CHEMICAL ENGINEERING TRANSACTIONS
VOL. 62, 2017

A publication of AIDIC
The Italian Association of Chemical Engineering
Online at www.aidic.it/cet

Guest Editors: Fei Song, Haibo Wang, Fang He

ISBN 978-88-95608- 60-0; **ISSN** 2283-9216

DOI: 10.3303/CET1762181

Application of Darcy Formula in Analyzing the Infiltration Mechanism of the Red Clay Considering the Effective Porosity

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There are closed pores and dead end pores in red clay, resulting in that the effective pores are less than the actual pores. Therefore, the permeability coefficient calculated according to the traditional Darcy law is greater than the actual value. Based on the traditional Darcy law theory, the empirical calculating formula of the effective porosity-Darcy permeability coefficient is derived by estimating the diameter of the soil particle and the effective porosity. According to the permeation test of different chloride solutions in red clay, the applicability of calculated permeability coefficient of different effective porosity-Darcy law and the actually measured value in red clay is compared. It is found that the permeability coefficient calculated by the traditional Darcy law has a larger deviation from the measured value, and the permeability coefficient calculated by the maximum compaction-Darcy law is consistent with the measured value the most.

1. Introduction

The water penetration capacity of the permeable reactive soil is one of the three major issues in soil mechanics research. At present, many scholars have studied the permeability of soil. They analyzed different types of osmosis deformation mechanism (Li and Zhang, 2006), carried out non-clay critical hydraulic gradient experiment and theoretical derivation (Mao et al., 2009), calculated the coefficient of permeability of the cohesive clay and found out the influencing factors of the coefficient (Xie et al., 2005). Nagaraj et al. found that the permeability coefficient is related to the soil moisture content and the liquid limit (Nagarj et al., 1994; Sivakumar et al., 1983). The pore structure, size and connectivity are the key factors to control the permeability of soil (Samarasinhe and Huang, 1982; Mesri, 1971; Ren et al., 2016). The existing empirical formulas of permeability coefficient are all derived from the permeability test of the coarse grained soil, and the permeability coefficient deviation is too large for calculating the cohesive soil. Therefore, on the basis of the traditional empirical formula, some scholars put forward the empirical formula based on the effective pore permeability coefficient to calculate the permeability coefficient of clay soil (Ren et al., 2016; Dang et al., 2015)
The permeability coefficient of the soil reflects the permeability of the permeable medium in the soil, which is influenced by the permeability medium and the properties of the soil. The Darcy law is derived from the law of permeability test of coarse grained soil. Many research results have proved that it is suitable for analyzing the permeability of fine grain soil. The calculation formula of Darcy permeability coefficient considered the porous media's properties (density, viscosity) and soil properties (specific surface area of the soil particles and pore ratio) and so on, ignoring the influence of pore structure, connectivity, soil particle shape and mineral composition on permeability. There are four types of pores in clay: channel pores, closed pores, dead end pores, and pores occupied by bound water membrane. Among them, the former is an effective pore. The seepage process of water in the soil is actually the process of free water moving in the channel pores. The clay particles are small and mostly charged. With the change of external conditions, the dispersed soil particles can coagulate, forming closed pores, dead end pores and pores occupied by bound water membrane, thus leading to high pore ratio and low permeability of red clay. The traditional Darcy law

Please cite this article as: Shanmei Li, Zhikui Liu, Jianping Meng, Tianjie He, Jiayu Chen, Chunmei Mu, 2017, Application of darcy formula in analyzing the infiltration mechanism of the red clay considering the effective porosity, Chemical Engineering Transactions, 62, 1081-1086 DOI:10.3303/CET1762181

originates from the coarse-grained soil and considers that water is percolating in the pore. Many researchers directly apply the traditional Darcy law to cohesive soil, ignoring the fact that a lot of invalid pores exist in cohesive soil. Therefore, this results in a large deviation between the results calculated by the traditional Darcy penetration formula and the measured values of permeability coefficient. Based on the traditional Darcy law and considering the effective pore, the effective pore-Darcy permeability coefficient calculation formula suitable for the clay soil is derived, and the applicability of the formula is verified by the permeability test of red clay.

2. Derivation of the Calculation Formula of Darcy Permeability Coefficient

2.1 Derivation of the calculation formula for Darcy permeability coefficient based on effective porosity

It is assumed that a micro unit is taken from the soil, whose area is *dA* and height is *ds*. The pore water pressure of the upper and lower surface of the soil column is *p+dp* and *p*, respectively. When the water flows through the soil column, the height of the water head changes by *dh*.
It is assumed that the effective porosity in the soil is *n*', and the gravity of the free water is $\gamma_{z\omega}$. The seepage force is F,It is positive along the seepage direction. The seepage flow in the soil is subjected to three kinds of forces, according to the equitation of the stress,

$$-\frac{dH}{ds}+\frac{F}{\gamma_{zw} n' dAds}=0 \tag{1}$$

The hydraulic gradient is recorded as J=dH/ds

Then, formula (1) is changed and the following formula is obtained:

$$F=\gamma_{z\omega}.n'.dA.ds.J \tag{2}$$

Clay minerals are mostly flaky or flat. Because of the electrification of clay minerals, clay particles exist in the form of aggregates. The aggregate is similar to a round ball. It is assumed that the soil particle is a spherical particle with a diameter of d, and the Stokes formula is used to calculate the laminar resistance on a particle:

$$t = \lambda.\mu.d\upsilon' \tag{3}$$

where, t is a drag force, d is the diameter of the particle aggregate, and the coefficient λ depends on the influence of the adjacent particles. Here, λ=3π, μ is the dynamic viscosity coefficient of the free water, and υ' is the average velocity in pore solution.
In the penetration test, it is assumed that velocity is υ. According to the principle that the amount of vadose water flowing through the section A is the same within a unit time, the following formula is obtained:

$$\upsilon'= \upsilon/n' \tag{4}$$

Substitute (4) into (3) and the following equation is obtained:

$$t=\lambda.\mu.d(\upsilon/n') \tag{5}$$

In Fig. 1, the total drag force imposed to the tiny unit body is:

$$T=\frac{(1-n')\ dAds}{\frac{\pi}{6}d^3 n'}\lambda\mu d\upsilon \tag{6}$$

Substitute (6) into (1) and the following equation is obtained:

$$\upsilon=\frac{\frac{\pi}{6}\left(n'\right)^2}{\lambda\ (1-n')}d^2\frac{\gamma_{zw}}{\mu}J \tag{7}$$

So, the Darcy permeability coefficient can be obtained.

$$k=\frac{\left(n'\right)^2}{18(1-n')}d^2\frac{\gamma_{zw}}{\mu} \tag{8}$$

The core problem of the application of formula (8) is to determine d and n '.

2.2 Determine the diameter of the aggregate d

1g soil is taken. Its moisture content is w, the soil density is ρ, and soil particle density is ρ_s. Then, the surface area of each particle aggregate is πd^2, and the volume of each particle aggregate is $\pi d^3/6$. According to the definitions of the three-phase indexes of soil mechanics, it is known that the volume of 1g soil particles is:

$$v_s = \frac{1}{(1+\omega)\rho_s} \tag{9}$$

The number (M) of soil aggregate contained in 1g soil:

$$M=v_s/(\pi d^3/6)=6/[(1+w).\rho_s.\pi d^3] \tag{10}$$

The specific surface area of 1g soil is s:

$$s=M\pi d^2/1=6\pi d^2/[(1+w)\rho_s\pi d^3]=6/(1+w).\rho_s.d \tag{11}$$

The mean diameter d of the soil particles can be calculated according to formula (17):

$$d = \frac{6}{(1+\omega)\rho_s s} \tag{12}$$

And $\begin{cases}(1-n)\rho_s = \rho_d \\ (1+\omega)\rho_d = \rho\end{cases}$ (13)

Substitute (13) into (12) and the following equation is obtained:

$$d = \frac{6(1-n)}{\rho s} \tag{14}$$

The formula (16) shows that the average diameter of the soil particles is related to the porosity, soil density and specific surface area of soil.
Determination of the effective porosity n'
The soil volume is set as v_s, the effective pore volume is set as v_t, the invalid pore volume is set as v_0, the total pore volume as v_k, the volume of pores occupied by the bound water membrane as v_j, and the volume of the closed pore and the dead end pore as v_f. It is assumed that in the process of infiltration, the total volume v of soil is fixed, and the soil particles do not condense and deform, that is, v_s is the fixed value.
Total pore volume of soil v_k
It is assumed that the following relationship exists between the invalid porosity and the total porosity:

$$\zeta=n_0/n \tag{15}$$

Then, $n'=n-n_0=(1-\zeta)\, n$ (16)

Substitute (14) and (16) into (8). The formula for calculating Darcy permeability coefficient, which takes into account the effective porosity, is obtained:

$$k=2(1-\zeta)^2(1-n)^2n^2\gamma_{zw}/\{\mu\rho^2s^2[1-(1-\zeta)n]\} \tag{17}$$

Formula (17) shows that permeability coefficient is related to soil density, specific surface area, pore solution's weight, dynamic viscosity coefficient, pore ratio and the structure, connectivity and size of pore. When other parameters are fixed, the permeability coefficient is inversely proportional to the square of the specific surface area. In the formula of (15) ~ (17), the value of ζ is related to the degree of compaction of the soil body, concentration of solution and the valence of ions. When the compactness of soil sample, the solution concentration and the valence of the ion are low, ζ takes the larger value; otherwise, it takes the smaller value. When the soil particles are absolutely dispersed, that is, there are no closed pores and dead end pores in the soil, and the invalid pore is the one occupied by the bound water membrane. At this time, the value of ζ is the smallest. Assuming that in the course of soil compaction, the deformation of soil particles is recoverable, i.e. under the condition of maximum compaction, the soil is most compact, then it can be considered that under the condition of maximum compaction, the porosity of the soil body is the largest invalid pore, and the value of ζ is the largest. The scope of the value of ζ can be determined as:

$n_j/n<\zeta< n_{op}/n$ (18)

In formula (18), n_j is the porosity rate of the pores occupied by the bound water membrane, and n_{op} is the porosity of the soil body under the condition of maximum compaction. The lower limit of the invalid porosity is the porosity of the pores occupied by the bound water film, and the upper limit is the porosity under the condition of maximum compaction.

(1) Calculation of the porosity of pores occupied by the bound water membrane

It is assumed that the aggregates of soil particles are homogeneous, and a layer of homogeneous and isopachous bound water membrane is formed at the periphery, which is not overlapped with the adjacent bound water membrane. The thickness of the bound water membrane is recorded as h.

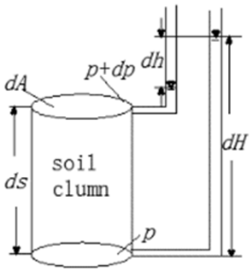

Figure 1: Penetration model of unit soil body

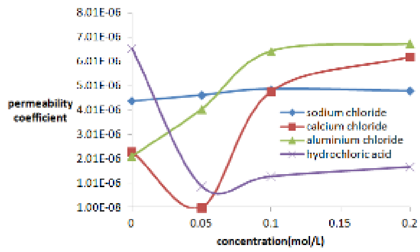

Figure 2: Permeability coefficient

Thickness of electric double layer:

$h=(DkT/8\pi n_0 e_2 v_2)^{1/2}$ (19)

In formula (25), D is the dielectric constant of the pore solution, k is Boltzmann constant ($k=1.38\times10^{-16}$ ergs/K), T is thermodynamical temperature, n_0 is ion concentration, e is 4.8×10^{-10} esu, and v is the valence of ion.

The volume of pores occupied by bound water membrane in 1g soil is:

$v_j=M\cdot\pi[(d+2h)^3-d^3]/6$ (20)

The porosity of pores occupied by water film is:

$n_j= v_j/v= v_j/(1/\rho)= \rho v_j$ (21)

Substitute (10), (14) and (20) into (21) and the following equation is obtained:

$n_j=\rho^2 s\{[\rho sh/3(1-n)]^2+\rho sh/3(1-n)+1\}h/[(1+w)(1-n)\rho_s]$ (22)

(2) Calculate the porosity of soil under the condition of maximum compaction

The maximum dry density of soil is denoted as ρ_{dmax}, and the optimum water content is set as w_{op}, then, $n_{op} = (v_s'- v_s'')/ v_s'=1-\rho_{dma\,x}/\rho_s$

Substitute (22) and (23) into (18), and ζ can be obtained. That is, the value range of the ratio between the invalid porosity and porosity is:

$\rho^2 s\{[\rho sh/3(1-n)]^2+\rho sh/3(1-n)+1\}h/[(1+w)(1-n)\rho_s]/n<\zeta< (1-\rho_{dma\,x}/\rho_s)/n$ (23)

Formula (24) shows that the upper and lower limits of ζ can be calculated according to the basic physical quantity ρ, s, h, n, w, ρ_s, and ρ_{dmax}.

3. Analysis of the Results of the Penetration Test of Red Clay

3.1 Production and installation of soil sample

The red clay is dried and crushed, and passes through the 2mm screen. The soil sample with moisture content of 28% is prepared with the deionized water, sealed with fresh-keeping bags, put into the moisture preserving cylinder, and taken out after 24 hours. 204g of wet soil which has been prepared was weighted. The soil sample in the shape of ring knives and with a density of 1.7g/cm^3 and a diameter of 6.18*4cm are pressed by a jack, which is fixed on the saturating device and saturated with distilled water.

The filter paper is taken out and soaked in the deionized water and attached to the upper and lower surfaces of the prepared soil sample. The specimen is installed in accordance with the variable head permeability test method based on soil testing regulations. 4 soil samples were shared during the test, and the batch number of the soil sample was the same as the sample preparation method, and the soil samples were numbered in turn. The Nacl, Cacl2, Alcl3 and Hcl solutions of 0M, 0.05M, 0.1M and 0.2M are prepared by the distilled water respectively. The No.1-No.4 soil samples were tested with the variable head permeability test method and the impacts of different cations with different concentrations on the permeability of red clay were compared.

3.2 Test result

The permeability coefficient of the red clay under the permeation of the chloride solution with different concentrations is shown in Fig. 2. It is known from the experimental results that, for the same soil sample, the effects of different concentrations of chloride on the permeability of red clay are different. With the increase of the concentration of Nacl and $Alcl_3$, the permeability coefficient increases and the permeability coefficient decreases first and then increases with the increase of concentration of $Cacl_2$ and Hcl.
The specific surface area test using the methylene blue titration method is carried out on the soil sample after the penetration test. The relation between the specific surface area and the permeability coefficient is shown in Fig. 3.It shows that the permeability coefficient is in inverse proportion to the specific surface area, which conforms to the influence law of formula (17).

Figure 3: Relationship between specific surface area and permeability coefficient of soil samples

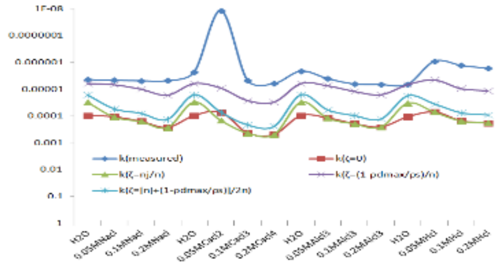

Figure 4: Comparison of permeability coefficient calculated by different methods

The type and concentration of the solution change the thickness of the electric double layer, which further affects the diameter of the aggregate and the effective pore of the soil, and affects the permeability of the red clay: As the diffusion layer increases, the soil particles are dispersed, the specific surface area increases, and the permeability coefficient decreases; on the contrary, the specific surface area decreases and the permeability coefficient increases.

4. Calculation of Permeability Coefficient of the Red Clay Using Darcy Law Which Considers the Effective Porosity

By using the permeability coefficient calculation formula (24) which is based on the Darcy law considering effective porosity and derived from the previous formula, the permeability coefficients of red clay under four different invalid pore conditions are calculated respectively: ① The invalid pore is not considered, that is $\zeta=0$. ②The invalid pores are the ones occupied by the bound water film, that is $\zeta=n_j/n$. ③The invalid porosity is the corresponding porosity under the condition of maximum compactness, that is $\zeta=(1-\rho_{dmax}/\rho_s)/n$. ④The invalid porosity is the average of the pores occupied by the bound water membrane and the pores under the maximum degree of compaction, that is $\zeta=[n_j+(1-\rho_{dmax}/\rho_s)]/2n$. The calculated values of the permeability coefficient are recorded as: $k(\zeta=0)$, $k(\zeta=n_j/n)$, $k(\zeta=(1-\rho_{dmax}/\rho_s)/n)$, and $k(\zeta=[n_j+(1-\rho_{dmax}/\rho_s)]/2n)$ respectively. The relevant parameters are substituted into formula (30). The permeability coefficients of the above four cases are calculated respectively, and the results are compared with those of the test results, as shown in Fig. 4.
By comparing the five curves in Fig. 4, it is found that the distribution curves of the permeability coefficients calculated with Darcy laws when the values of ζ are different are similar to the actually measured curves in terms of curve shape, but the values are much different. The deviation of the permeability coefficient calculated by the traditional Darcy law is the largest, which is 1~3 orders of magnitude larger than the measured value. The calculation value of the permeability coefficient under the influence of the thickness of bound water film is superior to the value calculated according to the traditional law, but there is still a large error. The permeability coefficient calculated with the porosity corresponding to the maximum degree of compaction is approaching the actually measured value the most. When the invalid pore is the average of the sum of the pores occupied by the bound water membrane and the pores corresponding to the maximum degree of compaction, the calculation value of the permeability coefficient is in the middle. The ζ value is sorted by the approximation degree of the permeability coefficient to the measured value:

$\rho_{dmax}/\rho_s)/n>[n_j+(1-\rho_{dmax}/\rho_s)]/2n>\zeta=n_j/n>0$ (24)

The above analysis shows that the traditional Darcy law can generate great error when calculating the permeability coefficient of red clay, and the Darcy law considering the effective porosity should be considered to calculate it. The contribution of pores occupied by bound water film to the invalid pore is lower than the sum of the closed pores and dead end pores, and the invalid porosity of red clay is approximately equal to the porosity corresponding to maximum compaction degree. Combining the seepage experiment results, the formula for calculating the permeability coefficient of red clay can be deduced by directly correcting the traditional Darcy law using the porosity corresponding to the maximum compaction degree, and the formula is called the maximum compaction-Darcy permeability coefficient formula.

$\zeta=(1-\rho_{dmax}/\rho_s)/n$ is substituted into formula (23), and the following equation is obtained:

$$k=2[1-(1-\rho_{dmax}/\rho_s)/n])^2 (1-n)^2n^2\gamma_{zw}/[\mu\rho^2s^2(2-n-\rho_{dmax} /\rho_s)] \quad (25)$$

Formula (26) is the maximum compaction-Darcy permeability coefficient formula, and the parameters are the same as those mentioned previously. Formula (26) shows that the permeability coefficient of the red clay is related to the factors such as the maximum dry density of soil, soil particle density, porosity, pore solution weight, viscosity coefficient and specific surface area of the pore solution.

5. Conclusion

The derivation of the calculation formula of the Darcy permeability coefficient based on the effective porosity is the correction of the calculation formula of the Darcy permeability coefficient of coarse grained soil. When $\zeta=0$, it is the traditional formula for calculating the Darcy permeability coefficient, which makes the formulas for calculating the permeability coefficient of coarse and fine grain soil unified. The results of variable head permeability test of the red clay show that the permeability coefficient is inverse proportional to the square of the specific surface area. The experimental results are consistent with the influence law of the calculation formula of Darcy permeability coefficient considering effective porosity in this paper. In red clay, the contribution of closed pores and dead end pores to invalid pore is large, and the contribution of pores occupied by bound water film is small, and the porosity corresponding to the maximum compaction degree is the closest to invalid porosity. In the calculation of permeability coefficient of red clay, the calculated value of permeability coefficient using the traditional Darcy empirical formula is 1~3 orders of magnitude larger than the measured value. The maximum compaction-Darcy permeability coefficient calculated in this paper is the closest to the measured value.

Acknowledgement

Fund Project: Chinese National Natural Science Foundation Project (51169004), Guangxi Natural Science Foundation in China (2017GXNSFAA198238, 2017GXNSFAA198213), Guangxi Key Laboratory of Geo-mechanics and Engineering in China (2015-A-01).

References

Dang F.N., Liu H.W., Wang X.W., 2015, Empirical formulas of permeability of clay based on effective pore ratio, Chinese journal of rock mechanics and engineering, 34(9), 1909-1917, DOI: 10.13722/j.cnki.jrme.2014.1583

Li C.J., Hu F., Zhang L.Q., 2006, Analysis of seepage stability and deformation of dam for Doupo Reservoir, Rock and Soil mechanics, 16(8), 1305-1311.

Mao C.X., Duan X.B., Chen L., 2009, Study of critical gradient of piping for various grain sizes in sandy gravels, Rock and soil mechanics, 30(12), 3705-3709.

Mesri G., 1971, Mechanisms controlling the permeability of clays, Clays and Clays Minerals, 19(1), 151-158.

Nagarj T.S., Pandian N.S., Narsimaha R.P.S., 1994, Stress state-permeability relationships for over-consolidate clays, Geotechnique, 44(2), 333-336.

Ren X.W., Yang Z., Peng Q.L., Kang J.Y., Li D.X., Wang D.B., 2016, A relation of hydraulic conductivity-void for soils based on Kozeny-Carman equation, Engineering Geology, 213, 89-97, DOI: 10.1016/j.enggeo.2016.08.017

Samarasinhe A.M., Huang Y., 1982, Permeability and consolidation of normally consolidated soils, Geotechnical Engineering, 108(6), 835-850.

Sivakumar B.G.L., Pandian N.S., Nagaraj A., 1983, Reexamination of the permeability index of clays, Canadian Geotechnical Journal, 30(1), 187-191.

Xie K.H., Zhuang Y.C., Li X.B., 2005, Laboratory investigation of permeability characteristics of Xiaoshan clay, Chinese journal of geotechnical engineering, 27(5), 591-594.