\overline{\tau}_{jk} = \theta_{jk} + 0.5 \cdot t_k \tag{35}$$

Here the extraction factor $\overline{\Lambda}_{jk}$ depends on the average time utilization of the felt $\bar{\tau}_{jk}$, according to Equation (35). In Equation (34), it is possible to observe that when $\bar{\tau}_{jk} \to \infty$, then $\overline{\Lambda}_{jk} \to 0$, which in Equation (33) implies that $m_k^{sp} \to m_k^e$, meaning that the felt is not able to remove more water from the paper.

The average time of the felt running in the machine, $\bar{\tau}_{jk}$, is equal to the time of felt *j* at the beginning of interval k, θ_{jk}, plus the average length of this time interval, t_k, as given by Equation (35). The value of θ_{jk} depends on whether the felt is the same one running in the machine ($y_{jk} = 0$), or if it is a new felt after replacement ($y_{jk} = 1$). The variable θ_{jk} measures the running time of the felt since the last replacement. Therefore, for a new felt $\theta_{jk} = 0$. The running times of operation at the felt in the press in the interval *k* are then given by:

$$\tau_{jk} = \theta_{jk} + t_k \qquad k = 1,\ldots,N \tag{36}$$

$$\theta_{jk} = \tau_{j,k-1} \cdot (1 - y_{jk}) \qquad k = 2,\ldots,N \tag{37}$$

$$\theta_{j1} = 0 \tag{38}$$

Equation (37) is not in a suitable form to use in a model for optimization, since it contains the product of integer and continuous variables, and so it can be replaced by the following equivalent constraints:

$$\theta_{jk} = \tau_{j,k-1}^{(2)} \tag{39}$$

$$\tau_{j,k-1} = \tau_{j,k-1}^{(1)} + \tau_{j,k-1}^{(2)} \tag{40}$$

$$0 \le \tau_{j,k-1}^{(1)} \le M_1 \cdot y_{jk} \tag{41}$$

$$0 \le \tau_{j,k-1}^{(2)} \le M_1 \cdot (1 - y_{jk}) \tag{42}$$

in which M_1 is parameter that is sufficiently large so as to include all possible values of all previous times.

6. OPTIMIZATION MODEL

In this model, the objective function includes the cost of new felts, the cost of energy in the press section, and the cost of energy in the drying section. Since the total time is fixed, the objective function uses the mass of water removed from the reels in the drying section at each interval k, m_k, and not the mass flow rate, $\dot{m}_k$. The objective function is then given by:

$$\min z = \sum_{k=1}^{N}\sum_{j=1}^{4} CF_j \cdot y_{jk} + \sum_{k=1}^{N}\sum_{i=1}^{N} CP_{ik} \cdot x_{ik} + \sum_{k=1}^{N} cs_k \cdot m_k \tag{43}$$

In this model, the time required to change a felt was neglected because it typically represents only 1%-3% of the total time. The operating cost of changing a felt can be included in the cost of purchasing a new one in the value of the parameter CF_j.

The unitary costs CF_j and CP_{ik} are parameters in the optimization (given values) and so the first and second terms in the right hand side of Equation (43) are linear. However, the third term is nonlinear and far more complicated. The cost of energy for drying in the interval *k* is given by:

$$cs_k = \sum_{i=1}^{N} CS_{ik} \cdot x_{ik} \tag{44}$$

The water removed in the drying section can be found from Equation (06):

$$m_k = m_k^e - m_k^s - m_k^{rp} \tag{45}$$

The water removed in the press section is given by the difference between the mass of water inside the paper that enters the press section and the water left in the paper after leaving the press section (and before entering the drying section):

$$m_k^{rp} = m_k^e - m_k^{sp} \tag{46}$$

These two equations result in:

$$m_k = m_k^{sp} - m_k^s \tag{47}$$

Following the model of water removal in the pressing developed in the previous section, the amount of water at the end of the press section, at interval *k*, is given by Equation (33).

The model given by objective function (43), restrictions (02)–(08), (27), (33)–(35), and (39)–(42) corresponds to a mixed-integer nonlinear program (MINLP) that involves binary and continuous variables mixed in a complicated way, such as in Equations (27) and (34), where integer variables multiply terms with continuous variables. To obtain a more tractable model, the equations are changed to regard separately linear terms with integer variables and nonlinear terms with continuous variables.

First, when Equation (47) is replaced in the term $cs_k \cdot m_k$ in the objective function, the second part can be rewritten as:

$$cs_k \cdot m_k^s = \left(\sum_{i=1}^{N} CS_{ik} \cdot x_{ik}\right) \cdot \left(\sum_{i=1}^{N} U_i^s \cdot A_i \cdot x_{ik}\right) = \sum_{i=1}^{N} CS_{ik} \cdot U_i^s \cdot A_i \cdot x_{ik} \tag{48}$$

since only one of the x_{ik} is equal to 1 in each summation.

The first part can be rewritten as:

$$\sum_{k=1}^{N} cs_k \cdot m_k^{sp} = \sum_{i=1}^{N} \sum_{k=1}^{N} CS_{ik} \cdot ma_{ik} \tag{49}$$

$$m_k^{sp} = \sum_{i=1}^{N} ma_{ik} \tag{50}$$

in which $ma_{ik} = 0$ if $x_{ik} = 0$. Equation (43) then becomes:

$$\min z = \sum_{k=1}^{N} \sum_{j=1}^{4} CF_j \cdot y_{jk} + \sum_{k=1}^{N} \sum_{i=1}^{N} CP_{ik} \cdot x_{ik} + \sum_{k=1}^{N} \sum_{i=1}^{N} CS_{ik} \cdot ma_{ik} - \sum_{k=1}^{N} \sum_{i=1}^{N} CS_{ik} \cdot U_i^s \cdot A_i \cdot x_{ik} \tag{51}$$

Second, Equations (33) and (34) for water extraction in pressing can be rewritten as:

$$\ln\left(m_k^{sp}\right) = \ln\left(m_k^e\right) - \sum_{j=1}^{4} \ln\left(1 + \overline{\Lambda}_{jk}\right) \tag{52}$$

$$\ln \overline{\Lambda}_{jk} = \ln \Lambda_{jk}^0 - \alpha_j \cdot \overline{\tau}_{jk} + \ln\left(1 - \varphi_{0,j}\right) - \ln\left(1 - \varphi_{0,j} \cdot \exp\left(-\alpha_j \cdot \overline{\tau}_{jk}\right)\right) \tag{53}$$

Defining the auxiliary variables:

$$v_k = \ln\left(m_k^{sp}\right) \tag{54}$$

$$u_{jk} = \ln \overline{\Lambda}_{jk} \tag{55}$$

the following constraints then result in an equivalent model:

$$v_k = \sum_{i=1}^{N} \left(\ln\left(U_i^e \cdot A_i\right)\right) \cdot x_{ik} - \sum_{j=1}^{4} \ln\left(1 + \exp\left(u_{jk}\right)\right) \tag{56}$$

$$u_{jk} = \sum_{i=1}^{N} \left(\ln \lambda_{ij}\right) \cdot x_{ik} - \alpha_j \cdot \overline{\tau}_{jk} + \ln\left(1 - \varphi_{0,j}\right) - \ln\left(1 - \varphi_{0,j} \cdot \exp(-\alpha_j \cdot \overline{\tau}_{jk})\right) \tag{57}$$

$$ma_{ik} = m_{ik}^{(1)} \tag{58}$$

$$m_{ik}^{(1)} + m_{ik}^{(2)} = \exp\left(v_k\right) \tag{59}$$

$$0 \le m_{ik}^{(1)} \le M_2 \cdot x_{ik} \tag{60}$$

$$0 \le m_{ik}^{(2)} \le M_2 \cdot \left(1 - x_{ik}\right) \tag{61}$$

in which M_2 is a parameter that is large enough to absorb the maximum amount of water that can be removed from reel i . Note that the amount of water m_k^{sp} in the exit of the press section cannot be less than the amount of water m_k^s settled for the end of the drying process. Therefore, the additional constraint must be added:

$$v_k \geq \sum_{i=1}^{N} \left(\ln\left(U_i^s \cdot A_i\right)\right) \cdot x_{ik} \qquad (62)$$

The MINLP model is then formulated as the minimization of the objective function given by Equation (51), subject to the constraints given by (02)–(05), (35), (39)–(42), and (56)–(62). In this model, the variables are: x_{ik} , y_{jk} , ma_{ik} , $m_{ik}^{(1)}$, $m_{ik}^{(2)}$, v_k , u_{jk} , $\bar{\tau}_{jk}$, τ_{jk} , $\tau_{jk}^{(1)}$, $\tau_{jk}^{(2)}$, θ_{jk} , t_k . The parameters (data that must be given) are: T_i , A_i , U_i^e , U_i^s , λ_{ij} , $\varphi_{0,j}$, α_j , CF_j , CP_{ik} , CS_{ik} , M_1 , M_2 .

7. RESULTS

To illustrate the application and computational effectiveness of the proposed MINLP model, we show the comparison of the results of the model with actual data from industry. There were 22 cases of felt exchange considered in this work. The values of the data are from industrial practice in one year of production, and include demands and length of time horizon. Each case was solved with the proposed MINLP model. We first solved the model with a fixed sequence of the reel production (the same one used in the real industrial case), and next we solved the same model but allowed the selection of the optimal sequence. The model was implemented in GAMS, and solved with the MINLP solver SBB using CONOPT3 and CPLEX10 on an Intel 3.2 GHz machine. The size of the MINLP was 564 single equations, 548 single variables, 92 discrete variables, and typically required 206 min of CPU time in order to find the global optimum. The results for water removal, felt lifetime, felt exchange and reel production are presented in the next section.

7.1. Water removal

The challenge faced by the proposed mathematical model was the reduction in the replacement of felts through the selection at an optimal sequence of the reel production, in this way increasing the water removal in the press section and thereby reducing the cost of energy in the drying sector. This represents an annual increase of the order of 5% in the water removal, which is a significant result *(Figure 2)*.

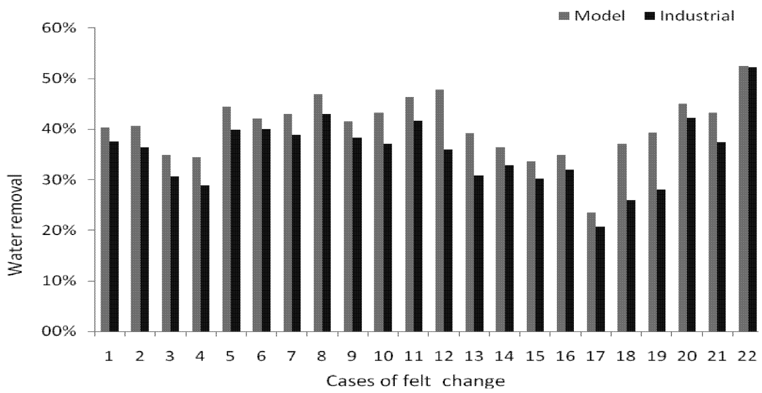

Figure 2: Water removal in the press section.

7.2. Felt lifetime

The lifetime of the felts can also have a significant impact on the cost. The main issue in the replacement of the felts is to find a point where they can stay in the machine without losing much their capacity of absorption.

Through the proposed model, we obtained an average increase of 15% in the potential useful life of the felt, using the optimal sequence predicted from the mathematical model. Hence, besides the increase of the water removal, we can have an increase of the useful life of the felt.

7.3. Felts replacement

The replacement of felts is one of the aspects of the paper machine where we have a higher cost of exchange, time loss and cost of purchase, beyond the “lost" time at production. Replacing the felts requires stopping the machine for approximately 1 hour and 30 minutes, and after this, additional time is required to start-up the machine for production. Therefore, with a better sequence in the production of the reel, we have improved conditions to minimize the exchanges carried through the machine of paper, and with this a significant increase in the production of the reels. We obtained a reduction of 46% in average in the reduction of exchange of the felts, which results in an increase of lifetime of the felts, because we have fewer replacements of felts in the machine.

7.4. Cost reduction

From the results we can also see a clear total cost reduction in the operation of the plant, the total cost was reduced by 4% (Table 1). Some costs have not changed, like the costs in the press reduction, but the cost reduction in the drying section and the lower cost in the replacement of felts were the main reason for the total cost reduction.

Table 1: Costs for the optimized problem and the industrial case study

Cost	Model	Industrial
Felts	US$ 5,178,960.03	US$ 5,833,774.48
Press	US$ 16,420.31	US$ 16,420.31
Drying	US$ 23,538,536.13	US$ 24,082,059.66
Total	US$ 28,733,917.03	US$ 29,932,254.45

8. CONCLUSIONS

The optimization to reduce costs and replacement of the felts were considered in the present paper. An MINLP model has been proposed to determine the replacement of felts in the press section in a paper machine. It was shown that the problem could be efficiently solved yielding better results. The improvement obtained for a case study with 22 individual cases resulted in a 5% increase in water removal, 15% increase in potential felt lifetime and 4% reduction in the total cost.

9. NOMENCLATURE

Sets

i reel
j felt or press
k time interval

Variables

e_f felt thickness
m_k amount of water removed in the drying section
m_k^e amount of water in the entrance of the press section in interval k
m_k^s amount of water in the exit of the dry section in interval k
m_k^{rp} amount of water removed for the pressing in interval k
m_k^{sp} amount of water in the end of pressing in interval k
ma_{ik} mass of water in the exit of pressing of the reel i in interval k

t_k time of interval k

u_{jk} auxiliary variable for the extraction coefficient of water in pressing

v_k auxiliary variable for water in the exit of the pressing in interval k

x_{ik} reel i processed in interval k

y_{jk} felt in press j exchanged in initial time k

τ_{jk} time of life of the felt j in interval k

$\overline{\tau}_{jk}$ average of the time of the felt j in interval k

θ_{jk} initial time of the felt j in interval k

Parameters

A_i total area of the processed reels

CS_{ik} steam cost in the drying section (US$/mass of water) in interval k

CF_j cost of buy/change (US$/felt)

CP_{ik} energy cost expense in the press section of i (US$/reel) in the interval k

e_p paper thickness

L_p length of paper in the reel

M_1 big number (time)

M_2 big number (mass of water)

T_i time to process reel i

U_i^e humidity of the paper in the entrance of the press section (mass of water/area)

U_i^s humidity of the paper in the exit of the drying sector (mass of water/area)

v_p paper velocity in the machine

α_j coefficient of reduction of the useful life of the felt

λ_{ij} extraction factor of the new felt j with reel i paper

$\xi_{0,j}$ volumetric fraction of fiber in the new felt j

$\varphi_{0,j}$ porosity of the new felt j

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