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Numerical Simulation of Magneto Hydrodynamic Natural Convection in a Vertical Cylindrical Crucible

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The purpose of this paper is a numerical study to investigate the effects of external horizontal magnetic field on heat transfer characters within a pure molten metal filled a vertical 3D cylindrical crucible. The inner and outer walls of annulus are kept at constant temperature and the upper and lower walls are thermally insulated. The model is based on magneto hydrodynamic natural-convection theory and low Reynolds number approximation. Finite volume approach and fully implicit scheme is adopted to discrete corresponding equations. Results have been conducted for three different geometrical sizes, Rayleigh and Hartman Numbers. The 3D simulation results indicate a strong relation between heat transfer, the magnetic field strength and the aspect ratio of the crucible. In addition, this model shows that the azimuthal distribution of velocity and temperature fields affected greatly in crucible in presence of external magnetic field. The model has been validated availiable results. The findings of this study are expected to be useful for enhancing the design of solidification systems.

1. Introduction

Solidification processing has been used extensively in metallic alloys and semiconductors industry for many years. From the view of production, the final product quality is dependent upon many parameters including composition, cooling rate, boundary conditions, body and external forces and the temperature field of the melt region. In fact, these parameters could affect the flow patterns in the melt by producing convection flows. In addition, the nature of these flows is so that plays significant role on melt/solid front morphology and solidified structure. Also, experimental evidences proved that during solidification process such fluid motions in molten metal affect performance of final product. Hence, controlling unwanted convection flows in melt is crucial. In practice, it is possible to overcome these limitations partially by several methods. An interesting solution is applying an external magnetic field to suppress the buoyancy driven flows which referred to Magneto hydrodynamics (MHD) problems in the literature.

Whatever we recognize as main figures of MHD theory refers to Hartmann, Alfvén, and other’s experiments in the first half of the twentieth century (Davidson 2001). (Sarris et al., 2010) carried out numerical investigations about the problem of transient and turbulent natural convection of the electrically conductive low-Prandtl number fluid that driven by horizontal temperature gradients in a vertical cylinder in the presence of a vertical magnetic field. An investigation of solidification and melting of gallium in presence of constant magnetic field in a rectangular cavity was performed (Charmchi et.al, 2004). (Sankar et al., 2008) studied the effect of magnetic field on the driven convection in combination of buoyancy and surface tension forces in a cylindrical annular enclosure. (Afrand et al., 2017) investigated a 3D numerical method to analyse the natural convection in a cylindrical annulus containing molten potassium under a magnetic field. An experimental study for free convection in a vertical annulus was presented by Ejaz and Manzoor (2018). Effects of a bottom heated rotating cylinder in a vertical annulus have been experimentally studied with respect to heat transfer parameters. (Vasanthakumari and Pondy, 2018) presented a numerical investigation of steady two dimensional laminar, boundary layer flow of incompressible, viscous nanofluid with MHD along with heat generation and suction effect is considered. (Wang et al., 2017) examined the natural convection heat transfer of Al_2O_3-water nanofluid in a differentially-heated three-dimensional cubic enclosure. The study focuses on the

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effects of the shape of nanoparticles, nanoparticles volume fraction, and Rayleigh number on the natural convection.

The objective of present study is to investigate the effect of magnitude of magnetic field, Ra number and aspect ratio on flow and temperature fields of the molten metal numerically. The other aim of this work is to investigate the effect of cancelling the electric induction field in calculations. Most of the MHD studies at present are based on regular rectangular shapes and limited studies are carried out using annulus shapes. The annulus cavity has received significant attention of researchers due to its applicability in various fields. In present study we tried to evaluate the effect of external magnetic field on the flow and temperature fields of an electrically conducting molten gallium in a vertical annulus.

2. Methodology

2.1 Physical model and assumptions

The problem that considered in this study is depicted schematically in Figure 1 and refers to the three-dimensional flow in an annulus whose height and width are given by H and D, respectively. It is filled with an electrically conducting molten metal. The inner wall (colder side) temperature set to be T_C and the outer wall (hotter side) is kept at T_H so that $T_H>T_C$. The other two surfaces (the upper and lower walls) are thermally insulated with rigid boundaries. Temperature gradients by two vertical walls cause density changes in molten metal and in presence of gravitational forces buoyancy flows are driven. In this problem a uniform external magnetic field B_0 also is applied in horizontal direction. In order to reduce calculation costs, the set of equations will be simplified by some assumptions. The melt flow behaves as a Newtonian incompressible fluid and considered to be laminar and steady. The thermo-physical properties of the melt are temperature independent except for density of melt that employs the Boussinesq approximation, $\rho = \rho_0(1-\beta(T-T_0))$ where β is volumetric thermal expansion, ρ_0 and T_0 indicate reference and temperature, respectively. Also, the heat dissipation, Joule heating, induced magnetic and electric fields are also neglected.

Invoking the previous assumptions, the governing three-dimensional mass, momentum, energy and Lorentz forces equations are as follows:

The continuity equation:

$$\nabla.\left(\rho\vec{V}\right) = 0 \tag{1}$$

The momentum equation

$$\rho\left(\vec{V}.\nabla\right)\vec{V} = \nabla P + \mu\nabla^2\vec{V} + \vec{F}_{\text{boyouncy}} + \vec{F}_{\text{magnetic}} \tag{2}$$

The energy equation

$$\left(\vec{V}.\nabla\right)T = \alpha\nabla^2 T \tag{3}$$

The magnetic body force in Eq (2) calculated as

$$\vec{F}_{\text{magnetic}} = \vec{J} \times \vec{B}_0 \tag{4}$$

Also, we assumed that the induced magnetic field produced by the motion of an electrically conducting fluid is negligible compared to the applied external magnetic field of B_o. The ratio is defined through the magnetic Reynolds number R_m, So R_m<< 1 (Davidson 2001). For small magnetic Reynolds numbers this assumption lets to use Ohm’s law for calculation of the electric current density as

$$\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B}_0) \tag{5}$$

Where, σ is the electric conductivity of the melt and $\vec{E} = -\nabla\varphi$ is induced electrical field by the motion of electrically conducting melt and assumed to be negligible. In present study this melt assumed to be molten gallium with thermo physical properties tabulated in table 1. The current density also can be expressed as

$$\vec{J} = \sigma(u_r\vec{r} + u_\theta\vec{\theta} + u_z\vec{k}) \times \vec{B}_0 \tag{6}$$

The initial and boundary conditions are specified as follows

$$\text{for } t < 0 \quad T = T_{ini} < T_m \tag{7}$$

$$\text{for } t \geq 0$$

$$T = T_C > T_m, \quad u_r = u_\theta = u_z = 0 \quad \text{at} \quad r = r_i \tag{8}$$

$$T = T_H > T_m, \quad u_r = u_\theta = u_z = 0 \quad \text{at} \quad r = r_o \tag{9}$$

$$\frac{\delta T}{\delta z}, \quad u_r = u_\theta = u_z = 0 \quad \text{at} \quad z = 0, H \tag{10}$$

Further, in MHD problems resulting flows generally depend on many dimensionless parameters. In this study results will discuss through the Rayleigh and Hartmann number and defined as

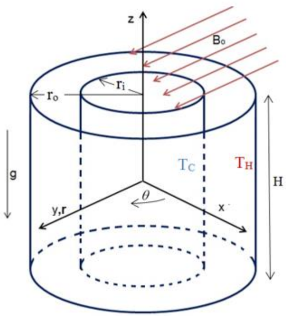
$$Ra = \frac{g\beta\Delta T D^3 \rho^2 C_p}{\nu\alpha}, \qquad Ha = DB_0\sqrt{\frac{\sigma}{\rho\nu}} \tag{11}$$

Where $D=r_o-r_i$ is characteristic length for Ra and Ha. For the present natural-MHD convection flow results validated by comparison the heat transfer rate that may be generally expressed as local and averaged Nusselt numbers on the hot wall in the form of

$$Nu = \frac{D}{T_H - T_C}\frac{\partial T}{\partial r}|r = r_i, \qquad \overline{Nu} = \frac{1}{2\pi}\int_0^{2\pi}\int_0^H Nu\,\partial\theta\,\partial z \tag{12}$$

2.2 Numerical procedure

Using the control volume approach the described equations discretized with non-uniform grid size on a staggered arrangement for the velocities. Pressure-velocity coupling is handled by the SIMPLE scheme. The convective terms are discretized by using the second order upwind differencing, and a second-order central difference scheme is preferred for the buoyancy and diffusion terms for numerical stability. The QUICK scheme employed in solution of energy equation. To solve the algebraic equations and related initial and boundary conditions we employed Guass-sieddel method. To find the accuracy and performance of the present numerical model, results are compared with the available numerical models. To ensure of present model performance, the results were validated with applying the external magnetic field. For this purpose, a constant external magnetic field was applied in x-direction. Figure 2 shows the predicted results of present model and those available in literature (Afrand et al., 2017) and (Shankar et al., 2008). It can be seen from this table that agreement between the 3D results is very good. It is noted that grid independence study has been done by a careful verification for different cases.



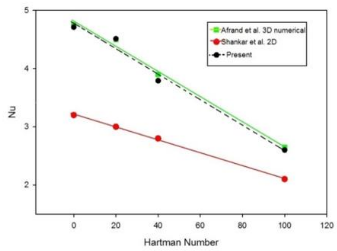


Figure 1: The physical model and the boundaries *Figure 2: Calculated Nusselt numbers for validation test*

3. Results and discussion

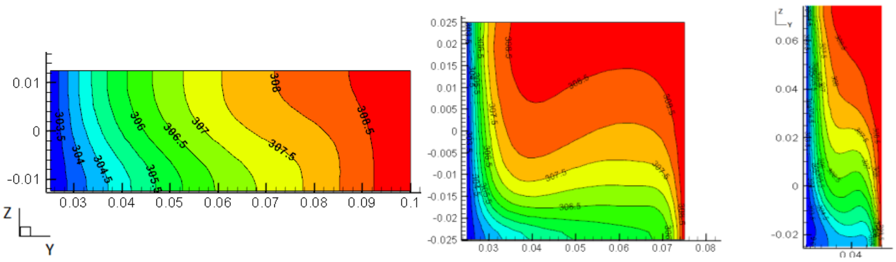
Because of expensive calculations of 3D models, most but not all of the works carried out in 2D axisymmetric models. However, when the simulated model is a vertical annulus and the applied magnetic field is in horizontal direction this assumption cannot be applied. In present numerical study the simulations are performed using gallium as molten metal that filled the 3D annulus with the thermo-physical properties that listed in Table 1. The Rayleigh number is varied from 10^4 to 10^6 and three magnetic field strengths of 0.01T, 0.05T, 0.075T are considered in calculations. Hartman number also is dependent upon the studied case and the Prandtl number is 0.0244. Three test cases with different radii ratio and heights are examined in simulations. Dimensions for the test cases are presented in Table 2. In all cases, the initial temperature of the gallium T_{int} is equal to 309 C. Also, in all simulations considered that temperatures of hotter T_H and colder T_C walls to be 309 K and 303 K, respectively. For convenience, $\lambda=r_o/r_i$ and $A=H/(r_o-r_i)$ are introduced as cavity aspect ratios. The temperature distributions and flow structures are represented for different Rayleigh numbers.

Table 1: Properties of the gallium (Charmchi et al., 2007)

Material Property	Value	
Density	6094.7	Kg/(m^3)
Heat capacity	397.6	J/(kg^{o}C)
Thermal diffusivity	1.29×10^{-5}	m^2/s
Viscosity	1.92×10^{-3},846,154.1	kg/(m.s)
Electrical Conductivity	1.29×10^{-4}	1/(Ω.m)
Thermal exp. coefficient	0.054	1/K
Prandtl number		-

Table 2: Dimensions of the annuluses considered

	Case 1	Case 2	Case 3
r_i, r_o 0.025,	0.1 m	0.025, 0.075 m	0.025, 0.05 m
H	0.025 m	0.05 m	0.1 m
$A=H/(r_o-r_i)$	1/3	1.0	4.0
$\lambda=r_o/r_i$	4.0	3.0	2.0



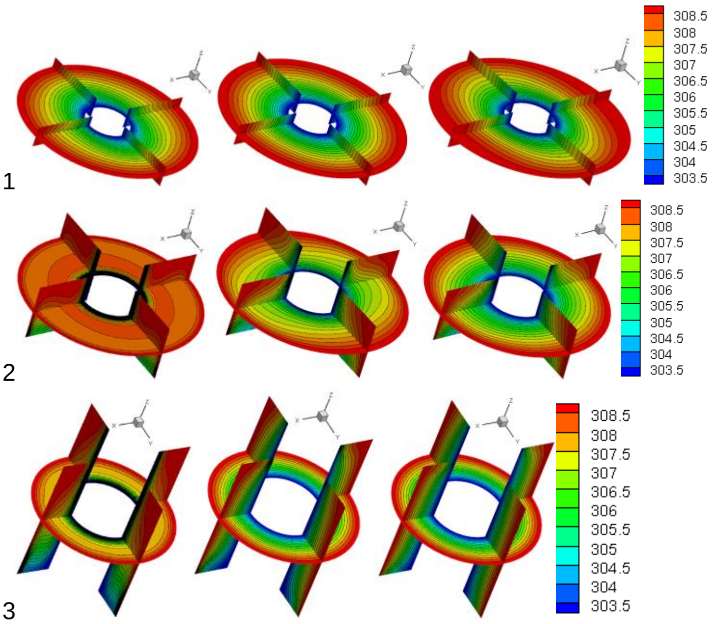
Case 1 Case 2 Case3

Figure 3: The isothermal contours in absence of magnetic field for Ha=0

Case 1

Case 2

Case 3

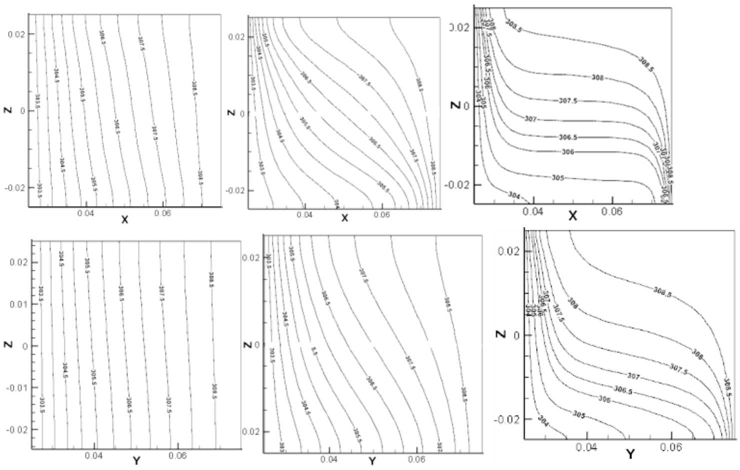


(a) B=0.01T (b) B=0.05 T (c) B=0.075 T

Figure 4: Comparison of the 3D isothermal contours for three cases in constant Rayleigh of $Ra=10^5$

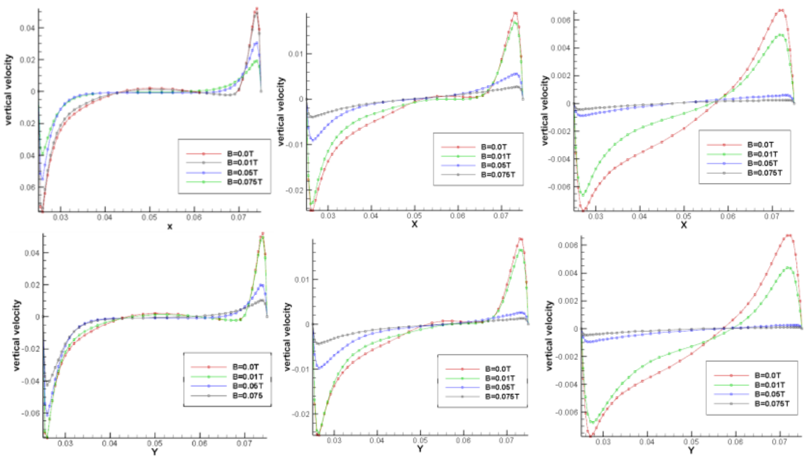
Figure 3 represents the results in a typical vertical cross section in three cases for Ra=10^5 and limiting case of pure thermal convection. Comparison of contours clears that convection structures in annuluses with high aspect ratio grow increasingly. Consequently, compared to short annuluses stronger magnetic fields are needed in suppressing the flows.

When magnetic field exists, the numerical computations are carried out to evaluate the effect of Lorentz force on the flow pattern and temperature distribution in liquid gallium. As noted previously, in present model the induced electrical field is negligible so the Lorentz force is generated only by the term of $V \times B_0$. Moreover, it can be found from Eq (10) that with an applied magnetic field along the y direction the resulting Lorentz force depends on direction of vertical plane. This force has a maximum value in z-y section thus is minimum at the z-x section. However, the convection structures of liquid gallium for different cross sections are represented for all test cases in Figure 4 for $Ra=10^5$. Figure 4 shows the 3D temperature fields for different Hartman numbers. These results illustrate clearly that in a constant Rayleigh number with increasing the magnitude of applied magnetic field the convection structures are damped greatly. As an interesting result it can be realized that with increasing the aspect ratio of annulus the stronger convection flows appear and obviously this suggests that suppressing the thermal stratifications in tall annuluses needs stronger applied magnetic field. This is due to intensity of convection flows.



(a) Ra=10^4 *(b) Ra=10^5* *(c) Ra=10^6*

Figure 5: Effect of Ra number on isotherms in x-z and y-z sections for B=0.075 T



(a) (b) (c)

Figure 6: Effect of magnetic field on vertical velocity profile in case 2 (a) Ra=10^4, (b) Ra=10^5, (c) Ra=10^6

To obtain the effect of Ra number the temperature isotherms are shown for case 2 for constant aspect ratio. This is accomplished by representing the results in x-z and y-z sections where minimum and maximum magnetic forces exist. Temperature profiles for $Ra=10^4$, 10^5 and 10^6 are shown in Figure 5. As Figure 5 shows for Ra=10^6 with applied magnetic field strength of B=0.075T thermal stratification still can be observed. The temperature distribution suggests the using of stronger magnetic field.

To further investigation of magnetic field influence on flow field, the vertical velocity profiles for case 2 are shown in Figure 6. The profiles indicate vertical component of velocity vector at the mid height of annulus in x-z and y-z sections. As can be seen effect of different magnetic field strengths were tested. Results indicate that with increasing in Rayleigh number vertical velocity increased considerably whereas with increasing the Hartman number (magnetic field) convection flow suppressed effectively. It is worthy to note that vertical velocity profiles in x-z and y-z sections have different magnitudes except for the case of B=0.

4. Conclusions

Understanding has been enhanced of the 3D thermo-magnetic convection within an electrically conducting fluid (gallium) filled an annulus in presence of a horizontal constant magnetic field. Three cases including tall, square and shallow shape annuluses are selected to study. Distribution of the temperature field and velocity vectors are given for different Rayleigh and Hartman numbers and results are compared to the case of no external magnetic field. The effect of annulus aspect ratio is also studied. The results of the simulation are as follows:

The natural convection inside the annulus greatly depends on Rayleigh number, magnetic field strength and annulus aspect ratio.

The results show that stronger magnetic field is needed to suppress convection flow in tall annuluses compared to the shallow ones, especially in high Rayleigh numbers.

The results indicate that when magnetic field applies, in shallow annuluses the azimuthal dependence of computed velocity and temperature fields are eminent compared to square and tall annuluses.

Present numerical results can be used as a comparative study on the validity and reliability of negligible induced electric fields assumption used in 3D MHD simulations.

Present numerical study provides a good prediction for thermo-magnetic convection in 3D annulus with negligible induced electric fields. These results are expected to be useful for enhancing the design of existing MHD solidification systems and to achieve high quality products.

References

Afrand M., Toghraie D., Karimipour A., Wongwises S., 2017, A numerical study of natural convection in a vertical annulus filled with gallium in the presence of magnetic field, Journal of Magnetism and Magnetic Materials, 430, 22–28. DOI: 10.1016/J.JMMM.2017.01.016

Charmchi M., Zhang H., and Li W., Faghri M., 2004, Solidification and melting of gallium in presence of magnetic field: experimental simulation of low gravity environment, IMECE2004-62365 Heat Transfer, 3, 581-589. DOI: 10.1115/IMECE2004-62365

Davidson P., 2001, Introduction to magneto hydrodynamic, Cambridge University Press.

Ejaz M.F., Manzoor S., 2018, Experimental investigation of heat transfer in a vertical annulus with a bottom heated rotating inner cylinder, International Journal of Heat and Technology, 36, 2, 730-740. DOI: 10.18280/ijht.360240.

Sankar M., Venkatachalappa M., 2008, Numerical study of double diffusive magneto convection in a cylindrical annulus, International Journal of Fluid Mechanics Research, 35, 19-38. DOI: 10.1615/InterJFluidMechRes.v35. i1.20

Vasanthakumari R., Pondy P., 2018, Mixed convection of silver and titanium dioxide nanofluids along inclined stretching sheet in presence of MHD with heat generation and suction effect, Mathematical Modelling of Engineering Problems, 5, 2, 123-129. DOI: 10.18280/mmep.050210.

Wang G., Zhang Y., Zhang J., Ma B., 2017, Effects of nanoparticles shape and concentration on natural convection of al2o3-water nanofluid in a cubic enclosure, Chemical Engineering Transactions, 61, 1171-1176. DOI: 10.3303/CET1761193.