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Reliability - Redundancy Allocation in Process Graphs

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Process graphs (P-graphs) have been proven to be useful in identifying optimal structures of process systems and business processes. The provision of redundant critical units can significantly reduce operational risk. Redundant units and subsystems can be modelled in P-graphs by adding nodes that represent logical conditions of the operation of the units. It is revealed in this paper that P-graphs extended by logical condition units can be transformed into reliability block diagrams and based on the cut sets and path sets of the graph a polynomial risk model can be extracted. Since the exponents of the polynomial represent the number of redundant units, the cost function of the reliability – redundancy allocation problem as a nonlinear integer programming model can be formalised, where the cost function handles the costs associated with consequences of equipment failure and repair times. The applicability of this approach is illustrated in a case study related to the asset-intensive chemical, oil, gas and energy sector. The results show that the proposed algorithm is useful for risk-based priority resource allocation in a reforming reaction system.

1. Introduction

The reliability of energy production and process systems is of crucial importance. The provision of redundant critical process units/components can significantly reduce the operational risk of these systems. As such modifications of the technology require additional investment and maintenance costs, it is beneficial to formalise the reliability – redundancy allocation problem as an optimisation task. The wide range of models that represent the structure of the systems, costs and resource constraints has led to several optimisation models. Due to the complexity of the problem, most of these approaches have been developed for specialised systems (Norani et al., 2017) or to utilise metaheuristic algorithms (Kuo and Prasad, 2000). Although the problem is actively studied, there is still a need for a general approach that efficiently combines the tasks of process synthesis and reliability modelling. The aim of this paper is to construct a general framework for reliability-based redundancy optimisation founded on the flexible P-graph representation of the process optimisation problems (Friedler et al., 1992). Varbanov et al. (2017) published an overview paper about the application areas and potential future opportunities of the P-graph framework such as supply chain optimisation, chemical process synthesis, risk management, resource planning, evacuation planning and business process modelling.
Our idea is rooted in the work of Süle et al. (2011), who created temporary nodes in the graph and applied a linear mathematical model for handling the uncertainty of the raw materials. A redundancy-based reliability improvement with regard to the P-graph-based optimisation of a biodiesel supply chain has already been proposed by Bertók et al. (2013). The importance of the P-graph-based reliability analysis of complex production systems was highlighted in a lecture by Orosz et al. (2016).
Our key goal is to formalise a nonlinear integer programming model of the reliability-redundancy allocation problem based on a polynomial risk model extracted from the path sets and cut sets of the P-graph.
To illustrate the proposed approach, the maintenance-related data of a reforming reaction system at Sinopec’s Luoyang Petrochemical Plant is studied (Hu et al., 2009).

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2. Methodology

Our focus is the safety critical optimal design of complex process systems. For this purpose, the reliability-redundancy allocation task is interpreted as a process network synthesis problem and a widely applicable method is proposed for the evaluation of the reliability of systems represented by P-graphs.

The process graph or P-graph is a directed bipartite graph used in process network synthesis (PNS) and workflow modelling (Friedler et al., 1992). The vertices of the graph can represent operations (*O*) and materials (*M*) that are the inputs and outputs of the operations. The PNS problem can be considered as a (P, R, O) triplet, where $P \subseteq M$ and $R \subseteq M$ are special material sets for product and raw-type materials, while $O \subseteq \wp(M) \times \wp(M)$ is the set of the operating units. Although the operations originally represent material transformations, recently the whole concept has been extended to the modelling and analysis of workflows. The analogy between P-graph and success trees (or reliability block diagrams) can easily be maintained when the "operating units" of the P-graph represent the logical connections and states of the functionalities of the components, and the "materials" are used to introduce the elementary faults into the model (see Figure 1). As represented in Figure 2, the reliability block diagram (a) in some cases can be transformed into fault tree (b), success tree (c) and P-graph (d). Although the original P-graph does not lend itself to reliability analysis, the cut set and path set-based analysis of the P-graph-related reliability block diagram allows the extraction of reliability estimation models.

It is assumed that the system is built from *c* components. Due to failures, some of these components do not perform their required functions within specified performance requirements, which can result in the whole system losing its functionality. The functioning-or-failed condition of components is represented as an $\boldsymbol{e} = [e_1, \dots, e_i, \dots, e_c]^T$ vector, where $e_i = 1$ represents that the i-th unit is functioning, while $e_i = 0$ represents the failure of the *i*-th component. The system structure function is a Boolean function that maps $\{0,1\}^c$ into $\{0,1\}$, which represents $e_0 = \varphi(\boldsymbol{e})$, assuming the whole system is functioning correctly. When the components of the system are in series then $\varphi(\boldsymbol{e}) = e_0 = e_1 \cdot \dots \cdot e_c$, but when in parallel $\varphi(\boldsymbol{e}) = e_0 = 1 - (1 - e_1) \cdot \dots \cdot (1 - e_c)$. The reliability of the system is equivalent to the probability of the system properly functioning, $P(\varphi(\boldsymbol{e}) = 1)$. The structure function is usually represented as reliability block diagrams.

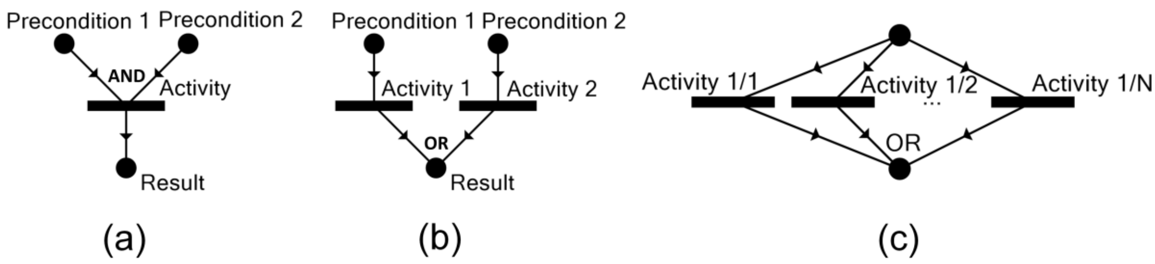

Figure 1: Representation of (a) AND, (b) OR dependencies and (c) redundancy of activities as OR connections

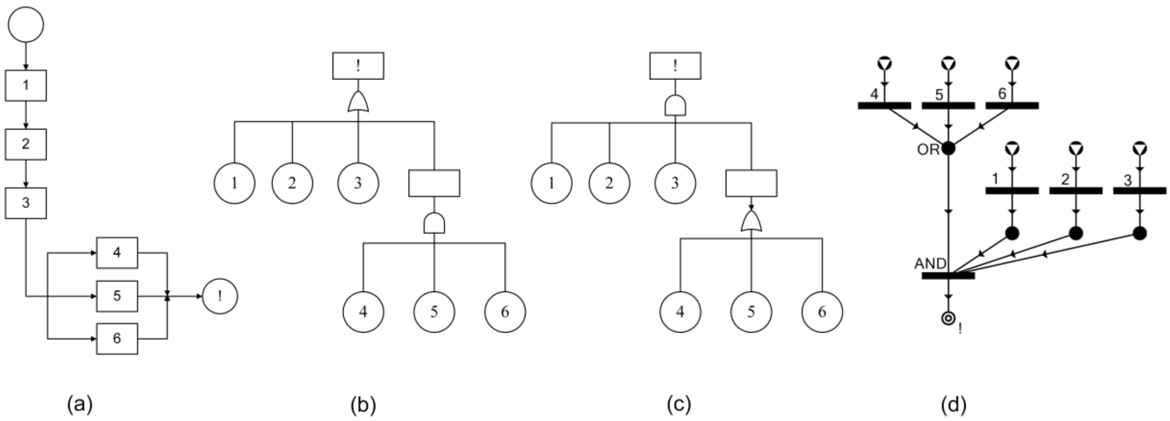

Figure 2: Example of a (a) Reliability block diagram, (b) Fault tree, (c) Success tree, and (d) P-graph representation. As can be seen, P-graphs can represent reliability block diagrams and success trees

The reliability block diagram of the system is a labelled random graph, where the nodes e_i represent the nodes of random variables indicating the *i*-th node is present in the graph. A path in a graph is a sequence of alternating adjacent nodes and the links joining them, beginning and ending with a node. Therefore, when a path to the end of the reliability block diagram exists through the sets of operating nodes/units, then the system is working properly. A path is referred to as minimal if it contains no proper subset that is also a path connecting the same two nodes. As a result, the set of minimal paths defines the set of operating units that ensure the operation of the whole system. Since there can be several minimal paths, $\pi_1, \dots, \pi_{n_p}$, the system functions when at least one path is available, so the (upper bound of) reliability of the system is:

$$P^{UB}(\varphi(\boldsymbol{e})) = 1 - \prod_{k=1}^{n_p}\left[1 - \prod_{i\in\pi_k} P(e_i = 1)\right] \tag{1}$$

A cut is a set of nodes and links whose removal from the graph disconnects the two nodes, so the sets of minimal cuts connect the sets of units whose failure results in the failure of the whole system. Namely, the system fails if at least one of the minimal cuts consists entirely of non-functioning units. Since several cut sets can exist, $\vartheta_1, \dots, \vartheta_{n_c}$, and the lower bound of the reliability of the system is:

$$P^{LB}(\varphi(\boldsymbol{e})) = \prod_{k=1}^{n_c}\left[1 - \prod_{i\in\vartheta_k}[1 - P(e_i = 1)]\right] \tag{2}$$

Note that the minimal path set of the example shown in Figure 2 is {{1, 2, 3, 4}, {1, 2, 3, 5}, {1, 2, 3, 6}}, while the minimal cut set is {{1}, {2}, {3}, {4, 5, 6}}.

Table 1: Minimal path set generation algorithm

Function Minimal Path Set Generator ((m, *o*): P-graph): minimal path sets

begin
min-path-sets := $\emptyset$
subproblems := $\{(P, \emptyset, \emptyset, \emptyset)\}$;
while *subproblems* $\neq \emptyset$ do
    let $(p, p^+, o^+, o^-) \in$ *subproblems*, where cardinality (o^+) is minimal
    subproblems := *subproblems* \ (p, p^+, o^+, o^-);
    if $isFeasible(o^+,\ min\text{-}path\text{-}sets\ )$ then
        if $p = \emptyset$ then
            min-path-sets := *min-path-sets* $\cup \{(\psi(o^+), o^+)\}$;
        else
            SubProbGen($(p, p^+, o^+, o^-), subproblems$);
        end if
    end if
end while
return *min-path-sets;*
end;
Function SubProbGen((p, p^+, o^+, o^-): subproblem, subproblems: set of subproblems)
begin
let $x \in \{\hat{x} |\ \hat{x} \in p \text{ and } (p, p^+, o^+, o^-) \in subproblems \text{ and } |\varphi^-(\{\hat{x}\})|\ is \text{ minimal}\}$;
$o_x := \varphi^-(\{x\}) \setminus o^-;\ o_{xb} := o_x \cap o^+;\ C := \wp(o_x \setminus o_{xb})$;
if $(o_{xb} := \emptyset)$ then
    $C := C \setminus \{\emptyset\}$;
end if
for all $c \in C$ do
    subproblems := subproblems $\cup \Big(\big(p \cup \psi^-(c)\big) \setminus\ p^+ \setminus \{x\} \setminus R, p^+ \cup \{x\}, o^+ \cup c,$
    $o^- \cup (o_x \setminus\ o_{xb} \setminus c\ )\ )$;
end for
end;
Function isFeasible(o^+: set of operating units, $min\text{-}path\text{-}sets$: set of minimal paths): bool
begin
for all $(m, o) \in\ min\text{-}path\text{-}sets$ do
    if $(o^+ \subseteq o)$ then
        return false;
    end if
end for
return true;
end;

In Table 1 an algorithm is presented which can automatically generate the minimal path sets. For the formal description of the minimal path set generation algorithm and its optimisation the following notations were introduced:

- $\psi^{-}(o)$ yields the set of materials of a process structure, each of which is an inlet to at least one operating unit given in set o. Formally: $\psi^{-}(o) = \cup_{(\alpha,\beta)\in o}\alpha$.
- $\psi^{+}(o)$ yields the set of materials of a process structure, each of which is an outlet from at least one operating unit given in set o. Formally: $\psi^{+}(o) = \cup_{(\alpha,\beta)\in o}\beta$.
- $\psi(o)$ yields the set of materials of a process structure, each of which is either an inlet to or an outlet from at least one operating unit given in set o. Formally: $\psi(o) = \psi^{-}(o) \cup \psi^{+}(o)$.
- $\varphi^{-}(m)$ yields the set of operating units of a process structure, each of which produces some materials found in set m as its outlets. Formally: $\varphi^{-}(m) = \{(\alpha,\beta) \in o\colon \beta \cap m \neq \emptyset\}$.

The reliability of the entire system can be characterised by a polynomial expression, as the reliabilities are multiplied when the elements are connected by 'AND' connections, while logical 'OR' connections aggregate the different sets. As an increase in the reliability of the system by introducing redundant elements is desired, the above equation can be written as follows:

$$P^{UB}\big(\varphi(\boldsymbol{e})\big) = 1 - \prod_{k=1}^{n_p}\big[1 - \prod_{i\in\pi_k} 1 - [1 - P(e_i = 1)]^{d_i}\big], \text{ or} \tag{3}$$

$$P^{LB}\big(\varphi(\boldsymbol{e})\big) = \prod_{k=1}^{n_c}\big[1 - \prod_{i\in\vartheta_k}[1 - P(e_i = 1)]^{d_i}\big], \tag{4}$$

where d_i represents the number of units.

The evaluation of the risk associated with the failure of the system requires the calculation of the economic consequence of equipment failures. In our study, the cost of the required maintenance cost (*MC*) and the cost of the production loss (*PL*) were calculated:

$$MC = Cfm\ + DT \cdot\ CV, \tag{5}$$

$$PL = DT \cdot PLPD, \tag{6}$$

where Cfm stands for the fixed cost of maintenance ($), DT denotes the downtime (number of days), CV represents the variable cost of maintenance per day ($\$\ d^{-1}$), and *PLPD* is the production loss per day ($\$\ d^{-1}$). The risk of each subsystem is the product of its failure probability and consequences of failure.

Based on this loss function and the polynomial reliability of the model, the following risk function can be determined, where o^* represents the set of materials and operating units involved in the optimal solution:

$$\sum_{(\alpha,\beta)=o_i\in o^*}(cfm_i + DT_i \cdot CV_i)\cdot(1 - P(e_i = 1)) + (DT_i \cdot PLPD_i)\cdot\big(1 - P(e_i = 1)\big)^{d_i} \leq Limit_{risk}^{Upper}\ , \tag{7}$$

whose risk is inversely proportional to the reliability of the system:

$$z^* = P^{UB}\big(\varphi(\boldsymbol{e})\big) = 1 - \prod_{k=1}^{n_p}\big[1 - \prod_{i\in\pi_k} 1 - [1 - P(e_i = 1)]^{d_i}\big]. \tag{8}$$

The risk always decreases by increasing the redundancy. However, the installation of additional components requires investment cost, resources for which are limited. As detailed information concerning the investment costs of the components is unavailable, the number of spare components is constrained:

$$\sum_{i=1}^{n} d_i \leq Limit_{component}^{Upper} \tag{9}$$

Based on these variables, a nonlinear integer programming model was defined, where the z^* objective function is maximised under the constraints related to the upper bound of the acceptable risk, $Limit_{risk}^{Upper}$, and the number of spare components (available investment costs) $Limit_{component}^{Upper}$.

3. Case study

The applicability of the proposed methodology is demonstrated using data from a real-life case study related to the reforming reaction system in Sinopec's Luoyang Petrochemical Plant (Hu et al., 2009). The reliability and cost parameters of the subsystems of the process are given in Table 2.

Instead of solving a process synthesis problem, in this study the P-graph of the process was obtained based on the success tree of the system (see Figure 3). Since the data were aggregated to the subsystems, the reliability-redundancy allocation problem was also defined at this level (see Figure 4).

Based on the P-graph, the path sets were determined by the proposed minimal path set generation algorithm. Because of the specific topology of the graph, the minimal path set contains all the activities in the graph, therefore, $P^{UB}(\varphi(\boldsymbol{e})) = \prod_{i\in\pi_k} P(e_i = 1) = 0.009$. The 'Nonlinear optimisation by Mesh Adaptive Direct Search' (NOMAD) black-box algorithm was used to solve this developed mathematical model. The algorithm defines a mesh with the discretisation of the space of variables and performs an adaptive search while the refinement of the meshes is also controlled (Audet and Dennis, 2006). The solutions were verified by BARON (Sahinidis, 1996) which is a computational system for solving nonconvex optimisation problems to global optimality. The reliability of optimal solutions for different constraints is presented in Table 3. The results show that by increasing the available budget, the reliability of the system is also increased, however, the number of redundant elements comprehensively determines the total cost and reliability. The results illustrate that the proposed methodology is applicable with regard to the risk-based resource allocation in the design of process systems.

Table 2: Reliability and cost parameters of subsystems (n=9)

#	Subsystem	Reliability ($P(e_i = 1)$)	cfm_i (\$)	DT_i (day)	CV_i (\$)	$PLPD_i$ (\$)
1	1st compressor subsystem	0.4208	2,173.9	1.5	144.93	43,478
2	Heating-reaction subsystem	0.4011	7,246.4	5.0	289.86	43,478
3	Heat exchanger subsystem	0.6088	2,898.6	3.0	289.86	43,478
4	Cooler subsystem	0.6801	1,449.3	2.0	289.86	43,478
5	Separation subsystem	0.9907	2,898.6	4.0	289.86	21,739
6	Pump subsystem	0.5722	724.6	1.0	72.464	0
7	2nd compressor subsystem	0.7874	1,449.3	1.0	144.93	0
8	Absorber subsystem	0.6984	1,449.3	4.0	144.93	14,493
9	Instrument subsystem	0.4141	724.6	1.0	72.464	0

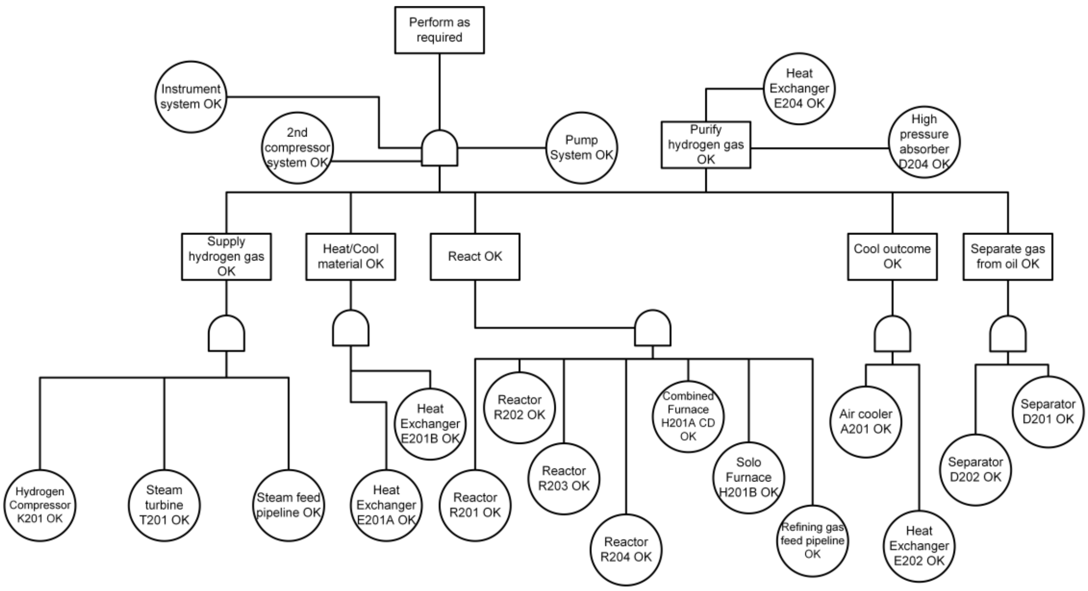

Figure 3: Success tree of reaction system published in (Hu et al., 2009)

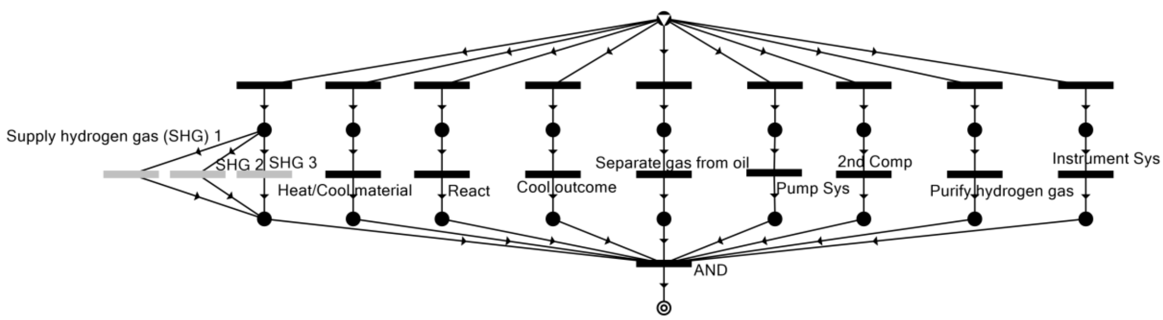

Figure 4: P-graph representing the subsystems of the reaction system. This figure also illustrates how redundancy is handled in the proposed framework

Table 3: Results of optimization

#	$Limit_{cost}^{Upper}$	$Limit_{activity}^{Upper}$	$d = (d_1, d_2, \ldots, d_9)$	Reliability of the system
1	110,000	15	(2,4,2,2,1,1,1,1,1)	0.0568
2	150,000	15	(2,3,2,1,1,2,1,1,2)	0,0879
3	180,000	15	(2,2,2,2,1,2,1,1,2)	0,0947
4	35,000	25	(5,6,4,3,1,1,1,3,1)	0,1514
5	50,000	25	(3,6,3,3,1,2,2,2,3)	0,3922
6	70,000	25	(4,4,3,2,1,3,2,2,4)	0,4563

4. Conclusions

In this paper, we presented a novel approach for safety-critical optimisation of process systems. To represent redundant process units and to calculate the reliability of the system we the added logical nodes to P-graphs. It was demonstrated that P-graphs extended by these logical condition units can be transformed into reliability block diagrams and based on the cut sets and path sets of the graph a polynomial risk model can be extracted. The cost function in terms of the reliability – redundancy allocation problem was formalised as nonlinear integer programming model, where the integers are the exponents of the polynomial model that represent the number of redundant units. With the help of the NOMAD algorithm, the reliability under the constraints related to the investment costs and the acceptable risks associated with the consequences of equipment failure and repair times was maximised. The applicability of this approach was illustrated by a case study related to a reforming reaction system. In our further work, how the time-dependent reliability of the units could be incorporated into the model and how the proposed toolset can be used for the prioritisation of the maintenance work will be focused on.

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