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Mathematical Modelling of the Thermal and Hydraulic Behaviour of Plate Heat Exchanger in the Fouling Conditions

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The mathematical model of Plate Heat Exchanger (PHE) subjected to fouling is proposed. It is represented by the system of ordinary differential equations. The model is accounting for the distribution of process parameters along the PHE channel that allows predicting fouling development in time at different locations along the channel length. The development of the fouling deposit is accounted with the fouling model presented by the equation in dimensionless form. The relative influence of different terms is characterized by empirical coefficients which can be identified with the data of monitoring the PHE thermal and hydraulic performance. The model allows also the prediction of pressure drop variation in PHE with the development of fouling deposition layer and respective reduction in channels cross-section area. The application of the model and its accuracy is demonstrated with two case studies considering the monitoring of PHEs thermal and hydraulic performance in the industry at sugar factory and in District Heating system.

1. Introduction

One of the important prerequisites for the sustainable development of mankind is the efficient use of resources, among which energy plays the leading part. It is generated nowadays mostly with the combustion of fossil fuels, not only exhausting their limited reserves but also emitting harmful combustion products into the environment. For efficient use of energy, the increasing of heat recuperation is of primary importance. It is possible with the process integration technique supplemented by the use of efficient heat transfer equipment (Klemeš et al., 2013).

Plate heat exchanger (PHE) is one of the modern efficient types of compact heat exchangers (Klemeš et al., 2015). The compact construction consisting of corrugated plates, stamped from a thin metal sheet, and intensified heat transfer in their channels allows a significant reduction of heat transfer area for the same duty compare to conventional shell and tube heat exchangers. The channels of PHEs are of a small hydraulic diameter and have an intricate geometry which promotes turbulence enhancing heat transfer coefficients. In such conditions, the correct accounting for fouling became of primary importance. The same amount of fouling deposit taken according to recommendations for shell and tube heat exchanger can lead to excessive heat transfer area and lower velocities in channels. It can be followed by a higher reduction in heat transfer performance and more severe blockage of narrow channels.

The correct accounting of fouling in PHE design requires the use of the sufficiently detailed mathematical models with the application of accurate enough models of fouling formation. As it is presented in a paper by Kapustenko et al. (2017), for the modelling of PHEs thermal performance, it is possible to account for local parameters of fouling development in PHE channels with one-dimensional model. However, besides PHE thermal behaviour the correct prediction of pressure drops and its time development is very important for estimation of its operational performance and maintenance schedule. The thermo-hydraulic fouling modelling

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for crude oil refinery heat exchanger was proposed by Yeap et al. (2004). There is presented an analysis of some fouling models of reaction and transport type based on the equation proposed for fouling deposition term by Epstein, that according to the paper of Epstein (2011) is written as follows:

$$\varphi_d = \lambda_f \cdot C_b \cdot \left(k_1 \cdot k_m^{-1} + \mu^{-1} \cdot k_2 \cdot \tau_w \cdot e^{E/R \cdot T_s} \right)^{-1} \tag{1}$$

where λ_f is thermal conductivity of fouling deposit, J/(m·K); μ is dynamic viscosity of liquid, Pa·s; τ_w is wall shear stress, Pa; E is activation energy, J/mol; R = 8.314 J/mol is universal gas constant; T_s is temperature of the fouling layer surface, K; k_m is mass transfer coefficient, m/s; k_1 and k_2 are dimensional constants; C_b is concentration of fouling precursors, mol/m^3

The fouling models based on Eq(1) are demonstrating good results in correlating the experimental data on fouling at intensified heat transfer surfaces, as shown in the paper of Yang and Crittenden (2012) for crude oil fouling in tubes with inserts and in the paper of Arsenyeva et al. (2013a) for water fouling in PHE channels. However, the presentation of the model in these papers in dimensional form requires obtaining empirical coefficients of complicated dimensional units and jeopardises the generalisation capacity of the model. The aim of the present paper is to modernize fouling model to dimensionless form and to develop on its base the mathematical model of fouling in PHEs accounting for local parameters of their thermal and hydraulic performance.

2. Mathematical modelling

The core issue of PHE modelling under fouling formation conditions is the selection of reliable and sufficiently accurate fouling model. As it was shown in the paper by Kapustenko et al. (2017) the good results on thermal modelling for water fouling in PHEs can be achieved with Kern and Seaton approach where fouling rate is described as a difference between fouling deposition term φ_d and fouling removal term φ_{rm}:

$$\partial \delta_f / \partial \theta = \varphi_d - \varphi_{rm} \tag{2}$$

In that paper the deposition term was presented in a form derived from Eq(1). In this study, the Eq(1) is presented in dimensionless form by the introduction of the set of dimensionless complexes with the use of following assumptions.

1. The analogy between heat and mass transfer holds the place and it can be written for the diffusional Nusselt number:

$$Nu_{D2} = k_m \cdot d_e / D = Nu_2 \cdot \left(Pr_{D2} / Pr_2 \right)^{1/3} \tag{3}$$

where Nu_2 is Nusselt number for heat transfer in cooling media; $Pr_2 = c_{p2} \cdot \mu_2 / \lambda_2$ is Prandtl number; $Pr_D = D \cdot \rho_2 / \mu_2$ is diffusional Prandtl number; ρ_2 is liquid density, kg/m^3; λ_2 is thermal conductivity of liquid, J/(m·K); c_{p2} is specific heat capacity of liquid, J/(kg·K); d_e is channel equivalent diameter, m; D is coefficient of diffusion, m/s.

2. The link between the coefficient of diffusion and dynamic viscosity for specific substances can be determined with modification of Stocks-Einstein equation (Einstein, 1906) in the following form:

$$D = \chi \cdot T_s \cdot k_B / \left(\mu_2 \cdot r_m \right) \tag{4}$$

where $k_B = 1.38048 \cdot 10^{-23}$ J/K is Boltzmann constant; T_s is temperature of the deposit surface, K; μ is dynamic viscosity, Pa·s; r_m is Van der Vaals radius of molecule, m, that is introduced as scale for molecule radius; χ is parameter depending on the solution substances nature which is accounting for radius of solute molecule and deviations from Stocks-Einstein equation for specific solution content. Such character of the relation between D, μ and T was confirmed experimentally by Wilke and Chang (1955) for different solutions in their study establishing the dependence of proportionality factor from solute properties. The same result is also following from more recent works, as e.g. Karunanithi and Bogeshwaran (2016). It can be concluded that for the specific solute the parameter χ can be regarded as not changing with concentration and temperature of the solution. As the radius of the molecule for different species is rather uncertain it is taken the radius for water molecule $r_m = 1.36 \cdot 10^{-10}$ m according to Zhang and Xu (1995). It is introduced as scaling factor and the differences in r_m are accounted in parameter χ.

Using Eq(3) and Eq(4), after rearranging the component terms, Eq(1) can be written in the dimensionless form as follows:

$$\varphi_d \cdot d_e \cdot \rho_2 / \mu_2 = \left\{ c_D \cdot K_D^{2/3} \cdot Pr_2^{1/3} / Nu_2 + c_R \cdot K_R \cdot \exp\left[E / \left(R \cdot T_s\right)\right] \right\}^{-1} \tag{5}$$

Here K_D and K_R are dimensionless complexes of variables which value can change with temperature and flow characteristics. These complexes are expressed as follows:

$$K_D = \mu_2^{2} \cdot r_m / \left(T_s \cdot \rho_2 \cdot k_B\right); \; K_R = \tau_w / \left(\rho_2 \cdot d_e \cdot g\right). \tag{6}$$

Another two dimensionless complexes c_D and c_R are:

$$c_D = k_1 / \left(\chi^{2/3} \cdot C_b \cdot \lambda_f\right); \quad c_R = \left(k_2 \cdot g\right) / \left(C_b \cdot \lambda_f\right). \tag{7}$$

These complexes are dependent only from constants k_1 and k_2 in Eq(1) and variables which are not changing for the same solution. For experiments with the same media and fouling substance c_D and c_R can be regarded as constants and determined by correlating the tests data.
The removal term in dimensionless form can be expressed using dimensional parameter B:

$$\varphi_{rm} \cdot d_e \cdot \rho_2 / \mu_2 = B \cdot \delta_f \cdot \tau_w \cdot d_e \cdot \rho_2 / \mu_2 = c_{rm} \cdot \delta_f \cdot Re^{*2} \cdot Pr_2 / d_e \tag{8}$$

Here Re* is Reynolds number calculated with the velocity of wall shear stress:

$$Re^{*} = \sqrt{\tau_w / \rho_2} \cdot d_e \cdot \rho_2 / \mu_2; \quad c_{rm} = B \cdot \lambda_2 / c_{p2} \tag{9}$$

Counting that for the same liquid the variation of c_{p2} and λ_2 are rather small, the parameter c_{rm} can be regarded as constant when correlating the experimental data on fouling for specific media.
The mathematical model of PHE thermal behaviour under conditions of fouling was presented in a paper by Kapustenko et al. (2017). However, there is used dimensional fouling model and dimensionless fouling model presented here require validation. It also requires an development to include calculation of the PHE hydraulic behaviour.
The basic differential equations of thermal design for countercurrent flows are:

$$\partial T_2 / \partial x = q \cdot \Pi / (g_2 \cdot c_{p2}) \tag{10}$$

$$\partial T_1 / \partial x = (\partial T_2 / \partial x) \cdot g_2 \cdot c_{p2} / (g_1 \cdot c_{p1}) \tag{11}$$

where T_1 and T_2 are temperatures of hot and cold streams, K; g_1 and g_2 are the mass flow rates of hot and cold streams through one channel, kg/s; c_{p1} and c_{p2} are specific heat capacities of hot and cold streams, J/(kg·K); Π is the channel perimeter, m; *x* is the distance from cold stream entrance, m; q is the specific heat flux, W/m^2.

$$q = \left(1/h_1 + 1/h_2 + R_f + \delta_w / \lambda_w\right)^{-1} \times (T_1 - T_2) \tag{12}$$

Here δ_w is the thickness of the plate metal, m; λw is the heat conductivity of the plate metal, W/(m·K); h_1 and h_2 are film heat transfer coefficients for hot and cold streams, respectively, W/(m^2·K).
The temperature at the outer surface of the deposited layer is:

$$T_s = \left[\left(1/h_1 + 1/h_2 + R_f + \delta_w / \lambda_w\right) \times h_2\right]^{-1} \times (T_1 - T_2) + T_2 \tag{13}$$

The coefficients h_1 and h_2 are calculated according to correlations presented by Kapustenko et al. (2011) for pressure drop and heat transfer at the main corrugated field depending on channel geometry and fluid thermo-physical properties. In general form:

$$h_j = h_j(W_j, T_j, T_s, \beta, \gamma, d_{ej}) \tag{14}$$

Here γ=2·b/S is the corrugation doubled height to pitch ratio; β is corrugation angle, degrees.
With fouling formation, the free cross-section area of the channel (f_{ch}, m^2) is becoming smaller inducing the increase of flow velocity that can be determined as follows:

$$W_2 = g_2 \big/ \left((f_{ch} - \delta_f \cdot \Pi) \cdot \rho_2 \right) \quad (15)$$

As it is discussed in a paper by Yeap et al. (2004), the deposited fouling layer is leading to increasing the pressure loss in the channel not only causing higher flow velocity but also creating a roughness on the flow boundary. For the majority of streams in heat exchangers, the thermo-physical properties are not much dependent on the change of pressure inside channels (especially for liquids). For this reason, the thermal part of the mathematical model can be solved without considering pressure loss in channel thus enabling to receive information about fouling layer development prior to consequent calculation of pressure losses.

The pressure drop in the PHE can be considered as a sum of pressure drop at the main corrugated field ΔP_{mf}, pressure drops at inlet and outlet distribution zones ΔP_{zin} and ΔP_{zout}, the pressure drop in port and collectors ΔP_{pc} (Arsenyeva et al., 2013b). The total pressure drop with expressions for corresponding constituent parts is as follows:

$$\Delta P_2 = \int_0^{L_p} \zeta_2 \cdot \frac{\rho_2 \cdot W_2^2}{2 \cdot d_e} dx + \zeta_{DZin} \cdot \frac{\rho_2 \cdot W_{2in}^2}{2} + \zeta_{DZout} \cdot \frac{\rho_2 \cdot W_{2out}^2}{2} + 1.3 \cdot \frac{\rho_2 \cdot W_{2p}^2}{2} \quad (16)$$

where W_{2in}, W_{2out} and W_{2p} are velocities at channel inlet, outlet and ports of PHE, m/s; ζ_2 is the friction factor in PHE channel determined depending on channel geometry by Eq (17) proposed by Arsenyeva et al. (2011) with the term A accounting for roughness created by fouling:

$$\zeta_2 = 8\left\{\left(\frac{12+p2}{\mathrm{Re}}\right)^{12} + \left[A + \left(\frac{37530\,p1}{\mathrm{Re}}\right)^{16}\right]^{-\frac{3}{2}}\right\}^{\frac{1}{12}};\; A = \left[p4 \cdot \ln\left(p5 \cdot \left(\left(\frac{7 \cdot p3}{\mathrm{Re}}\right)^{0.9} + 2.7 \cdot \frac{\delta_f}{d_e}\right)^{-1}\right)\right]^{16} \quad (17)$$

$$p1 = \exp(-0.157 \cdot \beta);\; p2 = \pi \cdot \beta \cdot \gamma^2 / 3;\; p3 = \exp\left(-\pi \cdot \beta / \left(180 \cdot \gamma^2\right)\right);\; p5 = 1 + \frac{\beta}{10};$$
$$p4 = \left(0.061 + \left(0.69 + tg\left(\beta \cdot \pi / 180\right)\right)^{-2.63}\right) \cdot \left(1 + (1-\gamma) \cdot 0.9 \cdot \beta^{0.01}\right) \quad (18)$$

The coefficient of local hydraulic resistance in inlet distribution zone is determined according to the paper by Arsenyeva et al. (2013), namely ζ_2 = 38. For outlet distribution zone to value 38 the correction for fouling roughness is taken as for friction factor at the end of the main corrugated field and also the velocity W_{2out} is calculated for cross-section area reduced by fouling.

The Eqs(2)-(18) with correlations for temperature dependences of thermal and physical properties of streams and geometrical relations for PHE represent the system of ordinary differential equations with nonlinear right parts. The numerical solution of this system with finite difference method is implemented for PC using Mathcad software. To check the validity of the developed model in the following section two case studies are considered.

3. Model validation and discussion of the results

For illustration of model application and its validation in this section are presented the comparisons of calculations results with data of tests in the industry.

3.1 Case study 1. PHE at the sugar factory

The PHE for heating the juice directed to evaporation by the heat from condensate after evaporation effect presented by Demirskiy et al. (2018) is considered. The PHE is of the M15M type produced by AlfaLaval. It consists of 151 plates forming 75 channels for thin juice. The heat transfer area of one plate is 0.62 m^2, the length of corrugated field is 1.38 m, cross section area of channel equals 0.00176 m^2, the height of corrugations is 4 mm, the corrugations inclination angle β = 35º, the doubled height to pitch ratio γ = 0.56. The results of monitoring operating parameters during 13 d of operation are presented in Table 1. By these data the temperatures of heated juice and cooled condensate after PHE are calculated and compared to experimental values in Table 1.

Table 1: The results of monitoring parameters and calculation by the model of PHE heating thin juice

Time θ, h		144	216	264	312
The flowrate of thin juice G_2, m^3/h		260	270	277	265
Inlet temperature of thin juice t_{21}, ºC		101	100.5	102	101.7
The outlet temperature of thin juice:	experimental t_{22exp}, ºC	105	106	107	106
	calculated t_{22clc}, ºC	104.9	105.9	106.8	105.9
Inlet temperature of condensate t_{11}, ºC		123.5	123.5	123.5	123.5
Outlet temperature of condensate:	experimental t_{12exp}, ºC	102.8	104.8	106.1	104.8
	calculated t_{12clc}, ºC	102.6	104.4	105.8	104.6

The calculated results are obtained for the following values of dimensionless empirical parameters in fouling model, which were determined by least squares method on experimental data: $c_D = 2.29 \cdot 10^6$; $c_R = 0.126$; $c_{rm} = 0.45 \cdot 10^{-15}$. The discrepancies between the calculated and experimental temperatures are not bigger than 0.3 ºC that confirms the validity of the thermal modelling. After rearranging the piping at PHE to make two connections for thin juice, the hydraulic parameters of operation were monitored for 93 d during the operation the next year. The comparison of experimental pressure drops and calculated is presented in Figure 1a.
The calculations are made for the average value of thin juice flow rate G_2 = 290 m^3/h, which was actually changed ± 7 % during the operation. Counting for uncertainties in industrial operation conditions the results of calculations and model accuracy can be regarded as acceptable for predicting the hydraulic behaviour of PHE under the fouling conditions of thin juice heating at the sugar factory.

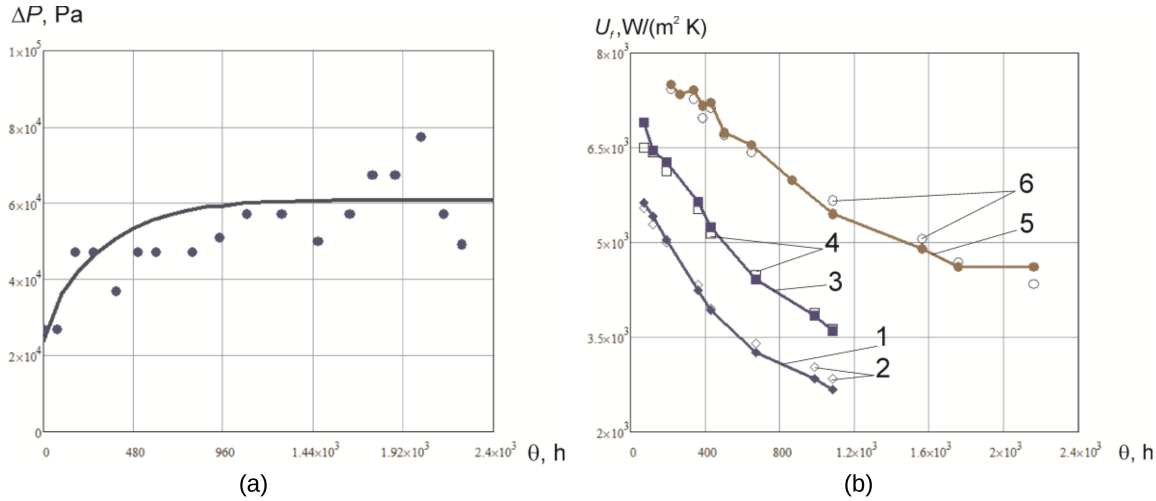

Figure 1: (a) Pressure drop for Case study 1; where solid line -calculated; dots - tests. (b) Overall heat transfer coefficient for Case study 2; where solid dots – calculation; empty dots – tests

3.2 Case study 2. PHE at District Heating system

Nowadays the District Heating systems are one of the biggest users of PHEs, which sake for efficiency and compactness practically completely overtaken new installations in this field from traditional shell and tube units. Demirskiy et al. (2018) have checked the validity of PHE thermal model employing a dimensional model of fouling by experimental data of Chernyshov (2002). These experiments performed with AlfaLaval M10B PHE for heating fresh water coming with temperature in the range 7.9 - 9.5 ºC to about 60 ºC by hot water from the boiler with temperature varied from 74 to 98 ºC. The inlet temperature of hot water was rising eventually to ensure the required temperature of the heated fresh water with fouling development.
The empirical values of dimensionless parameters in presented here fouling model were determined by least squares method using experimental data for one set of tests with flow velocity 0.57 m/s. These values are: $c_D = 8.18 \cdot 10^5$; $c_R = 0.045$; $c_{rm} = 0.17 \cdot 10^{-16}$. The comparison of the data for all experiments with overall heat transfer coefficients calculated by the model is presented in Figure 1b. The discrepancies between calculated and experimental results are not exceeding ±7 %. It confirms the thermal model validity and its ability to predict PHE fouling behaviour in an investigated range of flow velocities and temperatures. The absence of data on pressure losses does not permit to check the hydraulic part of the modelling in this case.

4. Conclusions

The mathematical model of PHEs thermal and hydraulic performance under fouling deposition on heat transfer surface is presented. The integral part of it is fouling model in dimensionless form with empirical dimensionless coefficients. It simplifies the use of the model and adds for prospects of its generalisation that should be checked in further research with different fouling conditions. There is a good agreement of the modelling results with obtained data on temperature program and pressure loss in heat exchangers. The calculation by the model enables more accurate design of PHEs to improve their performance and to implement fouling mitigation effects. It allows significant increase of the time between cleaning of heat transfer surface. The developed software can be used for the optimal calculation of PHEs subjected to fouling with an accounting of their capital cost and operational expenses including cost of cleaning.

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