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An Algebraic Analysis Modelling Method for Complex Reactions

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An algebraic analysis modelling method is introduced to analyse complicated reaction system, based on the relationship between substances studied from the atomic scope. Firstly, the feasible reaction network is identified based on the transformation of the atomic matrix. Then, the model is adjusted according to the kinetic parameters, and the most suitable grey-box model is identified, and can be applied to analyse the influence of reaction parameters (including feed, temperature, pressure and residence time) on the conversion, selectivity and yield, and provide theoretical basis for adjusting and optimizing the reaction system. The proposed method has less experimental data demand, lower kinetic parameter accuracy requirement and self-correction ability, and can be easily programmed into software. An ethylene cracking reaction is studied to illustrate its application.

1. Introduction

In the chemical process, the reaction unit has an apparent effect to both upstream and downstream units, especially the separation unit. The study on it can provide not only the information of what happened inside reactor and the product composition, but also useful data to the reaction network optimization and energy saving. However, all of these rely on a model which can describe the reaction correctly. A good reaction model not only can be applied to study the reactions in detail, but also can predict the reaction performance before the actual operation. This brings great opportunity for factory.

In 1981, Ninshida et al. (1981) had begun to seek the way of reaction modelling, and divided different approaches into two categories, the knowledge-based methods and the logic-based methods. The former greatly relies on databases or sets of known reactions to show different possible reaction transformation rules, such as LHASA (Corey et al., 1974) and ZENETH (Parenty et al., 2013). The latter is based on the idea of Ugi and Gillespie (1971), such as RAIN and IGOR (Ugi et al., 1988), use matrices to represent the chemical structure of molecules and reaction transformation, and can consider the breaking and making of individual atom-atom bonds, which can take all possible reactions into account. Its disadvantage is that some infeasible reactions might be included in the identified reaction system.

To identify which reactions are feasible or infeasible, and choose a set of reactions to describe the whole reaction system, Arise and Mah (1987) raised the idea of independent reaction by the support of stoichiometric number theory. Based on this, the number of reactions to describe the reaction system can be identified. However, the exact reactions cannot be identified.

In order to get a more accurate model, Franceschini et al. (2008) raised a MBDOE (model-based design of experiments) method with the model optimized by repeated experiments. Thereafter, Luna et al (2014) built a MBDOE method with the process optimization and product data taken into account. This method suit for the reaction system with parameters varying in a small interval. Because of this, Galvanin et al. (2010) added the lower limit of the experimental results into this method.

Based on the predecessors, Zhang et al. (2004) raised a systematic method with model building, experiment design and process optimization combined, and molecular dynamics considered. This method is efficient in identifying independent reaction, and suits for a broader reaction condition. However, it has limited flexibility and requires a large amount of computation.

The method of Zhang et al. (2015) is extended for modelling complex reactions and predicting the performance of the reaction system under different conditions. A grey-box model is built to analyse the influence of reaction

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parameters on the conversion, selectivity and yield, and provide theoretical basis for adjusting and optimize the reaction system.

2. Model building methodology

2.1 Rearranged atom-molecular matrix

To build the model of a reaction system, it is necessary to know all the involved molecular, which is represented by a dynamic set, S. Subset R and P represent reactants and products, respectively. To make the main reaction more clearly, the elements of S are rearranged in the order from R to P. Different from the work of Zhang et al. (2015), the elements can be sorted according to the decreasing order of mole fractions in the reactor. Their relationship is represented as $R \subset S$, $P \subset S$ and $S = R \cup P$. Each element in S can be represented by the number of corresponding atoms (or function groups) included in this element. All elements can be represented together by an arranged atom-molecule matrix A. Each line corresponds an atom, while each column corresponds an element. a_{i} represents the number of the ith atom/function group contained in the jth element, as shown below:

$$A_{M\times N} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & a_{ij} & \vdots \\ a_{M1} & a_{M2} & \cdots & a_{MN} \end{bmatrix} \qquad a_{ij} = \begin{cases} > 0 & \text{the number of the 4h atom} \\ contained in the jth element} \\ = 0 & \text{the 4h atom is not contained} \\ in the jth element \end{cases}$$
(1)

Where M is the total number of atoms involved in the reaction system, N is that of molecules.

2.2 Feasible reactions analysis

An n-dimensional vector, $X = (x_1, x_2, \dots, x_n, \dots, x_n)^T$, is defined to indicate the stoichiometric factor of the feasible reaction. If AX = 0, a feasible chemical reaction exists. However, there might be infinite solutions for *X*, and not everyone corresponds to a feasible reaction. Hence, it is necessary to identify whether a solution represents a feasible reaction or not. To solve this problem, reactions will be divided into two stages according to the difference of reactants, stage I and stage II. For stage I, the reactant of a reaction is a subset of set R, and the products should be a subset of set P. All of those subsets compose reactant set R₁ and product set P₁. For stage II, the reactants of a reactants should be a subset of set R and P) and the reactants should be a subset of the set P. All these subsets compose reactant set R₂ and product set P₂.

With all reactions assumed to be irreversible and all feed cannot appear as the product, all elements of reactant sets and products sets can be matched one by one, and their balance should be checked for stage I and stage II, respectively. If there is a balanceable reaction, a solution of *X* exists, and the stoichiometric factor of each molecular is the corresponding element of *X*. However, there are a large number of elements in the reactant set and product set, and the number of the reactant-product combinations, which is their multiply, is even more. It is difficult to identify all feasible reactions through checking the balance between reactant and product. This is the problems remaining in the method of Zhang et al. (2015). To solve this problem, an algebra method will be proposed to find feasible reactions.

In stage I, set R and P are written into a $M \times N$ arranged atom-molecule matrix A, in which R can be written as a $M \times T$ sub-matrix. If M > T, choose all line of R. If $M \le T$, choose M line randomly from R to replace the sub-matrix R, then a new atom-molecule matrix is constituted. In theory, there are C_T^M new atom-molecule matrix, and can be written as A_{0i} ($i = 0, 1, 2 \cdots C_T^M$). Then, sub-matrix, R_{0i} , can be identified through the row transform of A_{0i} . If a

column in sub-matrix P0 has no element less than 0, this column represents a feasible reaction X, whose product is the corresponding molecular of this column, and the reactants are the molecular corresponding the element greater than 0 in this row. With a randomly chosen line, P0, multiplied by i = 1,2,3 and added to another line, a new line is obtained. If there is no element less than 0, a feasible reaction is identified, and its products are the matters represented by both lines. Besides, according to the theory of liner algebra, no more than $N_{ind} = M - R$ (R is the matrix order of M₀) linearly independent reactions can be obtained.

In stage II, a line in sub-matrix P_0 is respectively selected from left to right and add to sub-matrix R_0 . Then, by the same method of stage I, the reactions of stage II can be identified.

2.3 Reaction network modelling

All identified X can be written in a stoichiometric coefficient matrix, φ :

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$$\varphi = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1R} \\ x_{21} & x_{22} & \cdots & x_{2R} \\ \vdots & \vdots & x_{jk} & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nR} \end{bmatrix} = \begin{bmatrix} X_{1}, X_{2}, \cdots, X_{R} \end{bmatrix} \qquad \qquad X_{jk} = \begin{cases} > 0 & \text{product} \\ < 0 & \text{reactant} \\ = 0 & \text{not included} \end{cases}$$
(2)

Where, x_{ik} is the stoichiometric coefficient of molecule j in the reaction k.

Then, the column vector of matrix will be divided into two parts, u vectors in stage I and v vectors in stage II. Meanwhile, the row vector will be divided into two parts, set R and set P. And, matrix φ will be divided into four parts: φ_{IR} , φ_{IR} , φ_{IR} , φ_{IR} and φ_{IR} .

If an element of matrix φ is greater than 0, rewrite it as 1; if it is less than 0, rewrite it as -1. In this way, the matrix φ will be transferred into the incidence matrix φ . Then, an u_{+v} dimension row vector *I* constituted by 0 and 1 is obtained. With φ multiplied *I* and the rank of 0 deleted, a new matrix, φ^i , is obtained. If φ^i satisfies the conditions below, *I* can be a feasible reaction network vector and φ^i can be a feasible reaction matrix.

- In the area $(\Phi_{_{I\!R}}, \Phi_{_{I\!P}})$, there must be at least one reaction. In other words, the feed must react.
- In the area (Φ_{IR}, Φ_{IIR}) , the sum of each row must be less than 0 after elementary transformation.
- In the area (Φ_{μ}, Φ_{μ}) , the sum of each row must be greater than 0 after elementary transformation.
- In the area of products (Φ_{IP},Φ_{IP}), each row must have one element not equal to 0.
- In matrix Φⁱ, not all elements within each row are 0 to make sure all molecular take part in the reaction.
- All elements of area Φ_{IR} should not be greater than 0.
- All elements of area $\Phi_{\mu\nu}$ should not be less than 0.

Once a feasible reaction network vector is identified, a feasible reaction network's rival model, φ^i , can be obtained according $\varphi \times I$ and with the rank of 0 deleted.

In this procedure, more screening is used. Hence, these rules are more succinctly than previous work.

2.4 Rival model option

In this part, a molecular dynamics way will be raised to calculate models and an imaginary number m will be used to estimate rival models. Furthermore, reactants will be assumed as well mixed and all molecules' residence time is long enough. To distinguish the rival model, whose result is in better agreement with the real reaction, the reaction rate constant of all feasible reactions (k) or the activation energy (Ea) should be identified. What's more, the real data of feed and products at given T and P are also important. The reaction rate can be calculated according to Eq(4).

$$V_i = k_i C_i$$

(4)

The concentration of each molecular can be identified according to the state equation, as shown by Eq(5).

$$C_i = \frac{P_i}{V} = \frac{P_i}{RT} = \frac{P_{X_i}}{RT}$$
(5)

Based on this, Eq(4) can be written as:

$$v_i = k_i \frac{P x_i}{RT}$$
(6)

Then, the reaction rate of each reaction can be substituted into the rival model. Firstly, all elements of 1 in vector I will be represented as the corresponding reaction rate and all elements of 0 will be deleted, then the new vector will be rewritten as a column vector, v. With φ^i multiplied v, column vector r is obtained, and its element represents the change rate of corresponding molecular (negative means consumption and positive means

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generation). Meanwhile, mi is used to represent the total molar variation calculated according to molecular i, and can be calculated according to Eq(7) based on the change rate of every molecular, g_i

$$m_i = \frac{r_i}{g_i} \tag{7}$$

In ideal situation, m_i should be a constant no matter which molecular is used to calculate. However, it may be different from each other because of the accuracy of the model. The dispersion degree of m_i shows the error between rival model and real reaction system. A weighted variance, W, can be used to evaluate the model, and can be calculated by Eq(8). The better a model is, the smaller the value of W will be.

$$W = \sqrt{(\overline{m} - m_1)^2 x_1 + (\overline{m} - m_2)^2 x_2 + \dots + (\overline{m} - m_n)^2 x_n}$$
(8)

Where \overline{m} is the weighted average of m_i .

3. Illustrative example: thermal cracking of ethane

Based on Wu's (2011) paper, the temperature and pressure of a thermal cracking reaction of ethane are 1,102.45 K and 1.55 bar respectively. The mole fractions of main molecular in feed and products are displayed in Table 1. What's more, the increasing rate of feed volume is 1.54.

Table 1: Feed and product data

	C ₂ H ₆ (a)	H ₂ (b)	C ₂ H ₄ (c)	CH ₄ (d)	C ₃ H ₆ (e)	C ₃ H ₈ (f)	$C_2H_2(g)$	Others
Feed	99.3	0	0	0.2	0	0	0	0
Product	26.7	35.2	33.1	3.6	0.3	0.3	0.2	0.6

3.1 Atom-molecular matrix analysis

According to de reaction mechanism, the basic atom or function group participating in the reactions are C and H, and the reactant set only include C_2H_6 . Then the molecular can be rearranged by their mole fraction, and the rearranged atom-molecular matrix A_0 is obtained.

(9)

	а	b	С	d	е	f	g	
$A_0 = C$	2	0	2	1	3	3	2]	
Н	6	2	4	4	6	8	2	

3.2 Generating feasible reactions

In this system, the number of molecular taken into consideration is 7, while the rank of A_0 is 2. Hence, the number of independent reactions should be no more than 5. In stage I, the reaction set only includes a, and only the first line should be simplified. Thereafter, A_0 is transferred into A_1 . According to this matrix, 5 independent reactions are identified in stage I and displayed in Figure 1a.

	а	b	С	d	е	f	g
$A_1 = C$	[1	0	1	0.5	1.5	1.5	1
Н	0	2	-2	1	-3	-1	-4

In stage II, the reactions can be identified in the similar procedure and are shown in Figure 1b. Among these 10 reactions, X_E is infeasible in chemical thermodynamics, and is deleted.

<i>X</i> ₁	a	=	С	+	b)	X_{5}	f	=	b	+	e]	
X_2	2a	=	С	+	2d	X_{6}	с	=	g	+	b	
X ₃ -	2a	=	d	+	f	X ₇ <	f	=	С	+	d	
X _E	3a	=	3d	+	е	$X_{_8}$	с	+ a	=	d +	е	
<i>X</i> ₄	a	=	g	+	2b)	$X_{_9}$	e	=	d	+	gj	
			а						b			

Figure 1: independent reactions for stage I and II

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3.3 Feasible reaction network modelling

According to the 9 identified feasible reactions, the stoichiometric coefficient matrix, φ can be obtained and transferred into the incidence matrix φ , as displayed in Figure 2. Thereafter, 3 rival models will be used to compare a lot of models. The feasible reaction network vector *I* is displayed in Table 2. For model 1, 2 and 3, the corresponding matrix, Φ^1 , Φ^2 and Φ^3 are displayed in Figure 3.

	<i>X</i> ₁	X ₂	<i>X</i> ₃	X_4	÷	X_{5}	X_{6}	<i>X</i> ₇	$X_{_8}$	<i>X</i> ₉	9	<i>X</i> ₁	<i>X</i> ₂	<i>X</i> ₃	X_4	÷	X_{5}	$X_{_{6}}$	<i>X</i> ₇	$X_{_8}$	X_{g}
а	[−1	-2	-2	-1	÷	0	0	0	-1	0]	а	[-1	-1	-1	-1	÷	0	0	0	-1	0]
		•••	•••	•••	•••			•••	•••		•••			•••	•••	•••		•••	•••	•••	
b	1	0	0	2	÷	1	1	0	0	0	b	1	0	0	1	÷	1	1	0	0	0
$\varphi = c$	1	1	0	0	÷	0	-1	1	-1	0	$\Phi=c$	1	1	0	0	÷	0	-1	1	-1	0
d	0	2	1	0	÷	0	0	1	1	1	d	0	1	1	0	÷	0	0	1	1	1
е	0	0	0	0	÷	1	0	0	1	-1	е	0	0	0	0	÷	1	0	0	1	-1
f	0	0	1	0	÷	-1	0	-1	0	0	f	0	0	1	0	÷	-1	0	-1	0	0
g	0	0	0	1	÷	0	1	0	0	1	g	0	0	0	1	÷	0	1	0	0	1

Figure 2: Matrix φ and incidence matrix φ

Table 2: A part of reaction network vector

Мо	del			I													
1				1	, 1, 0,	0, 1	1, 0, 0,	1, 1					_				
2				1	, 0, 1,	0, 0	0, 0, 0,	1, 1									
3				1	, 0, 1,	0, 1	1, 0, 1,	0, 1					_				
	<i>X</i> ,	X ₂	:	<i>X</i> ₅	X _s	X,		<i>X</i> ,	<i>X</i> ₃	:	<i>X</i> ₈	X,	x	, X ₃	: X ₅	<i>X</i> ₇	X
а	[-1	-1		0	-1	0	a	[-1	-1	÷	-1	0	a [-	1 –1	: 0	0	0
			•										.				
b	1	0		1	0	0	b	1	0	:	0	0	b .	0	: 1	0	0
Φ' = c	1	1		0	-1	0	Φ ² = c	1	0		-1	0	$\Phi_{s} = c$	1 0	÷ 0	1	0
d	0	1		0	1	1	d	0	1	:	1	1	d () 1	÷ 0	1	1
е	0	0		1	1	-1	e	0	0	:	1	-1	е () 0	: 1	0	-1
f	0	0		-1	0	0	f	0	1	:	0	0	f) 1	-1	-1	0
g	L o	0		0	0	1	g	0	0	÷	0	1	g	0 0	÷ 0	0	1
			а							b					С		

Figure 3: corresponding matrix

For model 1, it's easy to find out that it is infeasible as the sum of row f cannot be greater than 0. Model 2 is feasible although the area of $\Phi_{\mu\nu}$ has elements less than 0, as Φ^2 will satisfy all conditions with X₁, X₈ and X₉ added. Similarly, model 3 is feasible as Φ^3 can be made to satisfy all conditions through rank transformation.

3.4 Rival model compare

In this part, reaction system data taken from Basic organic chemical technology (Wu, 2011) will be used to estimate rival models. Based on the data of Sundaram and Froment (1976), the reaction rate constants under 1,102.45 K are inferred and displayed in Table 3. Reaction 4 and 6 are taken out of consideration as their reaction rate constants are too small. According to Eq(6), the reaction rate can be calculated, and are displayed in Table 3 as well.

Reaction number	Reaction rate constant (s ⁻¹ or $m^3 \cdot mol^{-1} \cdot s^{-1}$)	Reaction rate (mol⋅m ⁻³ ⋅s ⁻¹)	Reaction number	Reaction rate constant (s ⁻¹ or $m^3 \cdot mol^{-1} \cdot s^{-1}$)	Reaction rate $(mol \cdot m^{-3} \cdot s^{-1})$
1	24.9226	5.5198	7	0.0221	4.3657
2	0.9683	0.0475	8	0.1875	0.0074
3	0.9112	0.0447	9	0.0324	0.6386
5	0.0203	3.9982			

Table 3: Inferred reaction rate constant and reaction rates

For model 2 and 3, the corresponding matrix φ^2 and φ^3 are displayed in Figure 4a and 4b, respectively, and r^i are displayed in Table 4. The real change rates calculated according to Table 1 are listed in Table 4 as well.

Molecular	Model 2	Model 3	\boldsymbol{g}_i	m, (2)	<i>m</i> , (3)	Molecular	Model 2	Model 3	\boldsymbol{g}_i	<i>m</i> ; (2)	m, (3)
C_2H_6	-26.93	-26.75	0.378	71.25	70.75	CH ₄	1.13	0.97	0.0347	32.60	27.83
H ₂	24.92	24.94	0.352	70.80	70.76	C ₃	1.07	0.86	0.006	177.71	142.77
C_2H_4	24.74	24.94	0.328	75.41	76.05	C_2H_2	0.032	0.032	0.002	16.21	16.21

Table 4: Change rates of molecular ($mol \cdot m^{-3} \cdot s^{-1}$)

According to Table 4, the m for different molecular in every model can be identified. Furthermore, the weighted variance W of model 2 and 3 calculated according to Eq(8), are W2 = 2.02 and W3 = 2.27, respectively. Since W2 is less than W3, model 2 is better to describe the reaction under the condition of 155 kPa and 1,102.45K.

4. Conclusions

A new approach for reaction model building is proposed in this paper. Based on this method, all possible reactions and frameworks and a plenty of rival model can be identified to describe the reaction system based on only a set of reaction data. The variance can be used to judge every model, and the lower variance corresponds better model. The model can be used to predict the reaction system in a certain amount of reaction conditions and optimize the chemical production. However, if the reaction condition changes significantly, the reactions involved in the reaction system can be different, and the identified reaction model does not suit anymore. How to enlarge the scope of application or change the model along operating parameter will be considered in further.

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