CHEMICAL ENGINEERING TRANSACTIONS

VOL. 70, 2018

A publication of AIDIC
The Italian Association of Chemical Engineering
Online at www.aidic.it/cet

Guest Editors: Timothy G. Walmsley, Petar S. Varbanov, Rongxin Su, Jiří J. Klemeš

**ISBN** 978-88-95608-67-9; **ISSN** 2283-9216

DOI: 10.3303/CET1870310

# Linear and Nonlinear Control of an Upflow Anaerobic Sludge Blanket Reactor: Effects of IMC-PID Tuning and Uncertainty

Gustavo A. Rodriguez, Luis G. Obergon, Juan B. Restrepo*

Universidad del Atlántico, Chemical Engineering Program, Carrera 30 Número 8- 49 Puerto Colombia – Atlántico.
juanrestrepo@uniatlantico.edu.co

Upflow anaerobic sludge blanket (UASB) reactors are often used for wastewater treatment and microalgae production. UASB have a greater performance due to higher biomass concentrations. However, this comes at the cost of increased nonlinearities and therefore increased difficulties of control through linear methods (especially ones that make use of simplified models of the system). The methods developed to counter this type of problems include feedback linearization, which uses additional information on the system to create performance advantages. This paper performs a comparative study of both types of controllers applied to a UASB, first in their equilibrium points and their stability, secondly in the different stable regions (basins of attraction) of these points in the parameter space, and thirdly in the effects of identification uncertainty on over these regions. It is found that the two control loops have similar equilibrium points qualitatively and quantitatively. The kinetics of substrate inhibition limit the values of controller set point in the linear loop, and the uncertainty of identification gain has greater importance to that of the time constant.

## 1. Introduction

Biological processes are nonlinear (Zhai et al., 2017), especially in their kinetics (Lara-Cisneros et al., 2014), which is a challenge for linear control schemes (Garpinger et al., 2014). An example of this is the UASB type reactor, characterized by high biomass concentrations and a lot of parameters, representing an inconvenience to controller design (Nair and Ahammed, 2015). Many techniques are used to control bioreactors; they can be divided into linear and nonlinear methods. Linear methods usually use linear approximations, with their performance depending on the system complexity (El-Bardini and El-Nagar, 2014), while the nonlinear methods use additional information to create performance advantages (Hahn et al., 2004).
The PID control law (Proportional Derivative Integrative) is the most popular linear (Prasad et al., 2014). It has three adjustable parameters, usually determined by defined tuning methods, like the IMC-PID algorithm (JIN et al., 2013), this reduces the degrees of freedom of the PID to one parameter lambda ($\lambda$), being correlated to stabilization time (Bequette, 2003). An important nonlinear control method is Feedback linearization. This control law seeks to simplify systems by canceling nonlinearities (Lei and Khalil, 2016), the robustness has been studied by the use of bifurcation analysis to find the limitations of tuning and how these change with uncertainty (Hahn et al., 2008).
The main contribution of this paper is to do a comparative performance study of a linear PID tuned by an IMC-PID tuning rule and a feedback linearization controller coupled to a PID tuned by an IMC-PID tuning rule both applied to a bioreactor, by using bifurcation analysis to find and compare their stability regions of the closed loops in the process parameter space, using the stability regions of each controller to possibly derive truths about biological kinetics, PID control and IMC tuning, and how these relate to each other.
Explorations of the stability zone of a closed loop system under PID control and feedback linearization have been done before, especially in simple systems (Chang and Chen, 1984). Parametric stability zones are found and used for purposes like smarter tuning (Hahn et al., 2008), robustness analysis (Hahn et al., 2004), controller design (Zhusubaliyev et al., 2015), and fuzzy logic controller design (Galluzzo et al., 2008). These are often purely analytical studies and no deeper connection to phenomenological causes is found, this paper seeks to contribute in the deeper phenomenological understanding.

Please cite this article as: Rodriguez G.A., Obregon L.G., Restrepo J.B., 2018, Linear and nonlinear control of an upflow anaerobic sludge blanket reactor: effects of imc-pid tuning and uncertainty , Chemical Engineering Transactions, 70, 1855-1860 DOI:10.3303/CET1870310

## 2. The process

The study case is a UASB type reactor, with a wastewater inlet (with a dilution rate D), containing volatile fatty acids (VFAs) ($S_2$), and several chemical compounds which concentration is measured indirectly using Chemical Oxygen Demand (COD) ($S_1$). The bacteria inside the UASB (represented as $X_1$ and $X_2$) degrade the VFAs and the other chemical compounds reducing the COD, by Monod and Haldane kinetic expression respectively, producing $CO_2$ and $CH_4$, under the reactions $k_1S_1 \Longrightarrow X_1 + k_4CO_2 + k_2S_2$, and $k_3S_2 \Longrightarrow X_2 + k_6CH_4 + k_5CO_2$ (Bernard et al. 2001). The process model is presented in Eq(1).

$$\begin{bmatrix} \dot{X_1} \\ \dot{X_2} \\ \dot{S_1} \\ \dot{S_2} \end{bmatrix} = \begin{bmatrix} \mu_1 X_1 \\ \mu_2 X_2 \\ -k_1\mu_1 X_1 \\ k_2\mu_1X_1 - k_3\mu_2X_2 \end{bmatrix} + \begin{bmatrix} -\alpha X_1 \\ -\alpha X_2 \\ (S_1^{in} - S_1) \\ (S_2^{in} - S_2) \end{bmatrix} D \tag{1}$$

Where:

$$\mu_1 = \mu_{max1}\frac{S_1}{K_{S_1}+S_1} \quad \mu_2 = \mu_0\frac{S_2}{K_{S_2}+S_2+\left(\frac{S_2}{K_I}\right)^2} \tag{2}$$

And its parameters are listed in Table 1.

*Table 1: Model parameters*

| Parameter | Value | Parameter | Value | Parameter | Value | Parameter | Value |
|---|---|---|---|---|---|---|---|
| $D$ (days$^{-1}$) | 0.36 | $k_2$ ($\frac{mmolAGV}{gX_1}$) | 28.60 | $k_1$($\frac{gDQO}{gX_1}$) | 10.53 | $S_1^{in}$ ($\frac{g}{L}$) | 5.8 |
| $\mu_0$ (days$^{-1}$) | 0.74 | $K_{s2}$ ($\frac{mmolAGV}{L}$) | 9.28 | $\mu_{max1}$ (days$^{-1}$) | 1.20 | $\alpha$ | 0.5 |
| $K_{s1}$ ($\frac{gDQO}{l}$) | 7.10 | $K_I$ ($\frac{mmolAGV}{L}$) | 16 | $S_2^{in}$ ($\frac{mmol}{L}$) | 52 | $k_3$ ($\frac{mmolAGV}{gX_2}$) | 1,074 |

## 3. Controller design

### 3.1 Linear control by IMC-PID

An ideal PID has the form of Eq(3), where $K_c$, $\tau_i$, and $\tau_d$ are the controller parameters, $u$ is the input, $y$ the output, and subscripts $ss$ and $sp$ are stationary state and set point respectively.

$$u - u_{ss} = K_c(1 + (1/\tau_i)s + \tau_d s)(y_{sp} - y) \tag{3}$$

Using an approximate model $\widetilde{G_p}(s)$, then factorizing it into its invertible and non-invertible parts $G_p^-$, $G_p^+$ respectively, the IMC-PID method gives the tuning Eq(4) for Eq(3) (Bequette, 2003).

$$\mathrm{PID}(s) = G_p^-(s)^{-1}f(\lambda, s)/(1 - \widetilde{G_p}(s)G_p^-(s)^{-1}f(\lambda, s)) \text{ (Klemeš et al., 2017)} \tag{4}$$

Using a linear first-degree transfer function as $\widetilde{G_p}(s)$ to approximate the response of Eq(1) and applying the tuning Eq(4), produces the tuning parameters that can be seen in Eq(5). Where $I_k$ and $I_\tau$ are fractional identification uncertainties in the gain and temporal parameter.

$$K_p = 0.029(1 + I_\tau)/\lambda(1 + I_k) \text{ and } t_i = 0.33102(1 + I_\tau) \tag{5}$$

### 3.2 Input-output Linearization

This method determines an input $u$ such that the output $y^{(r)} = v$, $r$ being the minimum integer such that $y^{(r)}$ contains $u$ explicitly. The linearized system is then controlled with a IMC-PID using $v$ as a new input. The linearizing function can be found by applying Eq(6) - Eq(8) to the system (Isidori, 2013)

$$u = (\upsilon - L_f^r h(x))/\left(L_g L_f^{r-1} h(x)\right) \tag{6}$$

$$u = (\upsilon - L_f^r h(x))/\left(L_g L_f^{r-1} h(x)\right) \tag{7}$$

$$L_f h(x) = \sum_{i=1}^{n} f_i(x)\, \partial y/\partial x_i,\ L_{f(x)}^2 h(x) = L_{f(x)}L_{f(x)}h(x) \tag{8}$$

When the controlled variable is chosen to be $S_2$, the input-output linearization of the study case is Eq(9), and the PID controller used to manipulate $v$ is given by Eq(10)

$$D_l = -(v - k_2u_1x_1 + k_3u_2x_2)/(s_2 - s_2^{in}) \quad (9)$$

$$v = (sp - S_2)/\lambda \quad (10)$$

## 4. Parametric uncertainty and bifurcation analysis

Small values of $I_k$ and $I_\tau$ make a better control, the same happens with a correct choosing of $\lambda$ to create an adequate response, it's clear that knowing the relation of loop parameters to system properties is vital. An accepted approach to explore the parametric space is the bifurcation analysis (Kuznetsov, 2013). In simple terms, the bifurcation analysis of a dynamical system is the search for qualitative response changes (by monitoring the linearized system's eigenvalues) as the systems parameters are varied smoothly. Codimension two bifurcation analysis is used to map the stability behavior of the equilibrium points (steady states) of the closed loop UASB reactor as its control parameters are varied with the aim to assess its qualitative behavior and robustness, such equilibrium points are seen in Table 1 with $sp = 2.99$ and $\lambda = 0.1$, where $\beta$ and $\gamma$ are real parameters:

*Table 2: Equilibrium points controller loops (a) linear control (b) feedback linearization*

| (a) | 1st | 2nd | 3rd | 4th | (b) | 1st | 2nd | 3rd | 4th | 5th |
|---|---|---|---|---|---|---|---|---|---|---|
| $X_1$ $(\frac{g}{L})$ | 0.86 | 0 | 0 | −3.42 | | 0.86 | $\beta$ | 0 | −3.42 | 0 |
| $X_2$ $(\frac{g}{L})$ | 0.11 | 0 | 0.09 | 0 | | 0.11 | 0 | 0.09 | 0 | 0 |
| $S_1$ $(\frac{g}{L})$ | 1.25 | 0 | 5.8 | 23.84 | | 1.25 | 0 | 5.8 | 23.84 | $\gamma$ |
| $S_2$ $(\frac{mmol}{L})$ | 2.99 | 2.99 | 2.99 | 2.99 | | 2.99 | 2.99 | 2.99 | 2.99 | 2.99 |

### 4.1 Bifurcation surfaces of the linear control loop

The stability regions for each equilibrium point of the control loop in the parameter space are important to the design; these regions are separated from the unstable regions by a codimension two bifurcation. For the UASB reactor under IMC-PID, only the first and fourth equilibrium points have a codimension two bifurcation.

Figures 1 and 2 show the stable zones of the equilibrium points. In the case of the Figures 1(a) and 1(b), this zone is under the presented surface and in 2(a) and (b), is the side facing the origin limited in $sp$ by plane on top, and in its $\lambda$ by a surface.

It is important for the controller design to select parameters inside the stable zones in Figures 1(a) and 1(b), while choosing parameters outside the same zone in 2(a) and 2(b), given that it's an undesirable stable equilibrium point, from this it can be deduced that for this UASB reactor there is a superior limit in its $sp$, and an inferior one in $\lambda$.

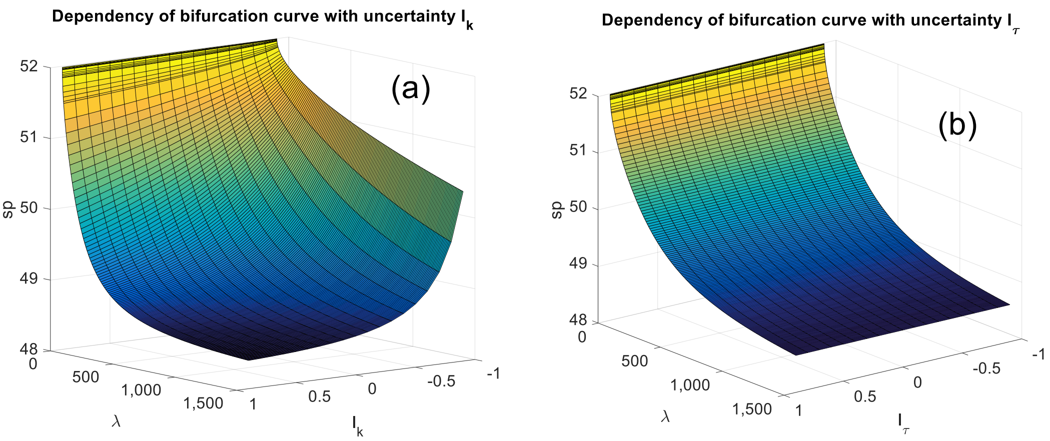


*Figure 1: Bifurcation manifolds of IMC-PID controller loop, where (a) and (b) are the bifurcation surface of the first equilibrium point on the $[\lambda \quad sp \quad I_k]$ space at $I_\tau = 0$ and $[\lambda \quad sp \quad I_\tau]$ at $I_k = 0$ respectively*

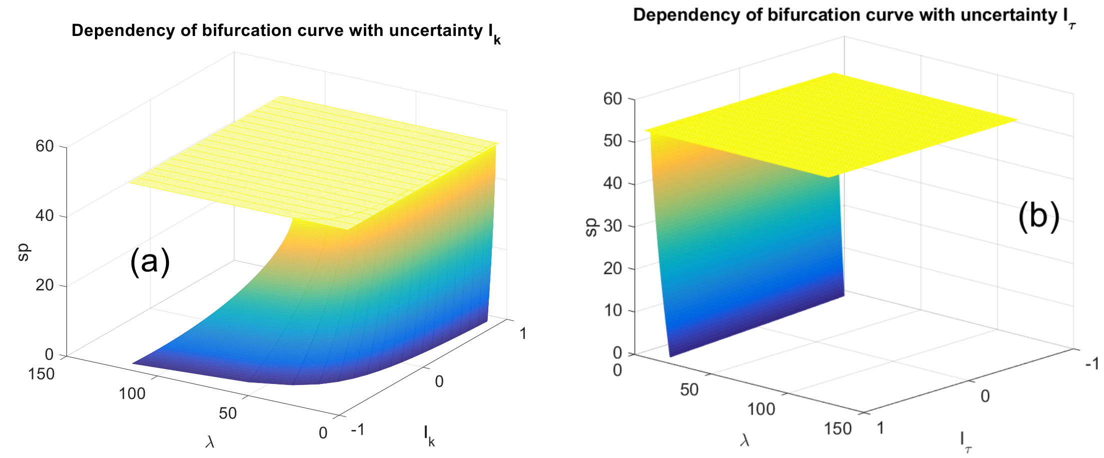


*Figure 2: Bifurcation manifolds of IMC-PID controller loop, where (a) and (b) are the bifurcation surface of the first equilibrium point on the $[\lambda \quad sp \quad I_k]$ space at $I_\tau = 0$ and $[\lambda \quad sp \quad I_\tau]$ at $I_k = 0$ for the fourth equilibrium points respectively*

As the value of VFA at the input current is 52, the bifurcation surfaces in (a) and (b) tend to the input value of VFA as $\lambda$ tend to zero, from this its deduced that the superior limit to $sp$ ought to be around $0.9S_S^{in}$, while the inferior limit to $\lambda$ is highly dependent on the uncertainty parameters.

For values of parameters in the surfaces (a) and (b), or values above them, a reactor washout is reached. The bifurcation surface is then shaped by the kinetics of substrate inhibition, in this case the set point $(sp)$ in the surface is lower at higher $\lambda$, because a higher $\lambda$ precludes a slower response and therefore a greater inhibition, and with it a lower flow necessary to washout the microorganisms, flow that is reached at lower controller set point $(sp)$. Fom this comes that $0.9S^{in}$ ought to be the recommended superior limit in any reactor with substrate inhibition.

When the bifurcation surfaces intersect the planes defined by $I_k = n$ or $I_\tau = n$, the resulting $R^2$ stable area is directly related to robustness. So, an increase between the stable area at a given uncertainty $A(I_k/I_\tau)$ and the area without it $A(0)$, is a sign that robustness is greater at $I_k/I_\tau$. If the increase happens in the stability area of the desired equilibrium point, it is a sign of diminishing robustness in any other stationary state, this is shown in Figures 3 and 4, for the linear control loop.

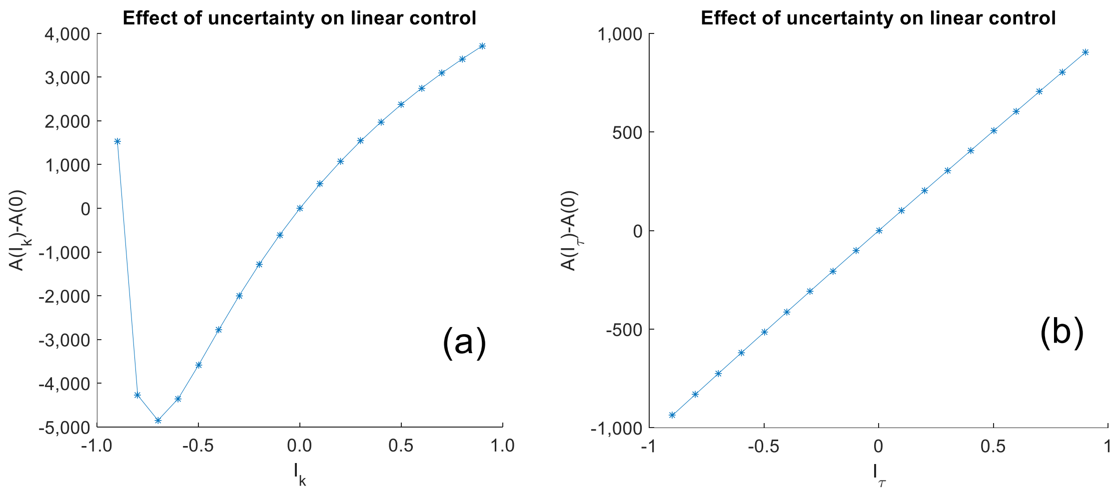


*Figure 3: Changes in stability area with uncertainty. (a) $A(I_k) - A(0)$, 1st equilibrium point. (b) $A(I_\tau) - A(0)$, 1st equilibrium point.*

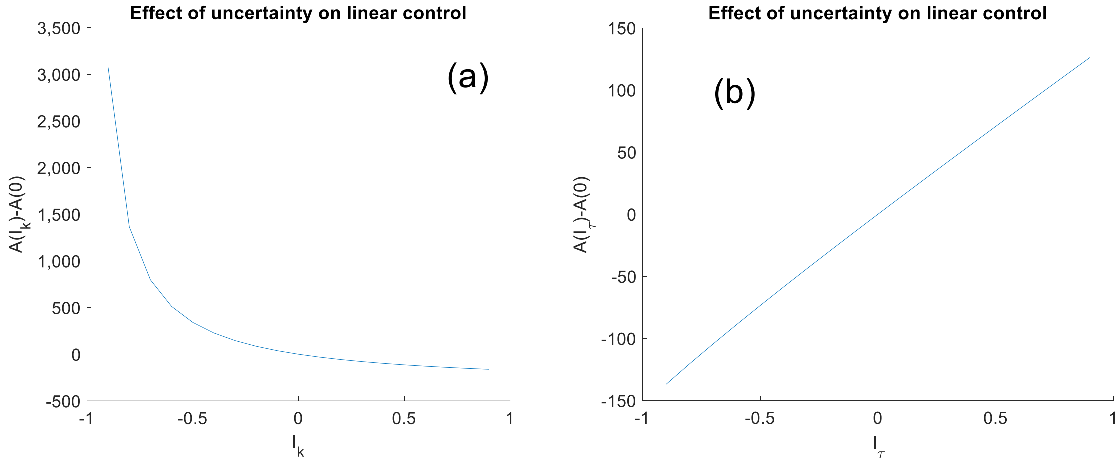


*Figure 4: Changes in stability area with uncertainty. (a)* $A(I_k) - A(0)$*, 4th equilibrium point. (b)*$A(I_\tau) - A(0)$*, 4th equilibrium point*

As it can be seen in Figures 3 and 4. an increase in $I_k$ increases the stable area of the desired equilibrium point (1st), while decreasing that of the 4th and undesired point, then it comes that small increases in $I_k$ improve system robustness. In the case of $I_\tau$ both areas are increased; however, the magnitude of their increase is magnitudes lower than for $I_k$

### 4.2 Bifurcation Surfaces of the Feedback Linearized Control Loop

An Input-output linearization was done on the UASB reactor, making it so that $S_2$ is linear with respect to an introduced input variable, which can be used to easily control $S_2$ by a regular IMC-PID, the equilibrium points of this loop were found and are shown in Table 2(b), however they possess no bifurcation, meaning the operational state is always stable regardless of parameter values, even when the set point takes on concentrations that are physically impossible to reach, such as $sp > S_2^{in}$

## 5. Phenomenological implications

Both loop equilibrium points with the same assigned number have very similar values, which points to open loop causes for closed loop equilibrium points. This is the case with simple enough controllers, with the controllers only modifying the systems dynamics. In the case of IMC-PID and feedback linearization, both modify the equilibrium points of the open loop system in the same ways.

In the case of the IMC-PID controlled system, its two stable equilibrium points are only stable in regions of the parameter space given by Figures 1 and 2. To have a stable loop its set point must be below the surface given in Figure 1 (a) and (b), and its controller time parameter must be above the surface in Figure 2 (a) and (b), while the 1st and 4th equilibrium points of the feedback linearized loop are stable regardless of parameters.

In the present case controller setpoint increases beget inflow increases (The speed of increase depends on parameter λ of the loop) washing away biomass which in turn increases $S_2$. If the $\mathrm{sp}$ increase enough, biomass will be washed out. However, the minimum $\mathrm{sp}$ of washout depends on the value of λ, because at slower flow responses the microbial reproduction is inhibited longer, and therefore a lesser flow is needed to cause a wash out, which means that at greater values of λ the setpoint value which causes a washout will diminish (see Figures 3 and 4). This is the cause of the differences in the parameter spaces of corresponding equilibrium points in the IMC-PID controlled system. On the feedback linearized system, the same phenomenon does not occur because the controller responses are faster.

The Figures 3 and 4 also draws another conclusion, a bigger $I_k$ increases the system robustness by increasing the stability area of the fourth equilibrium point, while $I_\tau$ with a smaller influence on robustness has mixed effects, on one hand, increases the stability area in the first equilibrium point, but on the other hand, it increases the area in the fourth point.

## 6. Conclusions

The results show that open loop dynamics influence closed loop equilibrium points, in some cases a determining one, and that closed loops under feedback linearization and PID control can be very similar regarding their equilibrium points, and their stability. It was also seen that there is plausible phenomenological explanations for bifurcation surfaces, and that in the case of biological systems under PID control the microbial kinetics of substrate inhibition can be this explanation, moreover, the results seem to indicate that, at least for bioreactors with substrate inhibition kinetics, when using the concentration of substrate at the output as a controller variable, that keeping the set point ($sp$) below 90 % the concentration of that same substrate at the input, in the present case $S_2^{in}$, can be a good thumb rule to keep controller parameters below any possible bifurcation caused by the kinetics; it's desirable to focus future research into this same phenomenon with other types of microbial kinetics, in order to develop similar heuristics for bioreactor operation. Lastly it was found that uncertainty in controller proportional constant has a bigger effect than uncertainty on controller time constant ($I_\tau$), and that a good heuristic for controller operation is trying to keep $I_k$ positive, given that a negative $I_k$ has more harmful effects on control

### References


Barontini F., Biagini E., Dragoni F., Corneli E., Ragaglini G., Bonari E., Tognotti L., Nicolella C., 2016, Anaerobic digestion and co-digestion of oleaginous microalgae residues for biogas production, Chemical Engineering Transactions, 50, 91-96.

Bequette B.W. (1ed), 2003, Process control: modeling, design, and simulation, Prentice Hall PTR, New jersey, US.

Bernard O., Hadj-Sadok Z., Dochain D., Genovesi A., Steyer J.P., 2001, Dynamical model development and parameter identification for an anaerobic wastewater treatment process, Biotechnology and bioengineering, 75, 424–438.

Chang H.-C., Chen L.-H., 1984, Bifurcation characteristics of nonlinear systems under conventional PID control, Chemical Engineering Science, 39, 1127–1142.

El-Bardini M., El-Nagar A.M., 2014, Interval type-2 fuzzy PID controller for uncertain nonlinear inverted pendulum system, ISA Transactions, 53, 732–743.

Galluzzo M., Cosenza B., Matharu A., 2008, Control of a nonlinear continuous bioreactor with bifurcation by a type-2 fuzzy logic controller, Computers & Chemical Engineering, 32, 2986–2993.

Garpinger O., Hägglund T., Åström K.J., 2014, Performance and robustness trade-offs in PID control. Journal of Process Control, 24, 568–577.

Hahn J., Mönnigmann M., Marquardt W., 2004, A method for robustness analysis of controlled nonlinear systems, Chemical Engineering Science, 59, 4325–4338.

Hahn J., Mönnigmann M., Marquardt W., 2008, On the use of bifurcation analysis for robust controller tuning for nonlinear systems, Journal of Process Control, 18, 408–420.

Isidori A. (3ed), 2013, Nonlinear control systems, Springer, Rome, Italy.

Jin Q., Liu Q., Wang Q., Tian Y., Wang Y., 2013, PID controller design based on the time domain information of robust IMC controller using maximum sensitivity, Chinese Journal of Chemical Engineering, 21, 529–536.

Kuznetsov Y. (1ed), 2013, Elements of applied bifurcation theory, Springer New York, Amsterdam, NL.

La Haura, A., Ma'mun K.Q., Sutikno J.P., Handogo R., 2017, Re-refinery used oil vacuum distillation column control by using internal model control, Chemical Engineering Transactions, 56, 1471-1476.

Lara-Cisneros G., Femat R., Dochain D., 2014, An extremum seeking approach via variable-structure control for fed-batch bioreactors with uncertain growth rate, Journal of Process Control, 24, 663–671.

Lei J., Khalil H.K., 2016, Feedback linearization for nonlinear systems with time-varying input and output delays by using high-gain predictors, IEEE Transactions on Automatic Control, 61, 2262–2268.

Nair A.T., Ahammed M.M., 2015, The reuse of water treatment sludge as a coagulant for post-treatment of UASB reactor treating urban wastewater, Journal of Cleaner Production, 96, 272–281.

Prasad L.B., Tyagi B., Gupta H.O., 2014, Optimal control of nonlinear inverted pendulum system using PID controller and LQR: performance analysis without and with disturbance input, International Journal of Automation and Computing, 11, 661–670.

Zhai C., Sun W., Palazoglu A., 2017, Analysis of periodically forced bioreactors using nonlinear transfer functions, Journal of Process Control, 58, 90–105.

Zhusubaliyev Z.T., Medvedev A., Silva M.M., 2015, Bifurcation analysis of PID-controlled neuromuscular blockade in closed-loop anesthesia, Journal of Process Control, 25, 152–163.