CHEMICAL ENGINEERING TRANSACTIONS

VOL. 70, 2018

A publication of AIDIC

The Italian Association of Chemical Engineering
Online at www.aidic.it/cet

Guest Editors: Timothy G. Walmsley, Petar S. Varbanov, Rongxin Su, Jiří J. Klemeš

ISBN 978-88-95608-67-9; **ISSN** 2283-9216

DOI: 10.3303/CET1870344

Mathematical Model of Plate Heat Exchanger for Utilisation of Waste Heat from Condensable Gaseous Streams

Petro O. Kapustenko[a,*], Oleksiy V. Demirskiy[a], Leonid L. Tovazhnyanskyy[a], Jiří J. Klemeš[b], Oleksandr A. Vasilenko[c], Sergiy K. Kusakov[c], Olga P. Arsenyeva[d], Alisher E. Khusanov[e]

[a]National Technical University “Kharkiv Polytechnic Institute”, Dep. ITPA, 21 Frunze st., 61002 Kharkiv, Ukraine
[b]Sustainable Process Integration Laboratory – SPIL, NETME Centre, Faculty of Mechanical Engineering, Brno University of Technology - VUT Brno, Technická 2896/2, 616 69 Brno, Czech Republic
[c]AO Spivdruzhnist-T LLC, 2 Kaplunovskiy per., 19, 61002 Kharkiv, Ukraine
[d]University of Paderborn, Chair of Fluid Process Engineering, Paderborn, Germany
[e]M.Auezov South Kazakhstan State University, Tauke hana Avenue, 160012 Shymkent, Kazakhstan
sodrut@gmail.com

The mathematical model of vapour condensation from the mixture with noncondensing gas in Plate Heat Exchanger (PHE) channels is presented. The model accounts for the change of process parameters along the heat transfer surface and local features of heat and mass transfer processes in PHEs channels with plates of different corrugations geometry. It consists of the system of ordinary differential equations with considerably nonlinear right parts. The software for its solution by finite difference method is developed. The validity of the model is confirmed by comparison with the experiment for steam-air mixture condensation in a PHE channel sample.

1. Introduction

A considerable amount of energy consumed in the industry is lost as waste heat to the environment. The substantial quantities of this are latent heat of condensable vapours leaving out in the mixture of gases exhausting after burning of fuels, after processes of drying different materials and other exhaust gases from the industry (Arsenyeva et al., 2016). Many processes such as volatile organic compounds recovery or ammonia condensation from synthesis gas after reaction also involve heat that is wasted with cooling streams. For efficient utilization of the heat in all such cases, there is a need for effective heat transfer equipment which is capable to perform required duty ineffective and economical way. Plate heat exchanger (PHE), as one of the modern efficient types of compact heat exchangers (Klemeš et al., 2015), is a good choice for this purpose. The compact construction consisting from a pack of corrugated plates, stamped from a thin metal sheet, and intensified heat and mass transfer in their channels allows a significant reduction of heat transfer area, size and weight for the same duty compare to conventional shell and tube heat exchangers.

For the successful economically grounded application of PHEs in processes of both sensible and latent heat recovery from gas mixtures the reliable methods for their modelling and optimal design are required. The process of condensation in the presence of noncondensing gases is substantially complicated by the additional resistance to transport of condensing substance to the surface of condensation, where the already condensed liquid is also creating thermal resistance for heat withdrawal to cooling media. The review of publications studying the processes of heat and mass transfer in tubular condensers in the presence of noncondensing gas is presented by Huang et al. (2015). The complexity of the process and substantial change of all its parameters along heat transfer surface requires the use of mathematical modelling and simulation for its adequate description (Kharangate and Mudawar, 2017). It should be supplemented by the experimental research of the process with accounting for its local parameters variation especially when enhanced heat transfer surfaces are considered (Famileh and Esfahani, 2017). The process is even more complicated in PHE channels due to the

Please cite this article as: Kapustenko P.O., Demirskiy O.V., Tovazhnyanskyy L.L., Klemes J.J., Vasilenko O.A., Kusakov S.K., Arsenyeva O.P., Khusanov A.E., 2018, Mathematical model of plate heat exchanger for utilisation of waste heat from condensable gaseous streams , Chemical Engineering Transactions, 70, 2059-2064 DOI:10.3303/CET1870344

complex structure of fluid flow. The influence of pressure drop during condensation in narrow passages can be also much more important than for smooth tubes (Wang et al., 1999). The considerable influence of plate corrugation geometry has led to development of a number of empirical correlations applicable for specific investigated commercial plates in the limited range of process conditions (Eldeeb et al., 2016). The adequate modelling of condensation processes in PHEs must account the change of local parameters along heat transfer surface with the use of reliable correlations for heat and mass transfer (Tovazhnyansky et al., 2004).
The objective of this paper is to improve accuracy of PHE design by development of the mathematical model of condensation with the presence of noncondensing gas accounting for the variation of local process parameters along heat transfer surface. The generalised correlations for friction factor and coefficients of heat and mass transfer are used. The model is validated with experimental results for condensation of gas-air mixture in a sample of PHE channel.

2. Mathematical model development

The mathematical model of PHE for condensation of vapour in the presence of non-condensable gases is developed based on the following assumptions:

1. The relations between process parameters on small parts of channel length are the same for any length of such part.
2. For conditions of phase change in flow core, the equilibrium concentration of vapour is assumed. In such conditions, the convective heat transfer is responsible besides the change of gaseous phase temperature also for condensation in the mainstream.
3. The condensate is wetting metal surface and film condensation is taking place.
4. The gas-vapour mixture is considered as a mixture of ideal gases.
5. The heat exchanger has one pass for both streams.
6. All channels in PHE are working under the same conditions.
7. The heat losses into the environment can be neglected.

Based on heat and material balances the process of vapour condensation from its mixture with noncondensing gas in case of countercurrent flow with cooling stream can be described by the following system of differential equations:

$$\frac{dG_v}{dx} = -\Pi \cdot j_v \tag{1}$$

$$\frac{dG_L}{dx} = \Pi \cdot j_v \tag{2}$$

$$\frac{dt_{cl}}{dx} = -\Pi \cdot q \tag{3}$$

$$\frac{dt_L}{dx} + \frac{dt_{mx}}{dx} + \Pi \cdot j_v \cdot r_v = \Pi \cdot q \tag{4}$$

$$\frac{dt_{mx}}{dx} = \Pi \cdot h_{cv} \cdot \left(t_{mx} - t_f\right) \tag{5}$$

$$\frac{dP_{mx}}{dx} = \frac{1}{d_e} \cdot \varsigma \cdot \frac{\rho_{mx} \cdot W_{mx}^2}{2} \cdot (1 + 2.9 \cdot X_{tt}^{0.46}) - \frac{d}{dx}\left(\frac{\rho_{mx} \cdot W_{mx}^2}{2}\right) - \frac{d}{dx}\left(\frac{\rho_{mx} \cdot g \cdot x}{2}\right) \tag{6}$$

Here G_v and G_L are the mass flow rates of condensing vapour and liquid condensate in one channel, kg/s; t_{cl}, t_L, t_{mx} and t_f are temperatures of cooling media, liquid phase, gas-vapour mixture and condensate film surface, respectively, °C; P_{mx} is the pressure of vapour-gas mixture, Pa; q is specific heat flux through heat transfer surface, W/m^2; j_v is specific mass flux of condensing vapour to the surface of condensation, kg/m; r_v is latent heat of vapour phase change, J/kg; h_{cv} is coefficient of convective heat transfer in gaseous phase, W/(m^{2}K); ρ_{mx} is the density of vapour-gas mixture, kg/m^3; W_{mx} is the velocity of vapour-gas mixture, m/s; Π is perimeter of channel, m; x is coordinate along channel length, m; d_e is equivalent diameter of the channel, m; ς is the friction factor for gas flow in channel; g is acceleration of gravity, m/s^2; X_{tt} is Lockhart-Martinelly parameter for turbulent regime of both phases movement, introduced according to paper of Tovazhnyansky et al. (2004) and determined by the following equation:

$$X_{tt} = \sqrt{dP_L / dP_G} \tag{7}$$

Beside differential equations, there are algebraic equations linking different variables. For the relations between the vapour saturation pressure and temperature $P_{sat} = P_{sat}(t)$ and $t_{sat} = t_{sat}(P)$ are taken according to literature data. The temperature of the condensate film surface is:

$$t_f = t_{cl} + q \cdot \left(\frac{1}{h_{cl}} + \frac{\delta_{wl}}{\lambda_{wl}} + \frac{1}{h_L} \right) \tag{8}$$

where δ_{wl} is the thickness of plate wall, m; λ_{wl} is the thermal conductivity of plate metal, W/(m K); h_{cl} is film heat transfer coefficient from to cooling media, W/(m^2 K); h_L is film heat transfer coefficient from a liquid film to the wall, W/(m^2 K). h_L is determined by the following equation presented by Arsenyeva et al. (2011):

$$h_L = \frac{\lambda_L}{d_e} Nu^* \cdot \left[1 + x_{tp} \cdot \left(\frac{\rho_L}{\rho_{mx}} \right) \right]^{0.48} \tag{9}$$

where λ_L is thermal conductivity of the condensed liquid, W/(m K); ρ_L is liquid density, kg/m^3, x_{tp} is mass vapour quality; Nu^* is the Nusselt number for liquid flowing in the channel with a total flow rate of two phases.
The partial pressure of vapour at liquid film surface is determined for saturation conditions:

$$P_{vf} = Psat(t_f) \tag{10}$$

The mass fraction of vapour at liquid film surface:

$$y_{vf} = \left(\frac{P_{mx} - P_{vf}}{P_{vf}} \cdot \frac{M_g}{M_v} + 1 \right)^{-1} \tag{11}$$

where M_v and M_g are molar masses of noncondensing gas and vapour, respectively, kg/kmol, (for steam M_v = 18.015 kg/kmol, for air M_g = 28.96 kg/kmol).
The mass fraction of vapour at the flow bulk:

$$y_{vb} = \frac{G_v}{G_v + G_g} \tag{12}$$

The partial pressure of the vapour at the flow bulk:

$$P_{vb} = \frac{P_{mx}}{\frac{M_v}{M_g} \cdot \frac{G_g}{G_v} + 1} \tag{13}$$

Here G_g is noncondensing gas flow rate in the channel, kg/s.
The saturation temperature at flow bulk:

$$t_{satb} = tsat(P_{vb}) \tag{14}$$

With the solution of Eq(5) it should be satisfied the condition:

$$t_{satb} = tsat(P_{vb}) \tag{15}$$

The specific mass flux of condensing vapour to the surface of condensation:

$$j_v = \beta_D \cdot \left(y_{vb} \cdot \rho_{mx} - y_{vf} \cdot \rho_{mxf} \right) \tag{16}$$

The coefficients of heat transfer h_{cv} and mass transfer β_D are determined using heat and mass transfer analogy with accounting for the influence of transverse mass flux in PHE channels according to Tovazhnyansky et al. (1989).

$$h_{cv} = \left(\lambda_{mx} / d_e\right) \cdot \Psi_H \cdot Nu_0 \tag{17}$$

$$\beta_D = \left(D_D / d_e\right) \cdot \Psi_D \cdot Nu_{D0} \tag{18}$$

where λ_{mx} is the thermal conductivity of gas-vapour mixture, W/(m K); D_D is the coefficient of diffusivity, m^2/s. The relative factors of heat and mass transfer:

$$\Psi_{H(D)} = 4 \cdot \left(1 + 0.85 \cdot b_{H(D)}\right) \cdot \left(1 + \sqrt{\rho_{mx} / \rho_{mxf}}\right)^{-2} \tag{19}$$

Here b_H and b_D are heat and diffusivity parameters:

$$b_H = \frac{c_{Pv}}{c_{Pmx}} \cdot \frac{j_v \cdot \mathrm{Re}_{mx} \cdot \mathrm{Pr}_{mx}}{\rho_{mx} \cdot W_{mx} \cdot Nu_0} \qquad b_D = \frac{j_v \cdot \mathrm{Re}_{mx} \cdot \mathrm{Pr}_D}{\rho_{mx} \cdot W_{mx} \cdot Nu_{D0}} \tag{20}$$

The heat and diffusional $_{(D)}$ Nusselt numbers without the influence of mass flux are determined by correlation for single phase flow at the main corrugated field of the channels of PHEs presented in the paper by Kapustenko et al. (2011):

$$Nu_{0(D)} = 0.065 \cdot \mathrm{Re}^{6/7} \cdot \left(\psi \cdot \zeta / F_x\right)^{3/7} \cdot \mathrm{Pr}_{(D)}^{0.4} \tag{21}$$

$$\psi = \left(\mathrm{Re}/A\right)^{-0.15 \cdot \sin(\beta)} \text{ at } Re>A;\ \psi=1 \text{ at } Re \le A \quad \text{where } A = 380/\left[tg(\beta)\right]^{1.75} \tag{22}$$

Here $Nu_D = \beta \cdot d_e/D_D$ is diffusional Nusselt number; Pr_D is diffusional Prandtl number. The friction factor is determined by following correlation presented by Arsenyeva et al. (2012):

$$\zeta = 8 \times \left[\left(\frac{12 + p2}{\mathrm{Re}}\right)^{12} + \frac{1}{\left(A+B\right)^{\frac{3}{2}}}\right]^{\frac{1}{12}} \tag{23}$$

$$A = \left[p4 \cdot \ln\left(\frac{p5}{\left(\frac{7 \cdot p3}{\mathrm{Re}}\right)^{0.9} + 0.27 \cdot 10^{-5}}\right)\right]^{16}; \qquad B = \left(\frac{37530 \cdot p1}{\mathrm{Re}}\right)^{16}$$

where p1, p2, p3, p4, p5 are parameters determined according to plates corrugations form as follows:

$$p1 = \exp\left(-0.157 \cdot \beta\right);\ p2 = \frac{\pi \cdot \beta \cdot \gamma^2}{3};\ p3 = \exp\left(-\pi \cdot \frac{\beta}{180} \cdot \frac{1}{\gamma^2}\right);\ p5 = 1 + \frac{\beta}{10};$$
$$p4 = \left(0.061 + \left(0.69 + tg\left(\beta \cdot \frac{\pi}{180}\right)\right)^{-2.63}\right) \cdot \left(1 + \left(1-\gamma\right) \cdot 0.9 \cdot \beta^{0.01}\right) \tag{24}$$

Here $\gamma = 2 \cdot b/S$ – the corrugation doubled height to pitch ratio; β is corrugation angle, degrees; Re is Reynolds number calculated for the equivalent diameter of the channel defined as $d_e = 2b$; F_x is area increase coefficient, equal to the ratio of actual heat transfer area of the plate to its projected area.
The correlations Eq.(19) - (24) are also used for calculation of heat transfer and pressure drop in single phase flow of cooling media, and in Eq.(6) and Eq.(9). It allows accounting in the mathematical model the influence of plate corrugations geometry on the process intensity.

The Eq.(1)-(24) supplemented by correlations for temperature and pressure dependences of thermal and physical properties of media and geometrical relations for PHE channel can be presented as the system of ordinary differential equations with nonlinear right parts. The analytical solution of this system is not possible because of strong nonlinearity. Its numerical solution with finite difference method is imploied. On its base the software for PC in Mathcad environment is developed.

3. Model validation and discussion of the results

The results obtained by calculations with the developed mathematical model are compared with data of experiments performed for condensation of the steam-air mixture in a sample of PHE channel. The experimental model consisted of four corrugated plates welded together to form three channels. The saturated steam-air mixture is directed to the central channel. It is cooled by water flowing in two outer channels thermally insulated on the outside surface of the outer plates. The temperatures of the steam-air mixture and cooling water are measured by copper-constantan thermocouples with accuracy ±0.1 ºC. The temperatures are measured at the inlet and outlet of the streams and at seven points along the channel length. The pressure of the steam-air mixture is measured at the inlet and outlet of the channel with accuracy ±0.005 bar by pressure gauges. The cooling water flow rate is measured by orifice flow meter with accuracy ±1 %. The flow rate of air before its mixing with steam is measured by the set or rotameters with minimal accuracy ±2 %. The mass flow rate of condensate is measured by a volumetric method with accuracy ±1 %. The mass flow rate of steam is calculated by adding to the condensate flow rate the amount of steam leaving channel with outgoing air at saturation conditions. The sample channel is 1 m long with the 0.225 m width. The height of corrugations is b = 5 mm, plate thickness δ = 1 mm, corrugation angle β = 60º, aspect ratio γ = 0.556 and area increase coefficient Fx = 1.15. The experimental study included 48 tests with different conditions of gas-vapour mixture condensation. The absolute pressure was varied from 2.95 to 1.02 bar; the volume fraction of the air in incoming mixture from 3% to 71 %; the local velocity of mixture from 45 to 4 m/s; the temperature of mixture from 88 to 115 ºC; the temperature of cooling water from 24 to 71 ºC.

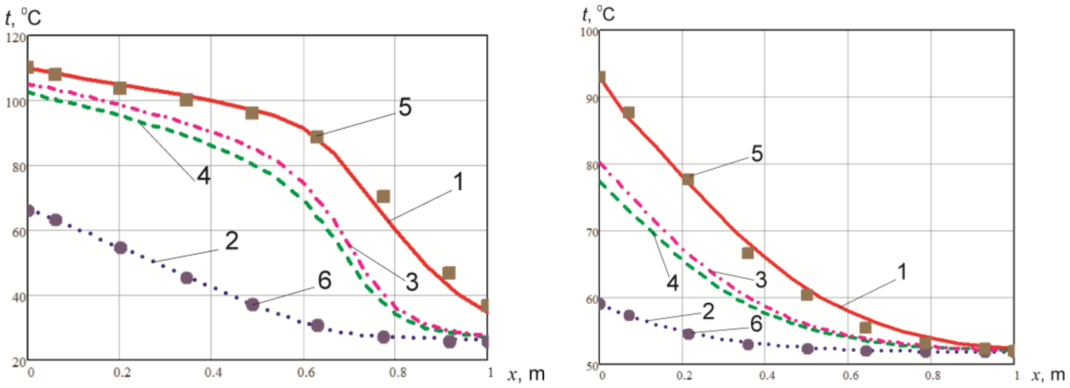

Figure 1: The distribution of temperatures in PHE channel. Calculated are 1 - air-steam mixture; 2 - cooling water; 3 - condensate film; 4 - wall. Experimental points are: 5 - air-steam mixture; 6 - cooling water.

The calculations of process parameters by the developed mathematical model were performed based on tests conditions with specified flow rates of steam, air, cooling water, temperature and pressure of incoming gas-vapour mixture and temperature of cooling water. The results of calculations are compared with the results of the tests. by developed model. The discrepancies between calculated and measured total heat loads were not exceeding ±2.8 % for all the tests. The differences in temperature of the outgoing gas-vapour mixture were less than ±2.5 ºC. It allows the conclusion about the satisfactory accuracy of calculations by the model of PHE overall performance. In Fig.1 are presented the results of calculated local temperatures in PHE channels for two tests with different content of air in the incoming mixture with steam. The accuracy of local temperatures calculation for gas-vapour mixture temperatures is also in the limits of ±3.1 ºC with the biggest discrepancies close to the end of the channel in case of low gas content at the entrance, when the most part of steam is condensed, and the relative errors in calculation of remaining small amounts of steam become more significant. It also reveals the importance of calculations by local parameters for the process with the significant change in all its characteristics along the channel length. It is very important for the use of PHE in the utilisation of heat from condensable gaseous streams and optimisation of their cost in cases like considered by Arsenyeva et al. (2016)

where the model was used for PHE selection. The accounting of the influence of corrugation parameters on PHE performance is also making the model an important tool for optimisation of PHEs plates geometry for the use in the processes of condensation in the presence of noncondensing gases.

4. Conclusions

The calculations of PHEs for utilisation of heat from condensing gaseous streams require the accounting for the change of local process parameters along heat transfer surface. It is possible with a proposed mathematical model represented by the system of ordinary differential equations with strong nonlinearity, for which solution numerical method implemented on PC is used. The model is validated by comparison with experimental data for condensation of the steam-air mixture in a sample of PHE channel. Accounting for the influence of plates geometry for the process intensity the model can be used for optimisation of PHE plate geometry for condensation processes, as well as for optimal selection of PHEs for heat utilisation form condensing gaseous streams in industry.

Acknowledgements

The support of Grant of Education and Science Ministry of the Republic Kazakhstan in-state program “Grant funding for research” is sincerely acknowledged. Olga Arsenyeva is grateful to the Alexander von Humboldt Foundation for the financial support. This research has been supported by the EU project “Sustainable Process Integration Laboratory – SPIL”, project No. CZ.02.1.01/0.0/0.0/15_003/0000456 funded by EU “CZ Operational Programme Research, Development and Education”, Priority 1: Strengthening capacity for quality research in a collaboration agreement with National Technical University “Kharkiv Polytechnical Institute” and AO Spivdruzhnist-T LLC.

References

Arsenyeva O.P., Čuček L., Tovazhnyanskyy L.L., Kapustenko P.O., Savchenko Y.A., Kusakov S.K., Matsegora O.I., 2016. Utilisation of waste heat from exhaust gases of drying process, Frontier of Chemical Science and Engineering, 10, 131–138.

Arsenyeva O.P., Tovazhnyanskyy L.L., Kapustenko P.O., Demirskiy, O.V., 2012. Heat transfer and friction factor in criss-cross flow channels of plate-and-frame heat exchangers. Theoretical Foundations of Chemical Engineering, 46(6), 634-641.

Arsenyeva O., Tovazhnyansky L., Kapustenko P., Perevertaylenko O., Khavin, G., 2011. Investigation of the new corrugation pattern for low pressure plate condensers. Applied Thermal Engineering, 31(13), 2146-2152.

Eldeeb R., Aute, V., Radermacher R., 2016. A survey of correlations for heat transfer and pressure drop for evaporation and condensation in plate heat exchangers. International Journal of Refrigeration, 65, 12-26.

Famileh, I.Z., Esfahani, J.A., 2017. Experimental investigation of wet flue gas condensation using twisted tape insert. International Journal of Heat and Mass Transfer, 108, 1466-1480.

Huang J., Zhang J. and Wang L., 2015. Review of vapor condensation heat and mass transfer in the presence of non-condensable gas. Applied Thermal Engineering, 89, 469-484.

Kapustenko P., Arsenyeva O., Dolgonosova O., 2011. The heat and momentum transfers relation in channels of plate heat exchangers, Chemical Engineering Transactions, 25, 357-362.

Kharangate C.R., Mudawar, I., 2017. Review of computational studies on boiling and condensation. International Journal of Heat and Mass Transfer, 108, 1164-1196.

Klemeš J. J., Arsenyeva O., Kapustenko P., Tovazhnyanskyy L., 2015. Compact Heat Exchangers for Energy Transfer Intensification: Low Grade Heat and Fouling Mitigation. CRC Press, Boca Raton, FL, USA.

Tovazhnyansky L.L., Kapustenko P.O., Nagorna O.G., Perevertaylenko O.Y., 2004. The simulation of multicomponent mixtures condensation in plate condensers. Heat Transfer Engineering, 25(5), 16-22.

Tovazhnyansky L.L., Kapustenko P.O., 1989, Theoretical foundations for design and development of welded plate cold exchangers for ammonia synthesis units. Chemical Industry (Khimicheskaya promyshlennost), 8, 17-22. (In Russian).

Wang L.K., Sunden B., Yang Q.S., 1999. Pressure drop analysis of steam condensation in a plate heat exchanger. Heat Transfer Engineering, 20(1), 71-77.