Stable Model Predictive Control tuning considering asymmetric bounded signals

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Abstract

In this paper the consideration of asymmetric bounded signals (disturbances, inputs) in a general procedure for tuning infinite horizon model predictive controllers (MPC) with constraints is presented. These measures are stated by means of an asymmetric objective functional, instead of the standard $l_1$ norm. The MPC is implemented with a terminal penalty to guarantee stability for all tuning parameters. Moreover, multiple models have been considered to ensure robust performance of the closed loop process, in the face of high non linear dynamics and load disturbances. The methodology has been applied to tune an MPC for the activated sludge process in a simulated wastewater treatment plant (WWTP). The problem is stated as a non linear programming problem with constraints and solved by Sequential Quadratic Programming (SQP).

Keywords: model predictive control, stability, $l_1$ norm, asymmetric signals, activated sludge process.

1. Introduction

Frequently, real plants are subject to constrained inputs (e. g. control signal saturations), and it is typical to have asymmetric input signals (control signals or disturbances), for example due to technological and safety reasons in the actuators or to a large variability of load disturbances. For those reasons, in this work an MPC tuning methodology developed by the authors [1, 2] is improved with the consideration of asymmetric bounded signals using a specific objective functional [3]. Moreover, the MPC implemented here operates over infinite horizon, and it is implemented as a finite horizon with terminal penalty, in order to solve finite dimensional QP problems. The terminal weight is obtained either as the solution of a Lyapunov equation or a Riccati equation, providing that the system is stable [4, 5].

The advantages of the development of a stable MPC tuning method are clear. MPC controllers have been tuned traditionally through a number of different parameters including the number of control moves, and input/output weights in the objective function. The tuning task is usually difficult because many of these parameters have overlapping effects on the closed-loop performance and robustness. Disturbance rejection capability is also a key issue in the controllability of a process, and therefore it is crucial to find a good controller that reduces the effect of disturbances.
The controllability indices used in this work are based on the $H_\infty$ and $l_1$ norms of different weighted closed loop transfer functions matrices of the system, representing the process disturbances rejection capability and control efforts [2, 6], together with an asymmetric objective functional considered in some constraints.

The tuning approach developed has been validated on a simulated activated sludge process based on a real WWTP [7]. The paper is organized as follows. First, the MPC formulation and the controllability indices are presented, including the specific asymmetric functional and the optimization problem for tuning. Then, the activated sludge process is explained and some results are discussed, to end up with some conclusions.

2. MPC formulation

The MPC formulation consists of the on-line calculation of the future control moves by solving the following constrained optimization problem subject to constraints on inputs, predicted outputs and changes in inputs. The prediction model is a linear discrete state space model of the plant obtained by linearizing the first-principles nonlinear equations of the process. The infinite horizon MPC is implemented with finite horizon and terminal penalty, guaranteeing closed loop stability. The objective function is the following:

$$\min_{\Delta u} V(k) = \min_{\Delta u} \left( \|x(k + Hc)\|_2^2 + \sum_{i=0}^{Hc-1} \left( \|x(k+i)\|_2^2 + \|\Delta u(k+i)\|_2^2 \right) \right)$$  \hspace{1cm} (1)

where $k$ denotes the current sampling point, $x(k+i)$ is the predicted state vector at time $k+i$, depending of measurements up to time $k$, $\Delta u$ are the changes in the manipulated variables, $Hc$ is the control horizon, $R$ and $Q$ are positive definite diagonal matrices representing the weights of the change of control variables and the weights of the set-point tracking errors respectively. In this work the reference is fixed to zero (steady state) and the outputs are equal to the states. Matrix $P$ is the terminal penalty calculated solving the following Lyapunov equation, where $A$ is the process state matrix:

$$P - A'PA = Q$$  \hspace{1cm} (2)

This formulation is based on [4], where an infinite horizon MPC is developed with constraints both on states and outputs. The feasibility of the constraints guarantees nominal stability of the closed loop system for any choice of the tuning parameters, because the objective function is a Lyapunov function. The implementation of this controller only requires the solution of finite QP (Quadratic Programming) problems to obtain the control increments in a receding horizon strategy.

The use of objective function with terminal penalty (1) guarantees MPC stability, but in some cases the performance might get worse. This behaviour improves considerably if from sampling time $Hc$, an optimal state feedback controller (LQR) is implemented [5]. For this reason, in the example of this work the terminal penalty comes from the solution of the Riccati equation:

$$P = A'PA - A'PB\left(B'PB + R\right)^{-1}B'PA + Q$$  \hspace{1cm} (3)
The MPC can be expressed as a combined feedforward-feedback control system, with the following output ($S_0(s)$ - $R_{d0}(s)$) and control ($M_0$) sensitivity functions to disturbances:

$$S_0(s) \cdot R_{d0}(s) = \frac{R_{00}}{1 + G_0 K_1} ; \quad M_0(s) = \frac{K_s - K_s G_{d0}}{1 + G_0 K_1} ; \quad R_{d0}(s) = G_{d0}(s) - K_s G_0(s)$$

where $K_i$ are the transfer functions between the control signal and the different inputs ($r(s)$-error signal, $d(s)$-disturbances) which depend on the control system tuning parameters ($Q$, $R$, $H_c$), and the nominal transfer functions are denoted by $G_0$ and $G_{d0}$.

### 3. Controllability indices and optimization for automatic MPC tuning

In this work, norm based indices are used in the tuning procedure to assess process controllability. Although those functions are only defined for linear control systems, it can be shown that it is also valid when the set of active constraints of the MPC is fixed [8, 2]. This assumption is sometimes a bit strong, and for that reason the constraints imposed are also used to keep the variables within the feasibility region. The tuning is stated as a mixed sensitivity optimization problem that takes into account disturbance rejection and control effort objectives in the same tuning function:

$$\min \| N_0 \| = \min \left( \max \left( N_0 \left( j \omega \right) \right) \right)$$

where $N_0 = \frac{W_p \cdot S_0 \cdot R_{d0}}{W_{esf} \cdot s \cdot M_0}$

The dependence on $s$ of the transfer functions has been omitted for brevity. $W_p(s)$ and $W_{esf}(s)$ are suitable weights to achieve closed loop performance specifications and to reduce the control efforts respectively.

In order to ensure disturbance rejection (considering normalized disturbances) the following constraint must be added to (5) [2, 9].

$$\| W_p \cdot S_0 \cdot R_{d0} \|_\infty < 1$$

The new functional proposed in [3] is also included as constraint in the MPC tuning methodology, giving information on the signal amplitudes, including the asymmetry. For a generic discrete signal $z(k)$ it is defined by the following expression, and the specific constraints will be defined in the paragraph of results:

$$\| z(k) \| = \begin{cases} 0, \max_k \{ z(k) \} \\ \max_k \{ 0, - \min_k \{ z(k) \} \} \end{cases} = \begin{bmatrix} z_{max} \\ z_{min} \end{bmatrix}$$

### 4. Activated sludge process and control problem

The plant layout is represented in Fig. 1, consisting of one aerobic tank and one secondary settler. The basis of the process lies in maintaining a microbial population (biomass) into the bioreactor that transforms the biodegradable pollution (substrate) when dissolved oxygen is supplied through aeration turbines. Water coming out of the reactor goes to the settler, where the activated sludge is separated from the clean water and recycled to the bioreactor to maintain there an adequate concentration of microorganisms.
The whole set of variables is presented also in Fig. 1. Generically, “x” is used for the biomass concentrations (mg/l), “s” for the organic substrate concentrations (mg/l) and “q” for flow rates (m$^3$/h). The complete set of non linear differential equations (the process order is 5) and model parameters are given in [7].

The control of this process aims to keep the substrate at the output ($s_1$) below a legal value despite the large variations of the flow rate and the substrate concentration in the incoming water ($q_i$ and $s_i$), which are the input disturbances. The recycling flow ($q_{r1}$) is the manipulated variable and the substrate ($s_1$) is the controlled variable. The biomass ($x_1$) is only a constrained variable for a good performance of the process. The different sets of disturbances used in dynamic simulations (Fig. 1) have been determined by the COST 624 European research program and its benchmark [10].

5. Results

The tuning methodology begins with the selection of a fixed plant with $V_f=7668$ m$^3$ (reactor volume) and $A=2970.88$ m$^2$ (settler cross-sectional area), together with a steady state working point. Then the MPC is automatically tuned using linearized state space models of the system, and the MPC obtained is tested on the linearized model of the plant, i.e., the prediction model and plant model are the same (in this case the variables are deviations from the steady state, and they are represented with upper bar notation). Finally, the MPC is tested on the nonlinear plant, using the differential equations of the activated sludge process.

In the WWTP, the influent disturbances variations are asymmetric. For example, for storm weather $q_{imin}=923.59$ m$^3$/h and $q_{imax}=1956.7$ m$^3$/h, whose normalized value (deviations from the steady state) is $\overline{d}_{max}=1; \overline{d}_{min}=0.2866$. Then, the $q_i$ domain is determined by $D' = [\overline{d}_{max}, \overline{d}_{min}] = [1, 0.2866]$. On the other hand the zero saturation of $u=q_{r1}$ puts a stricter bound $\overline{q}_{r1_{min}}$ while $\overline{q}_{r1_{max}}$ can be much larger depending on the pump characteristics. Then, the functional (7) is used, and the following constraint is included in the tuning procedure:

$$\|\overline{q}_{r1}\| \leq U$$, where $U' = [\overline{q}_{r1_{max}}, \overline{q}_{r1_{min}}]$ (8)
In order to solve the optimization problem, constraint (8) has been substituted by the approach in [3], which gives a simple condition (based on the impulse response of the system) that guarantees (8) for any disturbance defined in the asymmetric domain:

\[
\|d(k)\|_u \leq D', \text{ where } D' = \begin{bmatrix} \bar{d}_{\text{max}}, \bar{d}_{\text{min}} \end{bmatrix}
\]

(9)

In this work, only results for \( R \) tuning are shown, using the SQP method, because the influence of \( R \) in performance is more relevant. The value of \( H_c \) is empirically fixed to 10, large enough to provide a good response. However, the tuning of \( H_c \) could also be performed using a two iterative steps algorithm that combines a directed random search for tuning \( H_c \) (integer), and the SQP for tuning the weight \( R \) (real variable) [2, 6]. The sampling period is \( T=0.5 \) hours, and disturbances \( s_i \) and \( q_i \) are assumed to be measured. Multiple linearized models changing the substrate concentration in the reactor have been considered for robust performance tuning, imposing constraint (6) for every local model [2].

### Table 1: Results for different bounds \( U \) in \( qr_1 \) (dry weather disturbances)

<table>
<thead>
<tr>
<th>( R )</th>
<th>Unconstrained ( qr_1 )</th>
<th>( U' = [2200,2200] )</th>
<th>( U' = [2200,1450] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00259</td>
<td>0.00418</td>
<td>0.00444</td>
<td></td>
</tr>
<tr>
<td>( \max(qr_1) - \min(qr_1) )</td>
<td>735.09</td>
<td>712.06</td>
<td>707.56</td>
</tr>
<tr>
<td>( \max(s_1) - \min(s_1) )</td>
<td>2.71</td>
<td>3.36</td>
<td>3.46</td>
</tr>
<tr>
<td>[ \det \left</td>
<td>W \right</td>
<td>]</td>
<td>[534.3; 275.3]</td>
</tr>
<tr>
<td>[ | p_s \cdot R_\infty | ]</td>
<td>0.590</td>
<td>0.798</td>
<td>0.831</td>
</tr>
</tbody>
</table>

### Table 2: Results for different bounds \( U \) in \( qr_1 \) without \( H_c \) performance constraint (6) (storm weather disturbances)

<table>
<thead>
<tr>
<th>( U' = [5500,5500] )</th>
<th>( U' = [2200,2200] )</th>
<th>( U' = [2200,1000] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00997</td>
<td>0.03614</td>
<td>0.18222</td>
</tr>
<tr>
<td>( \max(qr_1) - \min(qr_1) )</td>
<td>1741.1</td>
<td>1472.9</td>
</tr>
<tr>
<td>( \max(s_1) - \min(s_1) )</td>
<td>23.18</td>
<td>30.19</td>
</tr>
<tr>
<td>[ \det \left</td>
<td>W \right</td>
<td>]</td>
</tr>
<tr>
<td>[ | p_s \cdot R_\infty | ]</td>
<td>1.391</td>
<td>4.857</td>
</tr>
</tbody>
</table>

First, the automatic tuning has been performed including the \( H_c \) constraint (6) for proper disturbance rejection, considering scaled disturbances for dry weather and different asymmetric bounds in \( qr_1 \). In Table 1 the numerical results are presented, including the optimal MPC. Then, in Table 2 and Figure 5 some results are presented changing the conditions over \( qr_1 \), for scaled storm weather disturbances. Here the \( H_c \) constraint (6) has not been considered to clarify the effect of constraint (8). In all results it can be seen that disturbance rejection is better when \( U \) is increased (\( \tilde{d}_{\text{max}} \) is relaxed), because the MPC obtained allows for larger control variations. The SQP convergence tolerances are \( 10^{-8} \) for all results, and it is reached in less than 20 iterations.

### 6. Conclusions

In this work a general method for tuning MPC considering asymmetric bounded signals for disturbance rejection has been developed. This method has been tested in MPC...
applied to a simulated activated sludge process, and the closed loop responses show that obtained controllers are properly tuned, taking into account the large magnitude of influent disturbances. The use of asymmetric bounded signals allows for a more realistic selection of the tuning controllability criteria, providing better control performance. The methodology proposed here is general, so any other chemical processes or performance criteria could be considered. Finally it is important to show that the developed method is particularly suitable for its inclusion in the resolution of the Integrated Design optimization problem, which determines the optimum controller and the optimum plant at the same time.

Fig. 4: Output substrate concentration ($s_1$) and recycling flow ($qr_1$) for the process with MPC tuned with $U^* = [2200, 2200]$ (dashed dotted line) and $U^* = [2200, 1000]$ (solid line).

7. Acknowledgements
The authors gratefully acknowledge the support of the Spanish Government through the MICINN project DPI2009-14410-C02-01.

References