

The Sizing of Plate-Fin Exchangers to Fixed Dimensions Within a Volume Design Region

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This paper shows the development of a design approach for plate and fin heat exchangers to meet fixed dimensions. This approach adopts the concept of volume design region that establishes the limits within which the physical dimensions (length, width and height) of a specific design problem can be set. The design region is determined by minimum and maximum dimensions. The heat exchanger volume is dictated by the problem specifications and the type of secondary surface used on each of the fluids. High density surfaces tend to produce small volumes, while the opposite applies to low density surfaces. In principle, if the heat transfer and friction factor correlations for secondary surfaces are expressed as a function of the geometrical parameters that define the fin density, then it possible to fix the surface density boundaries that give the smallest and largest exchanger volume. The design methodology presented in this work enables to include exchanger dimensions as a design objective along with the heat load and the pressure drop. To achieve these objectives, surface design is a central strategy. In this work, triangular, louvered, rectangular and offset surfaces are used to demonstrate the methodology.

1. Introduction

Plate and fin heat exchangers were originally developed for gas to gas applications. However, the new manufacturing techniques have made it possible to construct them in almost any kind of material and geometry (Hathaway et al., 2018), making them suitable for application with liquids and at higher temperatures and pressures (Mortean et al., 2016). In the plate and fin technology, fluids flow through channels separated by metal walls. Between these plates, secondary surfaces are placed to provide structural strength, to increase the heat transfer area, and for heat transfer enhancement. The thermal performance of a plate and fin heat exchanger depends mainly on the thermohydraulic characteristics of the heat transfer surface. A key issue in design, is their appropriate specification. Secondary surfaces tend to produce high heat transfer coefficients and pressure drop at low Reynolds numbers, and for the purposes of design, is of paramount importance the know the way the heat transfer coefficient and the friction factor behave as a function of Reynolds and the fin geometry. To date, a large amount of experimental data for compact surfaces has been published since the first largest collection reported by Kays and London (1984). Since then, studies have extended the availability of experimental data and semiempirical correlations (Rui et al., 2017). Further studies have demonstrated that higher fin densities improve the heat transfer performance of these exchangers (Yang et al., 2017). The design of plate and fin exchangers has been taken the route of optimisation studies seeking to find the design solutions for minimum heat exchanger volume by surface selection (Kunpeng et al., 2015) and by multi-objective optimisation (Khan and Li, 2017). Other authors have recognized the importance of surface selection when space is a limitation (Tao et al., 2017). Recent research developments in plate and fin heat exchangers have centred around the innovative production of new heat transfer surfaces aiming at improving heat transfer and friction performance. Such investigations demonstrate the important role that secondary surfaces play in the performance, size and cost of these devices. The present paper introduces a design approach for plate and fin heat exchangers where block dimensions become an additional design objective. The design approach is based on the engineering of secondary surfaces to meet a specific thermo-hydraulic performance. The work is organized as follows: The design principles for plate and fin exchangers is revised; then the thermohydraulic

aspects for the design of triangular, rectangular, offset and louvred surfaces are presented. Finally, the design approach is demonstrated on a case study.

2. Design equation

The geometry of a plate and fin heat exchanger requires the definition of the exchanger height, width, and length. For each stream, the type of secondary surface employed, and the number of passages must also be specified. The type of surface is a design element that must be fixed at the outset of a design approach. The general expression for the design of a heat exchanger is:

$$Q = U A F \Delta T_{LM} \quad (1)$$

Where U is the overall heat transfer coefficient (W/m^2K), A is the total surface area (m^2), F is the correction factor of the logarithmic mean temperature difference and ΔT_{LM} is the logarithmic mean temperature difference (K). For plate and fin heat exchangers, the total heat transfer area per unit volume is greater compared with other technologies; this feature is referred as area density β (m^2/m^3); for this reason, is common to express their dimensions as a function of the total exchanger volume, V_T (m^3). Similarly, the total surface area for the hot and cold sides may vary significantly with the type of secondary surface used. One way of dealing with this is by linking the total surface area for each side to the total volume of the heat exchanger (Picón-Núñez et al., 1999). This is represented by the term α , and is calculated as follows:

$$\alpha_i = \frac{A_i}{V_T}; i = 1,2 \quad (2)$$

Where α (m^2/m^3) is the ratio to the total surface area of one side of the exchanger to the total exchanger volume (V_T). The term i denotes the hot and cold side; for each side, α is calculated from the geometrical characteristics of the type of surface employed as:

$$\alpha_i = \beta_i \left(\frac{\delta_i}{\delta_1 + \delta_2 + 2F_{th}} \right); i = 1,2 \quad (3)$$

The term β is the area density and relates the surface area on one side of the heat exchanger to the volume on that side, δ is the plate spacing (m) and F_{th} is the plate thickness (m). The total surface temperature effectiveness of the fin can be determined from (Kays and London, 1984):

$$\eta_o = 1 + f_s \left\{ \frac{\tanh[(2h/kF_{th})^{1/2}(\delta/2)]}{[(2h/kF_{th})^{1/2}(\delta/2)]} - 1 \right\} \quad (4)$$

The term f_s is the ratio of the secondary surface area to that of the total surface area, for triangular surfaces is expressed as:

$$f_s = \frac{(\delta - F_{th})/\cos\theta}{[(a_t - F_{th}) + (\delta - F_{th})/\cos\theta]} \quad (5)$$

Where a_t is half of the base of the triangular fin (m) and θ is the characteristic angle ($^\circ$). Introducing α into heat transfer expression Eq(1), for a counter current arrangement ($F = 1$) and free of fouling, the resulting expression is:

$$V_T = \frac{Q}{\Delta T_{LM}} \left[\frac{1}{(\eta_o h A)_1} + \frac{1}{(\eta_o h A)_2} + R_w \right] \quad (6)$$

Where R_w is the resistance to heat transfer due to the thermal conduction through the metal wall (K/kW). For the design of a plate and fin exchanger, volume is a more precise variable to account for the size of the unit.

3. Surface engineering

For a heat exchanger to transmit the required heat load within the limitations imposed by the pressure drop and within a set of desired dimensions, surface geometry becomes a degree of freedom that can be manipulated to simultaneously achieve the three design objectives. Surface engineering is the procedure whereby the surface geometry that meets a specific thermal performance is found. The thermal performance depends on three terms: The heat transfer coefficient (h), the total surface area (A) and the total surface temperature effectiveness (η_o). The assumptions in the development of the approach for surface engineering are: steady state operation, single phase heat transfer process, constant fluid properties, adiabatic operation, negligible longitudinal conduction effects, uniform heat transfer coefficients and uniform flow distribution. Figure 1 shows the main geometrical

dimensions that determine the thermo-hydraulic performance of secondaries surfaces such as triangular, rectangular, offset, and louvered.

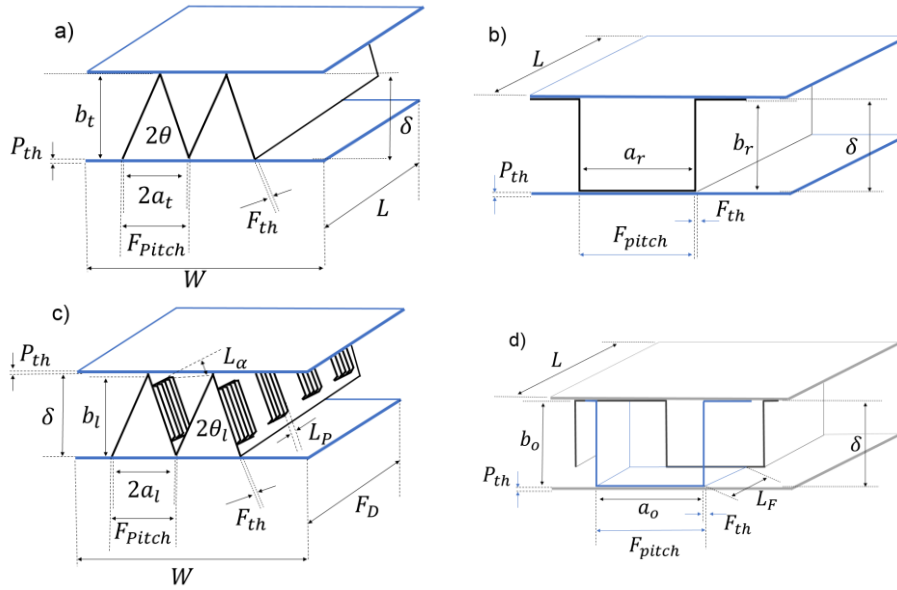


Figure 1: Geometry of secondary surfaces: a) Triangular, b) Rectangular, c) Louvered, d) Offset strip-fin

The heat transfer and friction performance of the secondary surfaces are determined from the expressions presented by several authors. The pressure drop due to friction across the core of the heat exchanger is expressed by:

$$\Delta P = \frac{2fLm^2}{\rho d_h A_c^2} \quad (7)$$

Where f is the friction factor, ρ is the fluid density (kg/m^3), L the flow length (m), m is the mass flow rate (kg/s) and A_c is the free flow area (m^2). For the complete specification of fin surface, d_h is the hydraulic diameter (m) and is calculated from the surface parameters as a function of the fin height and fin pitch.

4. Volume design approach

The volume design region represents the volume space where a feasible heat exchanger exists. A volume region has minimum and maximum boundaries. These are determined, when the highest surface area density (with the largest number of fins per inch) is used and when the lowest surface area density is used. For a two-stream heat exchanger the total volume is calculated using Eq(6). The type of secondary surface employed in design determines the shape and dimensions of the exchanger. For instance, a high-density surface results in a heat exchanger with a large frontal area and short flow length. With a low-density surface, the resulting exchanger exhibits low frontal area and long flow length. Table 1 shows heat transfer and friction correlation for different types of surfaces. As mentioned above, the relation between heat exchanger dimensions and fin geometry depends on the number of fins that can be accommodated per unit length in the flow direction. A high-density fin is designed when the values of the variables a_t, a_r, a_o , and a_l (Figure 1) take the smallest possible values. This is when the variables approximate the fin thickness: $a_t = a_l = F_{th}$ and $a_r = a_o = 2F_{th}$. A low-density surface is obtained when the number of fins per inch equals 1. In a pure countercurrent arrangement, only one of the streams can fully absorb the pressure drop allocated for design (Picon-Núñez et al., 1999). In this case, the stream chosen to maximise its pressure drop is referred to as the critical stream. In other arrangements such as the crossflow, both streams can fully absorb their pressure drop. For a given surface geometry, the pressure drop of the critical stream will fix the flow length and free flow area. To demonstrate the concept of volume design region, the maximum and minimum volumes are calculated using the same surface type and same surface density on a two-stream problem. The volume design region is calculated for triangular, rectangular, offset strip-fin and louvered surfaces. Figure 2 depicts the volume design region. The exchanger width is a degree of freedom that can be used to produce a design with a specific aspect ratio.

Table 1: Correlations for several heat transfer surfaces

Expression	Range of validity	Std dev	Notes
Rectangular surfaces			
$j = 0.233Re^{-0.48} \left(\frac{F_{pitch}}{\delta}\right)^{0.192} \left[\frac{F_{th}}{\delta}\right]^{-0.208}$	(8) 2,700 < Re < 10,000	±5.3%	(Diani et al., 2012)
$f = 0.029Re^{-0.09} \left(\frac{F_{pitch}}{\delta}\right)^{0.034} \left[\frac{F_{th}}{\delta}\right]^{-0.169}$	(9) 2,700 < Re < 10,000	±3.4%	(Diani et al., 2012) $j = \frac{hA_c}{mC_p} Pr^{2/3}$ (10)
Triangular surfaces			
$j = 0.718Re^{-0.625} \left[\delta/F_{pitch}\right]^{0.765} \left[F_{th}/F_{pitch}\right]^{0.765}$	(11) 100 < Re < 1,000	±12%	(Chennu, 2018)
$j = 0.789Re^{-1.1218} \left[\delta/F_{pitch}\right]^{1.235} \left[F_{th}/F_{pitch}\right]^{-0.764}$	(12) 1,000 < Re < 10,000	±12%	(Chennu, 2018)
$f = 3.12Re^{-0.852} \left[\delta/F_{pitch}\right]^{0.156} \left[F_{th}/F_{pitch}\right]^{-0.184}$	(13) 100 < Re < 1,000	±11%	(Chennu, 2018)
$f = 2.69Re^{-0.918} \left[\delta/F_{pitch}\right]^{0.355} \left[F_{th}/F_{pitch}\right]^{-0.175}$	(14) 1,000 < Re < 10,000	±11%	(Chennu, 2018)
Offset strip-fin surfaces			
$j = 0.6522Re^{-0.5403} \xi^{-0.1541} \delta^{0.1499} \eta^{-0.0678} (1 + 5.269 \times 10^{-5} Re^{1.34} \xi^{0.504} \delta^{0.456} \eta^{-1.055})^{0.1}$	(15) 300 < Re < 3,500		Rui et al., 2017 $\xi = \frac{a_o}{b_o}$ (16), $\delta = \frac{F_{th}}{L_{fo}}$ (17) $\eta = \frac{F_{th}}{a_o}$ (18)
$f = 9.6243Re^{-0.7422} \xi^{-0.1856} \delta^{0.3053} \eta^{-0.2659} (1 + 1.7669 \times 10^{-8} Re^{4.429} \xi^{0.92} \delta^{3.767} \eta^{0.236})^{0.1}$	(19) 300 < Re < 3,500		(Rui et al., 2017) $d_h = \frac{4a_o b_o L_{fo}}{2(a_o L_{fo} + b_o L_{fo} + F_{th} b_o) + F_{th} a_o}$ (20)
$\ln(j) = -0.0264136(\ln Re)^3 + 0.555843(\ln Re)^2 - 4.09241 \ln Re + 6.21681$	(21) 300 < Re < 3,500		(Rui et al., 2017) $d_h = \frac{2a_o b_o}{a_o + b_o}$ (23)
$\ln(f) = 0.132856(\ln Re)^2 - 2.28042 \ln Re + 6.79634$	(22) 300 < Re < 3,500		$\xi = \frac{a_o}{b_o}$ (24)
Louvered surfaces.			
$j = Re \left[-0.484 - \frac{1.887}{\ln Re} \right] \left[\frac{F_d}{L_p} \right]^{0.157} \left[2.24 - 0.588 \ln \left(\frac{F_{pitch} \sin L_\alpha}{L_p} \right) \right]$	(25) 100 < Re < 3,000		(Erbay et al., 2017) $Re = \frac{\dot{m} L_p}{\mu A_c}$ (27)
$f = Re^{-0.433} \left[\frac{F_d}{L_p} \right]^{0.185} \left(1.10 + 4.31 \left(\frac{L_\alpha}{90} \right)^2 + 0.836 \frac{\ln \left(\frac{F_{pitch}}{L_p} \right)}{\left(\frac{F_{pitch}}{L_p} \right)^2} \right)$	(26) 100 < Re < 3,000		(Erbay et al., 2017)

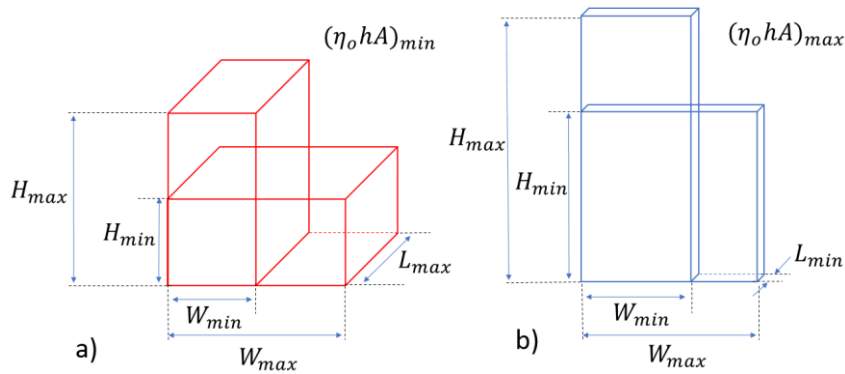


Figure 2: Pictorial representation of the volume design region: a) maximum volume, b) minimum volume

5. Case study

The case study refers to the design of a two-stream heat exchanger using plate and fin technology. Table 2 presents the operational data and physical properties of a problem taken from the literature (Smith, 1994). The design approach will provide the volume design region where feasible solutions exist. The fin and plate thickness used for the problem are 0.0003 m and 0.002 m, and the plate spacing (δ) is 0.0065 m.

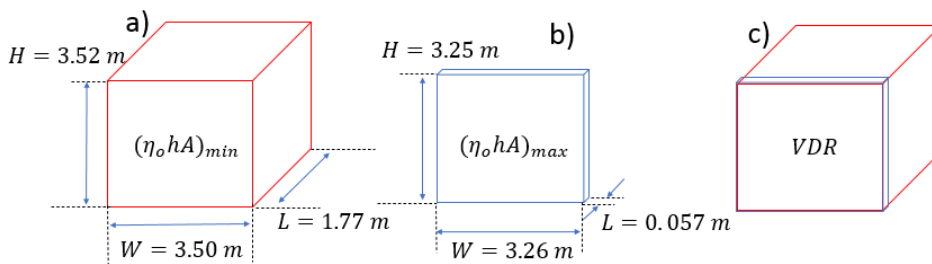
Table 2: Operating data and physical properties for case study

Flow stream parameters	Hot gas	Cold air
Mass flowrate (kg/s)	24.68	24.32
Pressure drop (Pa)	2,659.6	3,562.9
Inlet Temperature (K)	702.6	448.2
Outlet Temperature (K)	521.3	637.9
Physical properties mean values		
Prandtl number	0.670	0.670
C_p (J/kg K)	1,084.80	1,051.90
Viscosity (Pa·s)	0.000030	0.000028
Thermal conductivity (W/m K)	0.0488	0.0447
Density (kg/m ³)	0.577	5.827
Heat capacity mass flow rate CP (kW/K)	26.78	25.58

Using the operating information in Table 2 as input parameters, an iterative approach is implemented, and the design results are shown in Table 3. For each type of surface, the two columns represent the design using the lowest and highest fin density. As can be seen, for the case of the highest fin density, the louvered fin gives the lowest volume $V_T = 0.39 \text{ m}^3$ when compared with other geometries; however, it is not the case for the lowest fin density. In this circumstances, the offset strip-fin surface exhibits lower volume. This situation comes about as a result of the louvered fin having similar shape to the triangular surface, and, for a low fin density, the offset strip fin has a larger heat surface area. From the results in Table 3, it can be seen that the triangular surface gives the higher volumes for both conditions, $V_{T,max} = 21.81 \text{ m}^3$ and $V_{T,min} = 0.61 \text{ m}^3$. The flow length that corresponds to the minimum volume is $L = 0.057 \text{ m}$ and the free flow area is 3.37 m^2 . The flow length for the maximum volume is $L = 1.77 \text{ m}$ and the free flow area 2.97 m^2 . It is important to mention that the volume design region is case sensitive. Now, these results can be expressed in a different way: any flow length between $L = 0.057 \text{ m}$ and 1.77 m and free flow area between 3.37 m^2 and 2.97 m^2 , can be achieved if the fin density is modified accordingly. The fixing of the plate width (W) fixes the plate height (H), the shape of the frontal area (aspect ratio) can be accommodated to desired relative dimensions. For a near 1 aspect ratio, Figure 3 depicts the upper and lower limits for the volume design region of the feasible solutions.

Table 3: Volume and block dimensions for the case study.

Dimension	Triangular		Rectangular		Offset strip-fin		Louvered	
	F_{in} (fins/in)	F_{in} (fins/in)	F_{in} (fins/in)	F_{in} (fins/in)	F_{in} (fins/in)	F_{in} (fins/in)	F_{in} (fins/in)	F_{in} (fins/in)
	$F_{in}=1$	$F_{in}=28.2$	$F_{in}=1$	$F_{in}=28.2$	$F_{in}=1$	$F_{in}=28.2$	$F_{in}=1$	$F_{in}=28.2$
Volume (m ³)	21.81	0.61	7.03	0.35	6.24	0.51	9.88	0.39
Width (m)	3.5	3.25	3.40	1.04	2.70	2.8	2.38	3.24
Height (m)	3.52	3.26	3.41	1.05	2.67	2.7	2.37	3.25
Length (m)	1.77	0.06	0.07	0.3151	0.86	0.07	1.75	0.06
Surface area (m ²)	3,802	1,319	1,387.7	388.6	1,148	503.4	1,721	854.1
Pressure drop (Pa)	2,660	2,660	2,660	2,660	2,660	2,660	2,660	2,660
	161.10	161.10	127.20	127.20	217.70	156.6	778.0	778.0

Figure 3: Volume design region using triangular surfaces: a) $F_{in} = 1$, b) $F_{in} = 28.22$, c) Volume design region

6. Conclusions

This paper has introduced a design methodology for plate and fin heat exchangers where block dimensions are viewed as a new design objective. The main conclusions of this work are:

- Secondary heat transfer surfaces are a degree of freedom that can be used to achieve specific dimensions as a design objective.
- Surface engineering is a design strategy based on the selection of fin density through which, heat duty, pressure drop, and block dimensions can simultaneously be achieved.
- For a given design problem, the volume design region defines the minimum and maximum volume achievable and is case dependent.
- Any heat exchanger can be designed within the limits imposed by the volume design region.
- One of the limitations of this approach is the range of validity of the generalized expressions for heat transfer and friction factor. This is the case of very low viscosity fluids, that tend to exhibit large Reynolds numbers which go beyond the range of validity of the expressions.

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