Optimal Production Scheduling and Lot-sizing In Yoghurt Production Lines

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Abstract

The lot-sizing and production scheduling problem in a real-life multi-product yoghurt production line is addressed. A new discrete/continuous time representation mixed-integer linear programming model, based on the definition of families of products, is presented. It is mainly optimized the packaging stage while timing and capacity constraints are imposed with respect to the pasteurization/homogenization and fermentation stage. Packaging units operate in parallel and may share common resources. Sequence-dependent times and costs are also explicitly taken into account. An industrial case study is presented wherein production bottlenecks are revealed and several retrofit design options are proposed to enhance the production capacity and flexibility of the plant.

Keywords: Production scheduling, lot-sizing, dairy industry, MILP model

1. Introduction

Yoghurt production could be considered as a particular case of a batch or a semi-continuous production process. The PSE research community has studied these types of production processes during the last 20 years. One of the main features of batch processes is that a large number of products are produced from a few initial product recipes. The same holds for yoghurt production. Final yoghurt products may differ in at least one of the following features: (i) fermentation recipe type origin, (ii) total cup weight, (iii) number of cups per piece, (iv) labeling depending on their customer destination, (v) flavors, and (vi) packaging cup type (material, shape, etc.). Packing rates may significantly vary from one product to another.

A plethora of contributions addressing production scheduling problems can be found in the Operational Research and PSE communities’ literature. However, the use of optimization-based techniques for scheduling dairy plants is still in its infancy. This can be mainly attributed to the complex production recipes, the large number of products to be produced under tight operating and quality constraints and the existence of mixed batch and semi-continuous production modes. An excellent review covering the short-term batch and continuous process scheduling can be found in Méndez et al. (2006).

Few attempts regarding production planning in yoghurt production lines can be found in the literature (Entrup et al., (2005), Marinelli et al., (2007)). In most works, changeover times/costs or fermentation stage restrictions were ignored. To the best of our knowledge, this work is the first that address the main processing features in yoghurt production lines in tandem.
2. Yoghurt Production Processes Description

The two main yoghurt product types are set and stirred yoghurt. Both types are subsequently subjected to cooling and packaging. Additionally, fruit and nuts may be added to stirred yoghurt where applicable. The principal difference between these two yoghurt types is that set yoghurt first passes from the packaging lines and afterwards is fermented in the final retail container. Figure 1 illustrates the main processing steps for producing stirred yoghurt.

3. Problem Statement

3.1. Yoghurt production line description

The production line produces set, stirred or flavored yoghurt. Note that flavored yoghurt is stirred yoghurt with additional fruit (or other type) flavor. Thus, flavored yoghurt production should pass through fruit-mixer equipments in order to perform the addition and the mixing of fruit substances. The yoghurt production line consists of: (i) a set of cooling tanks (set yoghurt), (ii) a set of fermentation tanks (stirred and flavored yoghurt), (iii) 4 packaging units, and (iv) 2 fruit-mixer equipment units (see Fig. 2).

The short-term scheduling time horizon for yoghurt production is one week. Regular production is performed from Monday to Friday. Overtime is permitted on Sunday and/or on Saturday. Period production time is equal to 24 hours. Daily scheduled plant cleaning operations, shutdown, last 2 hours, while before the start of the fermentation stage (including pasteurization, homogenization, etc.), which reflect the total plant setup time, equal to 3 hours. Product demand data are packaging stage production targets and they are provided from the Logistics department.

3.2. Retrofitting alternatives

Some alternative retrofitting options revealed by having a closer look on the current yoghurt production line:

(i) Fruit-mixers are common resources that limit the total plant production capacity. For instance, packaging unit J3 and packaging unit J4 cannot package flavored yoghurt simultaneously (see Fig. 2); the same statement holds for packaging unit J1 and packaging unit J2. Therefore, a relatively low cost fruit-mixers investment seems an alternative to increase the yoghurt production line capacity.

(ii) Each product p and/or product family f can be produced only to one packaging unit.

Thus, the production process seems to be lacking of flexibility. Discussions with the plant manager revealed that it is possible to install a low-cost manifolds’ investment in packaging unit J4 in order to package more product families. With the current operating polocyte, packaging unit J4 can only process product family F23. Note that packaging unit J4 could process 5 product families (F6, F8, F9, F10, and F23) instead of just 1 product family (F23); if this investment takes place. The current plant configuration will be referred as NFM, the fruit-mixer retrofit design option as FM, and the joint fruit-mixer and manifolds investment as FM&M.
4. Conceptual Model Design

Production scheduling in dairy plants typically deals with a large number of products. Fortunately, many products illustrate similar processing characteristics. Products that share the same processing characteristics could be treated as a product family group. Products grouping significantly reduces the size of the underlying mathematical model and, thus, the necessary computational effort; without sacrificing any feasibility constraint. In the proposed approach products belong to the same product family if and only if: (i) they have the same fermentation recipe origin, (ii) there is no sequence-dependent setup time among them, and (iii) they share the same processing rate.

When changing the production between two products that are not based on the same recipe, it is always necessary to perform changeover cleaning and/or sterilizing operations. In dairy plants, a “natural” sequence of products often exists (e.g. from the lower taste to the stronger or from the brighter color to the darker) thus the sequence of products within a product family is known a priori. Therefore, when changing the production between two products of the same product family, the cleaning and sterilizing can be neglected. Hence, in dairy plants not only the sequence of products belonging to the same product family may be fixed but also the sequence of product families in each packaging line. In that case, different product families are enumerated according to their relative position in the production day. Note that a product families’ demand is the aggregated demands of the products that belong to it.

5. Mathematical Formulation

The proposed model is a crossbreed between a continuous-time and a discrete-time model. Concretely, the production horizon is divided to production periods (days) whose material balances are modeled with a discrete time representation, while within each production day a continuous-time representation is adopted. It follows a brief description of our model. For lack of space, are not presented all equations here.
5.1. Lot-sizing and timing constraints

Product families’ packaging times $T_{jn}$ lower and upper limits are given by Eq. (1). Packaging rates $\rho_j$ are fixed. Lower and upper bounds for the completion time $C_{jn}$ are given by Eqs. (2) and (3), respectively. Fermentation times $t_{j}^{ferm}$ are included in Eq. (2).

$$t_{jn}^{\min} \leq T_{jn} = \frac{Q_{jn}}{\rho_{jn}} \leq t_{jn}^{\max} \forall f, j \in (J_{pack} \cap F_{j}), n$$

$$C_{jn} \geq (\text{setup}_{jn} + t_{j}^{ferm})Y_{jn} + T_{jn} + \sum_{f' \neq f, f' \in F_{j}} s_{j}^{d_{fj}}X_{f'jn} \forall f, j \in (J_{pack} \cap F_{j}), n$$

$$C_{jn} \leq (\text{hor}_{jn} - \text{shutdown}_{jn})Y_{jn} \forall f, j \in (J_{pack} \cap F_{j}), n$$

5.2. Timing and sequencing constraints

Eq. (4) forces the starting time of a product family $f'$ that follows another product family $f$ on a packaging line $j$ (i.e. $X_{f'jn} = 1$) is greater than the completion time of product family $f$ plus the necessary changeover time $s_{df}$ between them. $M$ is a big number.

$$C_{jn} + s_{df} \geq C_{jn}' - T_{jn} + M(1 - X_{fjn}) \forall f, f' \neq f, j \in (J_{pack} \cap F_{j}), n$$

5.3. Allocation and sequencing constraints

Eqs. (5) and (6) state that if a product family $f$ is allocated to packaging unit $j$ at period $n$, at most one product family $f$ is processed after and/or before it, respectively.

$$\sum_{f' \neq f, f' \in F_{j}} X_{f'jn} \leq Y_{jn} \forall f, j \in (J_{pack} \cap F_{j}), n$$

$$\sum_{f' \neq f, f' \in F_{j}} X_{f'jn} \leq Y_{jn} \forall f, j \in (J_{pack} \cap F_{j}), n$$

Eq. (7) denotes that the packaging unit $j$ is used in period $n$, (i.e. $Y_{jn} = 1$) if at least one product family $f$ is assigned at period $n$. Eq. (8) states that the total number of sequencing binary variables plus the unit utilization binary variable should be equal to the total number of allocation binary variables in a packaging unit $j$ at period $n$.

$$Y_{jn} \leq Y_{jn} \forall f, j \in (J_{pack} \cap F_{j}), n$$

$$\sum_{f \in F_{j}} \sum_{f' \neq f, f' \in F_{j}} X_{f'jn} + Y_{jn} = \sum_{f \in F_{j}} Y_{jn} \forall j \in J_{pack}, n$$

5.4. Fermentation stage constraints

Fermentation and pasteurization stage restrictions must be included to the mathematical model in order to obtain feasible production schedules. Eq. (9) states that the cumulative packaged quantity of product families $f$ that come from the same fermentation recipe $r$ should be greater than the minimum produced fermentation recipe amount in the pasteurization/fermentation stage and lower than the maximum production capacity. Moreover, a fermentation recipe $r$ is produced at period $n$, (i.e. $Y_{rn} = 1$), if at least one product family $f \in R_{r}$ is packaged in a packaging unit $j$ at the same period $n$.

$$QR_{rn}^{max}Y_{rn} = \sum_{f \in R_{r}} \sum_{j \in (J_{pack} \cap F_{j})} Q_{jn} \leq QR_{rn}^{max}Y_{rn} \forall r, n$$
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\[ YR_{rn} \geq \sum_{j \in (J^{m} \cap RF_f)} Y_{j fn} \quad \forall r, f \in RF, n \]  

(10)

5.5. Common resources constraints
Appropriate common resource constraints regarding fruit-mixer equipments are also included in order to cope with the current production line configuration (NFM).

5.6. Mass balance constraints
In fresh food industry, backordering is not allowed; unsatisfied demand is lost. The total product family quantity produced at period \( n \) should not exceed the cumulative demand for the same product family \( f \) for all periods equal to or greater than actual period \( n \), as Eq. (11) states. Customers’ demand satisfaction is forced by Eq. (12). The inventory \( St_{fn} \) of product family \( f \) is estimated through the traditional mass balance of Eq. (13).

\[ \sum_{j \in (J^{m} \cap RF_f)} Q_{j fn} \leq \sum_{n=2}^{N} dem_{n} \quad \forall f, n \]  

(11)

\[ \sum_{j \in (J^{m} \cap RF_f)} Q_{j fn} \geq \sum_{n} dem_{n} \quad \forall f \]  

(12)

\[ St_{fn} \geq St_{fn-1} + \sum_{j \in (J^{m} \cap RF_f)} Q_{j fn} - dem_{fn} \quad \forall f, n \]  

(13)

5.7. Objective function
The optimization goal to be minimized is the total cost that includes several cost-related factors such as: (i) inventory costs, (ii) operating costs, (iii) fermentation recipes preparing costs, (iv) unit utilization costs, and (v) product families’ changeover costs.

6. Industrial Case Study
An industrial case study of 93 final products that have been grouped into 23 product families is addressed here. All models have been resolved using CPLEX 11.0 solver via the GAMS 22.8 interface. The optimal solution has been reached in all cases at a very low computational time. The bigger-size NFM model gave the optimal solution in 22 CPU seconds while FM and FM&M cases were solved in half a second.

FM model’s solution shows a 7.6% improvement over that of the NFM. The FM&M total cost is 10.8% lower than that of the NFM model. A visual representation is illustrated in Fig. 3. Note that FM and FM&M configurations lead to lower total inventory cost compared to that of the NFM (12.2% and 16.4%, respectively). Dairy plant opens on Sunday (\( n_0 \)) in the FM case, in order to achieve full demand satisfaction,
resulting into higher operating costs. The other alternatives are capable of satisfying the demand profile without overtimes. Fig. 4 presents the Gantt charts for all cases.

Figure 4: Production schedules for all plant configurations.

7. Final Considerations
This model aims at being the core element of a computer-aided advanced scheduling and planning system in order to facilitate decision-making in dairy plant industrial environments. A salient feature of the dairy industry is that the customers usually confirm (i.e. change) their order quantities just prior to dispatch, thus the production should be changed on the fly. Yoghurt is a perishable product and strategies of building up inventories are inappropriate since they compromise its quality, its selling price, its freshness, and its goodness. Therefore, a reactive production planning approach to address these problems in this kind of industries is a challenging research direction.

8. Acknowledgements
Financial support from the Spanish Ministry of Education (FPU grant) and project DPI2006-05673 is gratefully acknowledged. The authors would like to thank Mr. Nikolas Polydorides, production manager at KRI-KRI S.A., for the provision of data and the fruitful comments and suggestions.

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