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Design-Oriented Structural Controllability and Observability Analysis of Heat Exchanger Networks

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The control-relevant design and analysis of Heat Exchanger Networks (HENs) is an essential issue in terms of the design and intensification of sustainable production systems. The structural controllability and observability of HENs should be studied based on their dynamical model. Recently, a maximum matching based algorithm was developed to determine the locations of the minimum number of actuators and sensors needed to ensure the controllability and observability of linear dynamical systems. In this paper, the ability of concentrated parameter state-space models of the heat exchanger units to serve as building blocks of the network of state variables in HENs, and the use of the resultant network of state variables to study the control of relevant topologies and properties of HENs is highlighted. Based on the results of the systematic analysis, the structural patterns that facilitate the control and observation of HENs with a relatively small number of actuators and sensors were determined. Two methods were proposed to define the sets of additional actuators and sensors which are required to improve the operability of the network by decreasing the order of the controlled system. The first method is based on the interpretation of the placement of the sensors and actuators as a set cover problem, while the second one uses two network science-based measures: closeness centrality and betweenness centrality. The proposed methodologies are demonstrated on a benchmark example that is well covered in the literature.

1. Introduction

The process controllability, observability and flexibility of HENs can be studied by parameter sensitivity analysis of the system or simulation and structural analysis of the model. The resilience index (RI) quantifies the ability of a HEN to deal with disturbances based on a detailed model of the HENs (Saboo et al., 1985). Although RI can support the design by evaluating the alternative models, i.e. providing information about the process flexibility, operability and controllability, the index is calculated based on energy balances. The heat-transfer areas were introduced into this measure to produce a new controllability index (CI) (Westphalen et al., 2003). For a design-oriented analysis of controllability, a five-step procedure was proposed that uses performance relative gain array (PRGA) and partial disturbance gain (PDG) (Tellez et al., 2006). To deal with uncertainties and disturbances a controllability and resiliency (C&R) analysis based on relative gain array (RGA) and disturbance cost (DC) index was also introduced (Miranda et al., 2017).
Although the use of exact parameters can provide accurate information about the system, in many cases the exact parameters are unknown or not defined in advance, therefore, there is an urgent need for methods that are based solely on structural information concerning the design-oriented analysis of the controllability and observability of HENs.
The structural controllability and observability of the dynamical systems (Reinschke, 1988) and lumped and plug-flow HENs (Varga et al., 1995) have already been studied. The new wave of interest in structural analysis started with the work of Liu et al., who determined the location and minimum number of actuators by the maximum matching algorithm based on graph theory, which uses the network of the state variables of the dynamical system (Liu et al., 2011). Since then, the structural observability (Liu et al., 2013a), the control energy

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(Yan et al., 2015), the effect of the degree correlation on the controllability (Pósfai et al., 2013), and the robustness of the input configuration (Liu et al., 2013b) have also been analysed. The maximum matching-based approach has been applied to transcriptional networks (Müller and Schuppert, 2011), to process networks (Kang and Liu, 2017) and to ecological models (Leitold et al., 2016). Recently, the limitations and benefits of applying network theory to energy systems were also studied (Ruzzenenti and Basosi, 2017).
In this paper, it is highlighted that maximum matching of the network of state variables of HENs is an appropriate technique for the design of HENs to determine the optimal configuration of actuators and sensors. Since the minimum numbers of actuators and sensors needed to ensure controllability and observability are relatively small, more realistic and challenging objectives should be the focus of future research. To initialise such progress, the problem concerning the optimal placement of the sensors and controllers that ensures the order of the controlled system does not exceed a predefined limit, which is closely related to the operability of the process, is addressed. In Section 2, the structural controllability and observability analysis of HENs is introduced, and the typical structural patterns of the network of the state variables revealed. In Section 3, methods based on set covering and centrality measures to extend the theoretical minimal input and output configurations to decrease the input and output orders of the system are proposed. Finally, in Section 4 a benchmark problem is presented and the performances of these methods compared.

2. State-variable approach to the network analysis of HENs

A heat exchanger cell consists of two perfectly stirred tanks with inflows and outflows of hot and cold streams and a heat transfer area (Varga et al., 1995) (Figure 1a). To control a HEN, utility-heat exchangers are also commonly used. They have one inflow and outflow, and one input that can influence the outflow to the desired target. Two types of utility heaters can be distinguished. A utility cooler that cools the hot stream is shown in Figure 1b and a utility heater that heats the cold stream is presented in Figure 1c.
The linear ordinary differential equations of a simple heat exchanger cell can be seen in Eq(1) and Eq(2). Eq(1) describes the dynamics of a utility cooler, where x_2 stands for input u_1 which cools the hot stream, while Eq(2) outlines the dynamics of the utility heater, where x_1 represents the input u_1 which heats the cold stream.

$$\frac{dT_{ho}}{dt} = \frac{v_h}{V_h}(T_{hi} - T_{ho}) + \frac{UA}{c_{ph}p_hV_h}(T_{co} - T_{ho}) = -\left(\frac{v_h}{V_h} + \frac{1}{\tau_h}\right)x_1 + \frac{1}{\tau_h}x_2 + \frac{v_h}{V_h}z_1 = \frac{dx_1}{dt} \quad (1)$$

$$\frac{dT_{co}}{dt} = \frac{v_c}{V_c}(T_{ci} - T_{co}) + \frac{UA}{c_{pc}p_cV_c}(T_{ho} - T_{co}) = \frac{1}{\tau_c}x_1 - \left(\frac{v_c}{V_c} + \frac{1}{\tau_c}\right)x_2 + \frac{v_c}{V_c}z_2 = \frac{dx_2}{dt} \quad (2)$$

where $\frac{1}{\tau_\varphi} = \frac{UA}{c_{p\varphi}p_\varphi V_\varphi}$ and $\varphi \in \{h, c\}$. Based on Eq(1) and Eq(2), a structural matrix can be determined where only zero and non-zero elements are considered (Reinschke, 1988). This matrix yields the adjacency matrix of the structural analysis as well. The state variables are the output temperatures of the heat exchangers: in the case of the heat exchanger cell $x(t) = [T_{ho}, T_{co}]^T$, with regard to the utility cooler $x(t) = T_{ho}$, and concerning the utility heater $x(t) = T_{co}$. The input temperatures are regarded as disturbances, $z(t) = [T_{hi}, T_{ci}]^T$. According to Eq(1) and Eq(2) the network representation of a heat exchanger cell and the utility heat exchangers can be seen in Figure 1d-f.

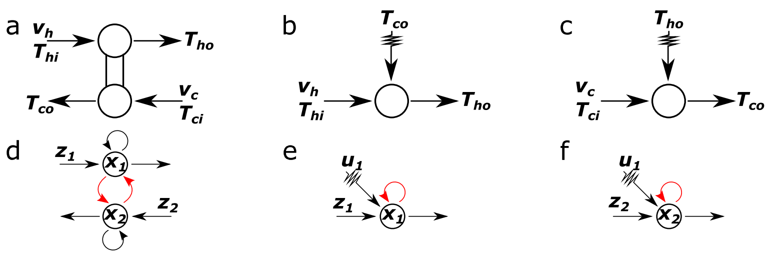

Figure 1: The simplest model and network representation of a heat exchanger cell (a) and (d), a utility cooler (b) and (e), and a utility heater (c) and (f).

In Figure 1, the edges in red denote the edges selected by the maximum matching algorithm and show the intrinsic dynamics of the elementary components as well. In the case of a heat exchanger cell, it is important that the hot and cold sides of the cell always belong to the same strongly connected component.
To determine the location of the minimum number of actuators one the maximum matching algorithm must be used. Since each state variable in the network processes a self loop, i.e. the diagonal elements are non-zero values in the adjacency matrix, maximum matching tends to select these edges, namely intrinsic dynamics

control the individual state variables. As a result, since each state variable is matched, then only one actuator can ensure the controllability if all state variables can be reached from the input. This strengthens the fact that if the maximum matching-based method is used to analyse a complex system, like a HEN (Kang et al., 2017), then the correct topology should be used to describe the system (Leitold et al., 2017). The second important result is that the aforementioned statements agree with a previous result (Varga et al., 1995), where the authors concluded that the problem concerning the structural controllability and observability was downgraded to only a reachability problem in the case of heat exchangers.

Although observability is the mathematical dual of controllability, a more straightforward algorithm was derived to determine the necessary sensor nodes of a complex system (Liu et al., 2013a). Accordingly, when the strongly connected components (SCCs) are identified in the inference diagram, then sensors should be placed at one of the nodes in each root SCC. SCC S is a root SCC if no edges originate from any node v, $v \notin S$ to any node w, $w \in S$. Liu et al. also highlight that the method can underestimate the number of sensor nodes if the system contains a family of symmetries. This phenomenon appears mostly linear systems, in nonlinear systems symmetries are extremely rare. The problem in the network representation can be considered to be the dilatation of a set of nodes, S, where the cardinality of the outgoing neighbourhood of S is higher than the cardinality of S (Liu et al., 2013a). Since in this paper HENs are represented by linear models, besides the SCC-based analysis the dilatations were analysed. Firstly, the SCCs in HENs were studied. As is shown in Figure 1, the SCCs contain two nodes in the case of a simple heat exchanger cell and one node for a utility heat exchanger. An SCC with more than two nodes appears in a HEN when there is a series of streams that possess a path in the network representation that ends in one of the state variables belonging to a heat exchanger where the path begins.

The effect of dilatations in HENs is studied by creating HENs from inducing dilatation in the network representations. The simplest approach utilises a diamond shape, where the top and bottom nodes are connected as can be seen in Figure 3b in green. Our results show that the dilatations cannot cause the underestimation of the sensor nodes, because of the self-regulatory dynamics. Moreover, the physical constraints play an important role, since a heat exchanger cell can connect to a maximum of two other heat exchangers (one by hot steam and one by cold steam). Since a heat exchanger cell is represented by two state variables, which create a complete subgraph, the upper bound in terms of out-degree is three if a bypass is not allowed. Naturally, with bypasses the out-degree can be increased, but the self-regulatory dynamics are valid in these cases also, thus, the component-based analysis is also reduced to a reachability problem as before. Therefore, the structural controllability and observability of HENs are not in question because HENs can be controlled and observed, the only requirement is for at least a path to exist between each state variable and at least one actuator or sensor to be present. It can be concluded that both the methods based on SCCs and maximum matching provide the same set of sensor nodes for the same HEN.

Although the HENs are observable and controllable, the time and energy required and the complexity of the necessary trajectories of the control inputs needed to transform the system into the desired state can be very high due to the relatively small number of actuators and sensors.

To deal with this problem, the following two methods were proposed to improve the operability of HENs.

3. Improvements in controllability and observability based on set covering and centrality measures

Before the introduction of the new algorithms that were developed to improve the operability of HENs, it must be noted that in the field of network science the problem of the energy demand required to control the network is also studied and the related measurement of energy demand is not identical to the actual energy requirement of the HENs.

Since the topology of the network of the state variables is being studied, the energy demand term has been adopted as it represents the order of the system, i.e. the distance between the actuator and a state variable, or the distance between an arbitrary state variable and a sensor.

More precisely, when the dynamical behaviour of the system between the input c and output o can be described by a transfer function in zero-pole-gain form

$$G_{co}(s) = \frac{Y_o(s)}{U_c(s)} = k_{gf}\frac{s^{m_z}+b_{m_z-1}s^{m_z-1}+\dots+b_1s+b_0}{s^n+a_{n-1}s^{n-1}+\dots+a_1s+a_0} = k_{gf}\frac{\prod_{i=1}^{m_z}(s-z_i)}{\prod_{i=1}^{n}(s-p_i)} = C_o(sI-A)^{-1}B_c \quad (3)$$

where k_{gf} stands for the gain, m_z is the number of zeros and n represents the number of poles, then the relative degree of the system, d, is the shortest distance between node c and node o in the network representation.

$$r_{rd} = d + 1 \quad (4)$$

The most critical actuator or sensor can be defined based on this relative degree. As it is assumed that the relative degree is related to the energy demand of the control problem, each state variable is assigned to its nearest actuator or sensor, so sets of nearest neighbours for state variables are defined by each actuator and sensor. Thus, to improve the operability of HENs, the component of the state variables with the highest relative degree should be enhanced by adding an actuator or sensor. From another perspective, the system can be upgraded by improvement of the largest component (Letellier et al., 2018). In this paper, two approaches are proposed to determine the new actuators and sensors for a given HEN.

The first approach is based on the well-known set cover problem. In the first step of this algorithm the maximum relative degree, r_{max}, of the system is defined. In the second step, a set of nodes, W_i, is generated as the nodes can be reached from node i in a maximum of $r_{\max}$ steps. Let U denote the set of all nodes, C the set of actuators, and O the set of sensors. In the case of the control task, let J represent the set of necessary driver nodes, such that P stands for the set of nodes that J covers, $P = \cup_{j\in J} W_j$, to create J with minimal cardinality such that $P = U$ and $C \subset J$, and $r_u \leq r_{max}$, $\forall\, u \in U$. This method easily can be transformed to create a robust input or output configuration. In this regard the set cover problem needs to be configured such that each node is covered at least twice by the minimum number of actuators or sensors. In this article, the greedy algorithm was used to solve the problem (Gori et al., 2010).

The second approach is based on the closeness and betweenness centrality measures. The closeness centrality of node i can be calculated by Eq(5), where N denotes the number of nodes in the network, and $d(i,j)$ represents the shortest path between nodes i and j. The number of the shortest path that intercepts node i ($\sigma_{st}(i)$) is calculated by the betweenness centrality and is divided by the number of all the shortest paths (σ_{st}) for each start (s) and target (t) node pair such that node i cannot be a start or target node. The betweenness centrality of node i is defined by Eq(6).

$$Cc(i) = \frac{N-1}{\sum_{j\neq i} d(i,j)} \tag{5}$$

$$Bc(i) = \sum_{s\neq i\neq t} \frac{\sigma_{st}(i)}{\sigma_{st}} \tag{6}$$

Each measure is normalised and in both cases a value of one means that the node is a central element in the network while a value of 0 indicates that the node does not play an important role in terms of the topology. Thus, an initial input configuration determined by maximum matching or an SCC-based method, C, can be exceeded by adding a node i to C as node i is the most central element, $C = C \cup \{i: \max(Cc(i) * Bc(i)), i \notin C\}$. This step is repeated until the maximum relative degree becomes smaller than or equal to the threshold parameter r_{max}, $r_i \leq r_{max}$, $i = 1,..,N$.

Although the two approaches introduced above are discussed with regard to the improvement of the input configuration, in the case of research related to observability only the direction of edges in the network should be reversed, and the same algorithms that were used to study the controllability can be directly utilised.

4. Results

The two approaches introduced above yield slightly different results. As a demonstration, an example network that is independent of heat exchanger networks (Figure 2), then a connected subnetwork of a HEN from (Westphalen et al., 2003) are analysed (Figure 3).

Firstly, the input and output configurations of the example network were determined. Both maximum matching and an SCC-based method determined that $u_1 = x_1$ and $u_2 = x_3$ as driver nodes (Figure 2a), and $y_1 = x_5$ and $y_2 = x_7$ as sensor nodes (Figure 2b). It is clearly visible that u_1 controls x_1 and x_2, while the remaining nodes belong to u_2 based on the relative degree. In terms of observability, x_7 and x_8 are observed by y_2, while the remaining nodes are observed by y_1. As a consequence, the relative degree is five in both cases. It can be seen that u_2 and y_1 are the sources of the critical components. The relative degree is five with regard to both components. To improve the input and output configurations, the reachable sets and the measurements of the closeness centrality and betweenness centrality are generated for each node. The threshold parameter of relative degree is determined as $r_{max} = 3$. Then the necessary nodes are added by both methods. The results can be seen in Table 1, the relative degree, i.e. the criticality of an actuator or sensor, is denoted in brackets.

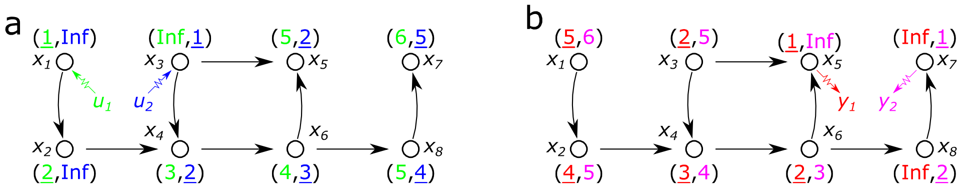

Figure 2: Example network of eight nodes and eight edges with the input configuration (a) and output configuration (b) that are determined by the maximum matching algorithm. The relative degrees are noted in brackets. The colours green, blue, red and purple denote the relative degree of the nodes from actuators u_1 and u_2 and sensors y_1 and y_2, respectively.

Table 1: Improved input and output configurations of the example network. Brackets contain the new relative degrees.

	Set covering	Centrality measures
Actuators	$x_1(2)$, $x_3(2)$, $x_6(3)$	$x_1(2)$, $x_3(2)$, $x_6(3)$
Sensors	$x_2(2)$, $x_5(3)$, $x_7(2)$	$x_4(3)$, $x_5(2)$, $x_7(2)$

The same set of driver nodes is provided by the improvement in the input configuration, while with regard to the output configuration the additional node was x_2 in the case of the set covering-based method and x_4 in terms of the centrality measures-based method. How the load is divided into the nodes is also determined by the relative degree. In order to not violate the properties of structural controllability and observability, the initial input and output configurations are expanded. The maximum relative degree in all cases was three.
The example HEN from (Westphalen et al., 2003) and its state-space network representation can be seen in Figure 3. The HEN in its typical form is presented in Figure 3a, while the state-space network representation with the identified SCCs is illustrated in Figure 3b, where green SCC contains more than two state variables.

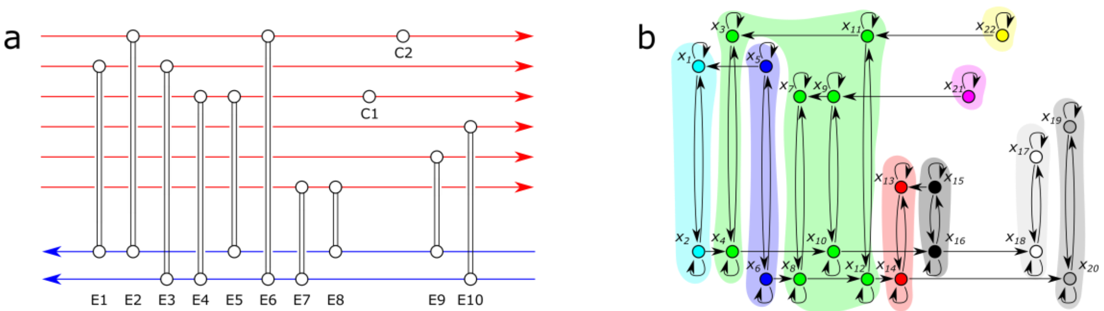

Figure 3: The heat exchanger network analysed by 10 heat exchanger cells and two utility coolers (a) and its state-space network representation with strongly connected components denoted by colours (b).

Since each SCC has at least one output edge, except for the SCCs in white and grey, actuators should be placed there in order to ensure structural controllability. Similarly, SCCs in yellow, purple and dark blue do not possess input edges, so sensors should be placed there to provide structural observability. In Table 2 the results when the improvement is executed with a threshold parameter of $r_{max} = 5$ can be seen.

Table 2: Improved input and output configurations of the example HEN.

	Minimal	Set covering	Centrality measures
Driver nodes	$x_{17}(7)$, $x_{19}(7)$	$x_3(4)$, $x_7(4)$, $x_{17}(4)$, $x_{19}(4)$	$x_{10}(4)$, $x_{12}(4)$, $x_{17}(4)$, $x_{19}(4)$
Sensor nodes	$x_5(4)$, $x_{21}(6)$, $x_{22}(6)$	$x_5(4)$, $x_{15}(5)$, $x_{21}(3)$, $x_{22}(3)$	$x_5(4)$, $x_{10}(4)$, $x_{12}(4)$, $x_{21}(1)$, $x_{22}(1)$

The relative degrees (energy demand) are the same in terms of the input configurations. In the case of the output configurations, the set covering-based method uses one node less, thus, the load is higher. (However, the cost of implementation is cheaper due to the smaller number of sensors). In conclusion, the minimal input and output configurations are improved evenly by each method, and both are suitable in terms of extending an existing configuration in order to improve the operability of HENs.

5. Conclusions

The topologies and properties of HENs related to control can be easily studied based on the structural analysis of the network of the state variables. Based on the maximum matching of the network we determined the structural patterns that make the HEN controllable and observable with a relatively small number of actuators and sensors. Two methods were proposed to define the sets of additional actuators and sensors required to decrease the energy demand of the control by reducing the order of the controlled system. The first method is based on the set covering interpretation of the problem, while the second algorithm uses closeness centrality and betweenness centrality measures of network science.
With the set covering-based method, the robustness of input and output configurations can also be determined, so our future work will focus on the simultaneous and multi-criteria optimisation of robust control configurations.

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