CHEMICAL ENGINEERING TRANSACTIONS
VOL. 71, 2018

A publication of AIDIC
The Italian Association of Chemical Engineering
Online at www.aidic.it/cet

Guest Editors: Xiantang Zhang, Songrong Qian, Jianmin Xu

ISBN 978-88-95608-68-6; **ISSN** 2283-9216

DOI: 10.3303/CET1871161

Hydromagnetic Oscillatory Slip Flow of a Visco-elastic Fluid through a Porous Channel

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An investigation is carried out to study the effects of suction/injection on the slip flow of oscillatory visco-elastic hydromagnetic flow through a vertical porous channel. It is considered that the fluid has small electrical conductivity and the electro-magnetic force produced is also very small. The flow is subjected to suction at the cold wall and injection at the heated wall. Under the Boussinesq's approximation, the governing equations are solved using multiparameter perturbation technique. The effects of pertinent flow parameters on fluid velocity, temperature, shearing stress, the rate of heat transfer is obtained and illustrated graphically. This study plays an important role in physiological, industrial and hydrological problems under suitable conditions.

1. Introduction

Unsteady oscillatory flow of electrically conducting fluid has grabbed the attention of many researchers due to its admirable eruption in chemical engineering, turbomachinery and in aerospace technology etc. (Mishra et al., 2016) have investigated the MHD oscillatory channel flow with porous medium in presence of chemical reaction. (Makinde et al., 2005) have studied the heat transfer to MHD oscillatory flow in a channel filled with porous medium. (Adesanya et al., 2014) have investigated the effect of slip on the free reactors. Also, (Hamza et al., 2011) have investigated the unsteady heat transfer and effects of slip condition of a conducting optically thin fluid through a channel filled with porous medium. (Palani et al., 2009) have studied the combined effects of MHD and radiation effects on free connection flow past an impulsive isothermal vertical plate using Rosseland approximation. Also, (Kumari et al., 2017) have investigated the viscous dissipation and mass transfer effects on MHD oscillatory flow in a vertical channel with porous medium. In the above studies, the channel walls are assumed to be impervious. Again, for other suction/injection-controlled applications, several authors have investigated the connective heat transfer through porous channel. (Umavathi et al., 2009; Tang et al., 2009; Khramtsov et al., 2016) have contributions in this domain. The modelling in visco-elastic fluid has been studied extensively in many fluid flow problems. A good number of researchers viz. (Choudhury et al., 2012; Falade et al., 2017; Singh et al., 2014; Karunakar et al., 2013; Raptis et al., 1981; Choudhury et al., 2016; Priya et al., 2014; Benazir et al., 2015; Jha et al., 2010) have shown their interest in this field for its applications in chemical and petroleum engineering, Hydrology, Geo-physics, Paper and Pulp technology etc.

2. Mathematical Formulation

The unsteady laminar visco-elastic flow of an incompressible electrically conducting fluid through a channel with slip at the cold plate has been investigated (Figure 1). An external magnetic field is placed normal to the channel.
Under the usual Boussinesq approximation the equation governing the flow are as follows:

Equation of continuity:

$$\frac{\partial v'}{\partial t'} = 0 \Rightarrow v' = -v_0 \tag{1}$$

Please cite this article as: Choudhury R., Dey B., Das B., 2018, Hydromagnetic oscillatory slip flow of a visco-elastic fluid through a porous channel, Chemical Engineering Transactions, 71, 961-966 DOI:10.3303/CET1871161

Equation of motion:

$$\frac{\partial u'}{\partial t'} - v_0 \frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{dp'}{dx'} + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{k_0}{\rho}\left(\frac{\partial^3 u'}{\partial t' \partial y'} - v_0 \frac{\partial^2 u'}{\partial y'^2}\right)$$
$$-\frac{\nu}{k}u' - \frac{\sigma_e B_0^2}{\rho}u' + g\beta(T' - T_0) \quad (2)$$

Equation of energy:

$$\frac{\partial T'}{\partial t'} - v_0 \frac{\partial T'}{\partial y'} = \frac{k_f}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{4\alpha^2}{\rho C_p}(T' - T_0) \quad (3)$$

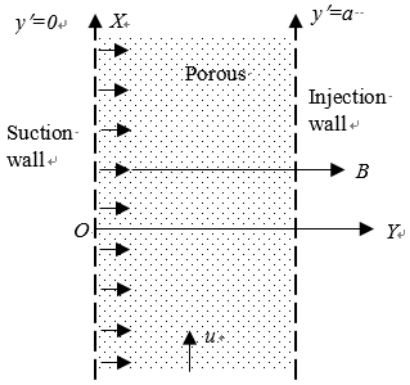

Figure 1: Flow configuration

corresponding boundary conditions are:

$$y' = 0; u' = \frac{\sqrt{k}}{\alpha_s}\frac{du'}{dy'}, T' = T_0$$
$$y' = a; u' = 0, T' = T_1 \quad (4)$$

where u' is the axial velocity, t' is the time?
We introduce the following dimensionless parameters and variables

$$x = \frac{x'}{h},\ y = \frac{y'}{h},\ u\ \frac{hu'}{\nu},\ t = \frac{\nu t'}{h^2},\ p = \frac{h^2 p'}{\rho \nu^2},$$
$$Gr = \frac{g\beta(T_1 - T_0)h^3}{\nu^2},\ \Pr = \frac{\rho C_p \nu}{k_f},\ \theta = \frac{T' - T_0}{T_1 - T_0},$$
$$\delta = \frac{4\alpha^2 h^2}{\rho C_p},\ \gamma = \frac{\sqrt{k}}{\alpha_s h},\ Ha^2 = \frac{\sigma_e B_0^2 h^2}{\rho \nu},\ Da = \frac{k}{h^2}$$
$$S = \frac{v_0 h}{\nu} \quad (5)$$

Where G_r denotes the Grashof number, P_r denotes Prandtl number, δ represents the thermal radiation, γ indicates Navier slip parameter, H_a designates magnetic parameter, D_a signifies permeability, S symbolizes suction parameter. The equations (2) and (3), are transformed into

$$\frac{\partial u}{\partial t} - S\frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial^2 u}{\partial y^2} - k_1\left[\frac{\partial^3 u}{\partial t \partial y^2} - S\frac{\partial^3 u}{\partial y^3}\right] - \left(Ha^2 + \frac{1}{Da}\right)u + Gr\theta \quad (6)$$

$$\frac{\partial \theta}{\partial t} - S\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + \delta\theta \tag{7}$$

the modified boundary conditions are:

$$y = 0 : u = \gamma \frac{du}{dy}, \theta = 0$$
$$y = 1 : u = 0, \theta = 1 \tag{8}$$

We assume an oscillatory pressure gradient, such that solutions of the dimensionless equations (6) and (7) under the boundary conditions (8) are in following forms:

$$-\frac{dp}{dx} = \lambda e^{i\omega t}, u(t, y) = u_0(y)e^{i\omega t}, \theta(t, y) = \theta_0(y)e^{i\omega t} \tag{9}$$

where λ is any positive constant, and ω is the frequency of oscillation?
Introducing (9) into the equations (6) and (7) we get the transformed equations as follows:

$$k_1 S u_0''' + (1 - k_1 i\omega)u_0'' + S u_0' - \left(Ha^2 + \frac{1}{Da} + i\omega\right)u_0 = -\lambda - Gr\theta_0 \tag{10}$$

$$\theta_0'' + S\Pr\theta_0' + (S - i\omega)\theta_0 = 0 \tag{11}$$

subject to the boundary conditions:

$$y = 0; u_0 = \gamma u_0', \theta_0 = 0$$
$$y = 1; u_0 = 0, \theta_0 = 1 \tag{12}$$

3. Method of solution

The solution of equation (11) subject to the boundary conditions (12) is

$$\theta_0 = c_1 e^{a_1 y} + c_2 e^{-a_2 y} \tag{13}$$

To solve (10), we use multiparameter perturbation scheme following Nowinski and Ismail (as k_1<<1 for small rate of shear) as

$$u_0 = u_{00} + k_1 u_{01} + O(k_1^2) \tag{14}$$

Using (14) in (10) and equating the coefficients of like powers of k_1 with the neglect of higher order terms, we get
Zeroth -order equation:

$$u_{00}'' + S u_{01}' - a_3 u_{00} = -\lambda - Gr\theta_0 \tag{15}$$

First-order Equation:

$$S u_{00}''' + u_{01}' - i\omega u_{00}'' + S u_{01}' - a_3 u_{01} = 0 \tag{16}$$

the relevant boundary conditions are:

$$y = 0; u_{00} = \gamma u_{00}', u_{01} = \gamma u_{01}' \quad and \quad y = 1; u_{00} = 0, u_{01} = 0 \tag{17}$$

Solving (15) and (16) subject to the boundary conditions (17) we get

$$u_{00} = a_6 e^{a_4 y} + a_7 e^{-a_5 y} + a_8 - a_9 e^{a_1 y} - a_{10} e^{-a_2 y} \quad u_{01} = a_{19} e^{a_4 y} + a_{20} e^{-a_5 y} + a_{15} y e^{a_4 y} + a_{16} e^{-a_5 y} + a_{17} e^{-a_1 y} - a_{18} e^{-a_2 y}$$

Using the above solutions, the expressions for velocity and temperature are obtained as follows:

$$u=[\left(a_6+k_1a_{22}\right)e^{a_4y}+\left(a_7+k_1a_{230}\right)e^{-a_5y}+a_8+\left(k_1a_{17}-a_9\right)e^{a_1y}-\left(a_{10}+k_1a_{18}\right)e^{-a_2y}k_1a_{15}ye^{a_4y}+k_1a_{16}ye^{-a_5y}]e^{i\omega t}$$

$$\theta=\left(c_1e^{a_1y}+c_2e^{-a_2y}\right)e^{i\omega t}$$

4. Results and discussion

In this study, the effects of visco-elasticity are examined using multiparameter perturbation technique. The software MATLAB is used for computational work. The real part of the solution is implied throughout the computation. Figures 1 to 7 depict the fluid velocity u against y for different pertinent flow parameters viz. magnetic parameter Ha, permeability parameter D_a, suction parameter S, thermal Grashof number G_r, thermal radiation δ, Navier slip parameter γ. Figures 8 to 13 show the variation of shearing stress σ against different fluid flow parameters. The numerical calculation is carried out by considering S=1, P_r=3, δ=1, H_a=4, D_a=1, γ=8, G_r=7, γ=0.1 unless otherwise stated. In all the cases, the fluid velocity reveals an accelerating trend for visco-elastic fluid in comparison with simple Newtonian case but an opposite pattern is demonstrated in case of shearing stress against flow parameters. The velocity distribution u against y (figure 5.1) indicates that the flow velocity first enhances and then diminishes with the increase in y for both Newtonian (K_1=0) and non-Newtonian (K_1=0.02, 0.04) cases for different flow parameters. Also, the fluid velocity enhances with the decrease of magnetic parameter H_a in both Newtonian and non-Newtonian fluids (figures 2 and 3). It is observed that maximum flow occurs when magnetic parameter is less and this is due to the generation of Lorentz force. Figures 2 and 4 show the effect of permeability parameter D_a of the fluid velocity and we observe that with the increase in permeability parameter the fluid velocity shows slightly increasing trend and this is due to that of free flow of fluid with the increase of D_a. Figures 2 and 5 reveal that due to the increase of suction/injection parameter S, the fluid velocity decreases in both Newtonian and visco-elastic fluids. With the enhanced value of Grashof number G_r the fluid velocity does not show a diminishing trend in fluid flow region (figures 2 and 6). The variation of fluid velocity for the enhancement of thermal radiation parameter δ is exhibited in figures 2 and 7. Figures 2 and 8 present the accelerating trend of fluid velocity for the growth of the slip parameter. The variation of shearing stress σ against magnetic parameter H_a, permeability parameter D_a, pressure gradient λ, Grashof number G_r thermal radiation δ, Navier's slip parameter γ are illustrated in figures 9 to 14. With the enhancement of H_a and γ, the shearing stress shows a decelerating trend in both Newtonian and visco-elastic fluid flows but and accelerating trend is observed with the variation of D_a, λ, G_r and δ.

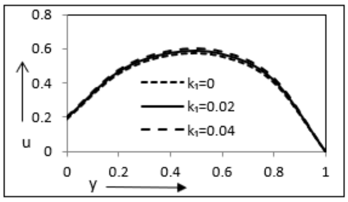

Figure 2: Fluid velocity u against y for S=1, Pr=3, δ=1 Ha=4, γ=0.1, ω=π

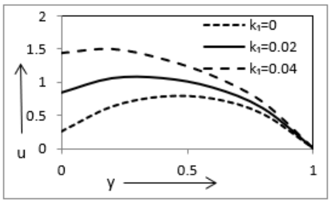

Figure 3: Fluid velocity u against y for S=1, Pr=3, δ=1, Da=1, λ=1, Gr=7, γ=0.1, ω=π

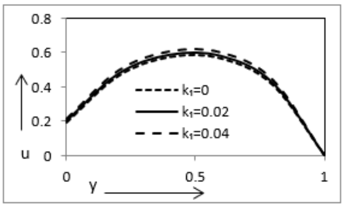

Figure 4: Fluid velocity u against y for S=1, Pr=3, δ=1, Da=1, λ=1, Gr=7, γ=0.1, ω=π

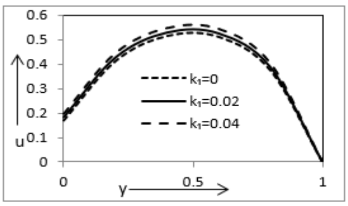

Figure 5: Fluid velocity u against y for S=2, Pr=3,δ=1 Ha=4, Ha=4, Da=2, λ=8, Gr=7, γ=0.1, ω=π

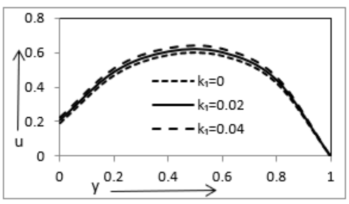

Figure 6: Fluid velocity u against y for S=1, Pr=3,δ=1 Ha=4, Da=1, λ=1, Gr=7,γ=0.1, ω=π

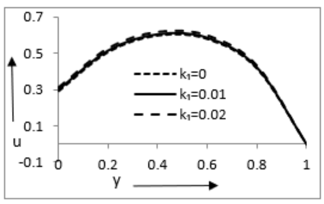

Figure 7: Fluid velocity u against y for S=1, Pr=3,δ=2, Ha=4, Da=1, λ=1, Gr=8,γ=0.1, ω=π

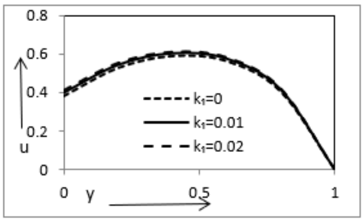

Figure 8: Fluid velocity u against y for S=1, Pr=3,δ=1, Ha=4, Da=1, λ=1, Gr=8, γ=0.4, ω=π

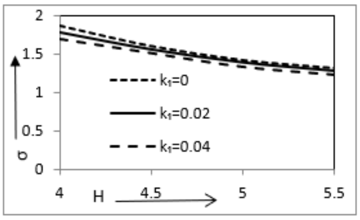

Figure 9: Shearing Stress σ against Magnetic parameter Ha

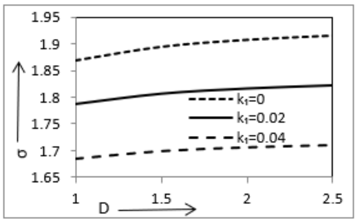

Figure 10: Shearing stress σ against Darcy parameter Da

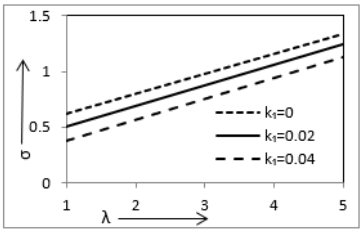

Figure 11: Shearing stress σ against pressure gradient λ

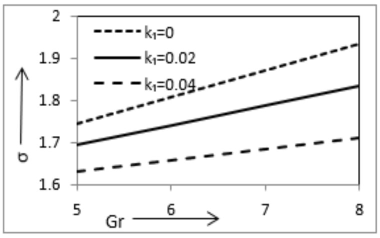

Figure 12: Shearing stress against Grashof number Gr

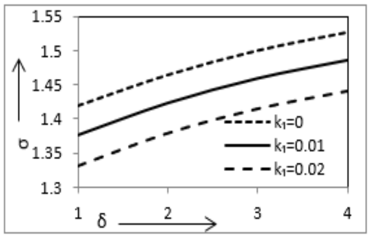

Figure 13: Shearing stress against thermal radiation δ

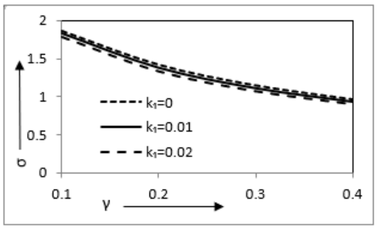

Figure 14: Shearing stress against Navier's slip parameter

5. Conclusions

From this study, some explicit conclusions are highlighted below:
This analysis establishes a significant role of visco-elastic parameter on the fluid velocity in the entire fluid flow region.
The fluid velocity is parabolic in nature in both Newtonian and visco-elastic fluids.
Velocity slip parameter significantly influences the velocity of the flow within the considered region.
The shearing stress diminishes with the increase of magnetic parameter and Navier's slip parameter but shows a reverse trend with the increase of permeability parameter, pressure gradient, Grashof number and thermal radiation parameter

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