Mathematical modeling of Spark Plasma Sintering:  
A sensitivity analysis of materials rheological parameters

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Highlights
- A novel mathematical model of Spark Plasma Sintering is developed.
- A generalized description of powder viscosity is provided.
- Temperature, electric current and density distributions are obtained.
- A systematic investigation of model parameters effects is performed.

1. Introduction
Among thousands of studies related to Spark Plasma Sintering (SPS) [1], only less than two hundred are devoted to mathematical modelling of this process. On the other hand, scalability and optimization of SPS depend on the availability of reliable process models. In particular, model ability to properly describe density evolution and distribution within the consolidating sample is still of high demand. In this regard, it is worth noting that only few tens of papers concerning SPS modeling are able to simulate densification phenomena. Moreover, the latter studies considered almost exclusively the power law creep relation as constitutive equation for describing the non-linear viscous behavior of porous materials [2]. However, it is known that power law suffers some limitations [3]. Therefore, a generalized powder viscosity constitutive equation is adopted in this work for the description of powders densification during SPS. A systematic investigation of the effect of model parameters on temperature and density temporal-spatial profiles is carried out and the obtained results are presented.

2. Methods
The proposed mathematical model is based on the following dimensionless balance equations:

\[
\frac{\partial \rho^*}{\partial t^*} + \nabla^* \cdot (\rho^* \vec{v}^*) = 0 
\]

\[
\frac{\partial (\rho^* \vec{v}^*)}{\partial t^*} + \nabla^* \cdot \mathbf{T}^* = 0 
\]

\[
\frac{\partial \vec{v}^*}{\partial t^*} = \vec{v}^* 
\]

\[
\frac{\partial \mathbf{T}^*}{\partial t^*} + \vec{v}^* \cdot \nabla^* \mathbf{T}^* - \alpha \nabla^* \cdot (k^* \varphi^* \mathbf{T}^*) = \beta (\mathbf{\pi}^* \cdot \nabla^* \vec{v}^*) + \chi (\mathbf{I}_Z^* \cdot \mathbf{E}^*) + \delta (\nabla^* \cdot \vec{v}^*) 
\]

\[
\nabla^* \cdot \sigma^* (\nabla^* \mathbf{V}^*) = 0 
\]

Specifically, Eq. (1) represents the total mass balance (continuity) equation used for the description of the temporal-spatial evolution of the density of samples undergoing SPS. The latter variable is strongly related to the velocity and displacement fields given by the linear momentum balance equation (2) and the equation (3), respectively. Materials properties affecting the sintering process are dependent upon the temperature field that is described by the internal energy balance equation (4). Finally, the electric charge balance equation (5) defines the evolution and distribution of the electric current flowing inside the SPS ensemble. It is worth mentioning that the powder viscosity is described by the Bird-Carreau-Yasuda model [4]:
\eta^* = \eta_0 \varphi \left(1 + f_\eta (T^*)^{1/m-1} W^* \right)^{m-1} \tag{6}

3. Results and discussion
The study conducted in this work evidenced the main effects produced when powder material parameters are systematically changed. An example of temperature spatial profiles within the SPS ensemble is reported in Figure 1 at different time intervals. From this figure, it is apparent that the moving mesh approach is adopted. The effect of the self-diffusion dimensionless activation energy \(Q^*\), which affects the temperature dependence of \(f_\eta\) in eq. (6), on relative density time profile is shown in Figure 2.

![Figure 1. Temperature distribution within the SPS ensemble at different time intervals: a) \(t^*=0.1\); b) \(t^*=1\); c) \(t^*=2\).](image)

![Figure 2. Relative density time profile for three different values of the dimensionless self-diffusion activation energy.](image)

4. Conclusions
A novel dimensionless mathematical model of the Spark Plasma Sintering process based on total mass, linear momentum, internal energy and electric charge balance equations is developed. Powder viscosity is expressed by the Bird-Carreau-Yasuda model, which represents a generalization of the power-law creep model. The influence of model parameters on temperature and density distribution has been thoroughly investigated. It is found that the effect of pressure and temperature significantly depends on the powder material parameters as well as the actual level of strain rate and the adopted heating procedure.

References

Keywords
Spark Plasma Sintering; Mathematical modeling; Powder viscosity.