

## Thermohydraulic Transport Characteristics In Wavy Microchannel Under Pulsating Inlet Flow Condition

Tapas Kumar Nandi<sup>a</sup>, Suvanjan Bhattacharyya<sup>\*b</sup>, Sampad Gobinda Das<sup>c</sup>, Arnab Banerjee<sup>b</sup>, Himadri Chattopadhyay<sup>c</sup>

<sup>a</sup>Department of Mechanical Engineering, Techno India College of Technology, Kolkata, West Bengal, India.

<sup>b</sup>Department of Mechanical Engineering, MCKV Institute of Engineering, Liluah, Howrah, West Bengal, India.

<sup>c</sup>Department of Mechanical Engineering, Jadavpur University, Kolkata, West Bengal, India.

\*suvanjan@gmail.com

The numerical investigation on developing unsteady laminar fluid flow and heat transfer inside a 2D wavy microchannel, due to sinusoidal varying velocity component at inlet is done. The thermohydraulic flow was developing while the microchannel walls were kept at constant uniform temperature. The transient solution of 2D Navier-Stokes equation was attained using the SIMPLE algorithm with the momentum interpolation technique. The deionized water is used as working fluid and Reynolds number ranging from 0.1 to 100. The simulation results reports, mainly the heat transfer and pressure drop for wavy microchannel are compared with straight microchannel by keeping the cross sectional area same. By comparing with steady flow in wavy channel it was seen that imposed sinusoidal velocity at inlet can improved performance in terms of heat transfer at different amplitude (0.2, 0.5, 0.8) and frequency (1, 5, 10) while keeping the friction factor within tolerable limits.

Keywords: simultaneously developing; pulsating flow; heat transfer; laminar; microchannel; numerical.

### 1. Introduction

The continuous as well as rapid advancement in the field of micro fabricated devices and integrated electronic circuits which leads to generate a huge amount of heat to be cooled down using some integrated cooling facility and microfluidic devices on which researchers have a tremendous interest to explain the heat and mass transmit processes in micro channels.

Some techniques to enhance heat transfer by using turbulators proposed by Bhattacharyya et al., (2016, 2016, 2017). One can the wavy type micro channels are easy to build and reasonably cost-effective way of transfer of heat passively. Hence, wavy profiled channels have been regarded in several earlier studies as a way to improve heat transfer when utilized in conventional high Reynolds number (Re), presently for low Re and organizing of microchannels (Xin et al., 1988; Wang 1981; Bhatti et al., 2016). Basically traditional microchannel heat sinks (MCHS) are commonly utilized in case of straight channel in which the streamlines of the coolant are almost one directional and straight. Whereas, self-persistent flow have oscillation which developed in case of a wavy profiled channel. These self-persistent oscillation leads to the undermining of the laminar boundary layer which improve the assimilation between the core and fluid near the wall. The thought of cooling using MCHS was initially proposed by Tuckerman et al., (1981). An investigational study was accomplished by Sui et al., (2011) throughout which they have established that the effectiveness of heat transfer in case of the wavy shaped micro channels are judged against those of straight baseline micro channels having the equal cross section and length. Wu et al., (2003) reported on friction factor for the case of smooth micro channel with trapezoidal profile made of silicon with dissimilar aspect ratios (AR). It confirmed that the friction factor of liquid flowing into micro channel consisting of equal hydraulic diameter but having dissimilar cross-sectional shape might be highly diverse as a consequence of the profile of the section of the flow path. The technique has been performed and well thought-out by many researchers to obtain enriched heat transfer in microchannel (Wang et al., 2004; Patankar et al., 1974; Yang et al., 2010). Fletcher et al.,

(2005, 2006) have numerically analyzed heat transfer in episodic serpentine channel having various cross sections for a fully laminar flow. Kowalewski et al. (2008) considered the transport process for microfluidic system and hence concluded that the diffusion is the only mode of transport of heat at very low Re (Creeping flows). In some cases researchers utilized some simulated ways to progress the heat transfer for creeping flows in wavy shaped micro channel Quddus et al., (2005) and Xia et al. (2009). Hsieh et al., (2008) reported that several varieties of ribs inside wavy shaped micro channel for the enrichment of the heat transfer at very low magnitude of Re (<1 values). Mohammad et al., (2011) performed a numerical simulation of heat transfer enrichment in wavy shaped MCHS and reported that both the coefficient of heat transfer and shear stress at wall increases with the amplitude of the wavy shaped micro channel. Gong et al., (2011) numerically studied the flow and heat transfer in a wave shaped micro channel having hydraulic diameter of 500 $\mu$ m and considering Re to be within Re=50-150. According to their indication that wavy surface of a microchannel can be a prospective element for heat transfer upgrading with the appropriate choice of shape and selection of flow parameter devoid of utilizing any external assimilating assists. Chattopadhyay et al., (2006) worked on the concurrently developing flow for channel in laminar regime.

It has been observed that at very low Re, wavy pathway does not endow with any major amount of heat transfer enrichment because of the flow is steady, predominantly. If anyhow the flow is prepared unsteady by using any exterior effects, major enrich in heat transfer may be detected exclusive of roughness of elements, swing to turbulence or convoluted geometry. The effect of the superimposition of heat transfer of sinusoidal pulsating flow component on the mean channel flow through a wavy shaped micro channel is been studied in the recent work.

## 2. Problem Formulation and Governing Equation

A schematic of wavy shaped micro channel used in the current investigation is represented in Figure 1. The length of the microchannel (L) is assumed to be extremely long compared to the height (H). Therefore the problem can be considered to be 2-D. The geometry and flow properties of the problem are non-dimensionalized by using the channel height H, the fluid density ( $\rho$ ), Specific heat ( $C_p$ ), dynamic viscosity ( $\mu$ ) and thermal conductivity (k). The conventional serpentine shaped channel is designed where crest and trough facing each other alternately by a phase of 1800.i.e.  $\phi=\pi$ . The height of the designed channel changes sinusoidally is described by the function:

$$Y = A_w \sin\left(\frac{2\pi}{\lambda} x\right) \quad (1)$$

The parameters amplitude of wave  $A_w$  and wavy length  $\lambda$  are kept fixed ( $A_w=0.2$ mm and  $\lambda=1$ mm) for this case, with an aim to understand the impact of pulsating flow at inlet on thermal performance at this geometry. The length of the channel was 23 with a straight section at the inlet. The wavy section spanned at the middle having length 22 in the channel of a hydraulic diameter of 1mm. The numerical simulation was executed by solving the time dependent, continuity momentum and energy equations for a incompressible fluid with the following suppositions made; (Eq.2) Continuous Newtonian fluid, Re with unsteady laminar flow and heat transfer (Eq.3) Specific heat, thermal conductivity and viscosity are single variable functions of temperature (Eq.4) Negligible gravity and radiation heat transfer. Therefore the governing equations based on these guesses are:

Continuity equation:

$$\frac{\partial u_i}{\partial t} + \nabla \cdot (\rho u) = 0 \quad (2)$$

Momentum equation:

$$\frac{\partial u_i}{\partial t} + \frac{\partial(u_i u_j)}{\partial x_i} = -\frac{\partial p}{\partial x_i} + \frac{1}{\text{Re}} \nabla^2 u_j \quad (3)$$

Energy equation:

$$\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \frac{1}{\text{Re} \cdot \text{Pr}} \nabla^2 T \quad (4)$$

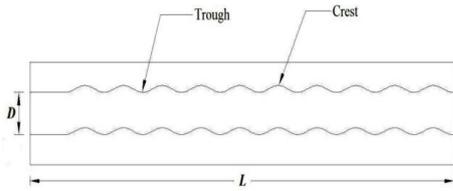


Figure 1: Schematic diagram of the flow channel of test section

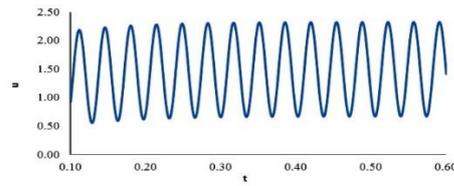


Figure 2: Periodic axial velocity at a location (18.5, 0.4) for  $Re=10$ ,  $A=0.8$  and  $St=10$

Adiabatic circumstances are engaged at the entire microchannel walls. Both the top and bottom wave shaped surfaces are sustained at an isothermal temperature of 330 K. The common no-slip boundary conditions are engaged at the wave shaped surface wall. At the inlet, the velocity profile is developed by engaging the consistent uniform velocity profile having a sinusoidal pulsation. Thus the velocity at inlet profile is specified by the subsequent equation.

$$U_{in} = U_m [1 + A \cdot \sin(2\pi ft)] \quad (5)$$

Where  $A$  is the non-dimensional amplitude and  $f$  is the frequency, then  $f$  can be defined as  $f = St u_m / H$  where  $St$  is non-dimensional strouhal number,  $H$  is the channel height and  $u_m$  is mean velocity. At the outlet the continuative boundary condition, where the second derivative of the primitive variables are set at zero, is engaged. This ensures that there is no abrupt transition at the outlet Chandratilleke et al., (2010).

The governing equations were worked out by means of the SIMPLE algorithm, a finite volume -formulation of Patankar (1980). Although the staggered-grid is more popular for such computations, a collocated grid arrangement for the primitive variables was used. While the application of collocated grid is relatively clear-cut, many researchers used the theory of staggered grid to stay away from the 'checkered-board' pattern of pressure field. However, in the current work, the momentum interpolation method according to Rhie et al., (1983) was used, which can acceptably overcome the problem of pressure oscillation.

A structured mesh was developed for the solution domain by using the mesh generation facility was preferred as  $250 \times 75$  for all cases after performing a rigorous grid independence check. The length of the computational domain was assorted for four different levels of  $Re$ , as the growing length is a function of  $Re$ . Again, different grid distributions were needed for different  $Re$ , depending upon the  $L/D$  ratio. Typically ( $250 \times 75$ ) and ( $250 \times 50$ ) grids were used for  $Re=0.1$ , 1, 10 and 100 for this wavy shaped microchannel. Grid independence was made certain by computing as a minimum at three dissimilar grid levels. The bulk Nusselt number at  $Re$  value of 100 for straight channel were compared with available literature Cebeciet al., (1984) and found to agree within 2%.

### 3. Results and discussion

The numerical model was tested by computing growing up flow in wavy shaped microchannel. As such, the consequences for such a condition are not broadly covered in literature. In the current work, analysis was performed with Strouhal number ( $St$ ) range 1 to 10 and Amplitude range 0.1 to 1. The records plotted in Figure 2 depicts the instantaneous axial velocity represents at an arbitrary location (4.5, 3.5) and (18, 3.5) and several such monitors were used to ascertain that the fully periodic flow field was established after monitoring the data at different location, subsequent the cyclic pulsation at the inlet. After that the periodical time-averaging was established.

For technical estimates, the most imperative parameters are the time-averaged data such as time averaged values of Nusselt number and PD value in the developing region. Figure 3 depicts the assessment of time-averaged  $Nu$  value with different  $Re$  for  $A = 0.2, 0.5, 0.8$  and steady case. It is found that at too much low  $Re$  and pulsation of low amplitude the  $Nu$  is roughly close to the steady case ( $Re \leq 10$  and  $A = 0.2$ ). This is for the reason that the surface structure of the wavy shaped channel has no effect on the main flow at too much low  $Re$  and the flow appears to be dominated by viscous forces and no recirculation could be observed at low  $Re$ , the flow in the wavy shaped passages is distinguished by steady flow as in refs Stone et al., (1991). However, as  $Re$  is augmented ahead of a reserved magnitude, the pulsating inlet flow at all amplitude ( $0 \leq A \leq 1$ ) prevails viscous force which is overlaid with the main flow and the flow becomes unsteady with the assimilation of the shear layer with the channel wall fluid. The unsteady flow increases the assimilation procedure linking the core and near-wall fluid ensuing in significant increase in heat transfer compare to steady case.

When  $Re$  is very low, the average  $Nu$  value is roughly the same as steady case. As  $Re$  is increasing, the difference of average  $Nu$  value between the steady and unsteady case is more significant. In the laminar flow

way, the efficiency of heat transfer of microchannel robustly depends on the assimilation of fluid elements within the channel. The degree of convective fluid assimilation is straightly exaggerated along the path of flow. Thus it is essential to investigate the flow field inside the wavy shaped microchannel which works for the augmentation of the heat transfer. A difficulty that comes up in steady flow is that the flow becomes normal and the boundary layer thickness grows up, which can deteriorate the heat transfer efficiency by the flow. However, when a sinusoidal pulsating flow is imposed at the inlet along with a wavy surface in the microchannel, it is coming up that the vortices can rapidly develop along the path of the flow and disturb in the boundary layer. Thus better heat transfer efficiency can be predicted in wavy shaped microchannels with inlet pulsation.

The pulsation effect is considered by the enrichment proportion which is the ratio of time averaged  $Nu$  value for unsteady case to the local  $Nu$  value considered for the case except pulsation. We define the enrichment proportion as:  $\eta = (Nu_{avg}/Nu_{st})$ ; where  $Nu_{avg}$  is time averaged magnitude of local  $Nu$  at some position and the  $Nu_{st}$  denotes the case when the inlet profile is steady for the same position. Thus a value of  $\eta$  over 1.0 specifies heat transfer enhancement ratio at the particular position.

A difficulty that happens in straight microchannel is that the flow becomes normal and the thickness of the boundary layer increases, which can deteriorate the heat transfer effectiveness along the flow path. However, for wavy microchannel at a reasonable  $Re$ , it is monitored reverse flow and small circulation zone develop quickly along the flow path and brings instability in boundary layer. So enriched heat transfer efficiency can be predictable in wavy shaped microchannel. Figs. 4 & 5 show the heat transfer enhancement with varying amplitude at different  $St$  for microchannel at  $Re = 100$ . It is monitored that at high  $Re$ , the rate of enhancement is less compare to the low  $Re$  flow. This enhancement is prominent after 50% of pulsating amplitude and at low  $Re$  ( $Re = 10$ ) the result depicts enhancement ratio increases gradually.

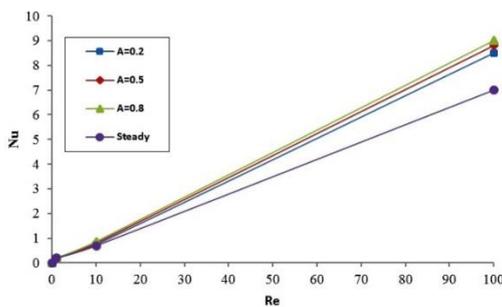


Figure 3: Comparison of time average bulk  $Nu$  with  $Re$  for different amplitude and steady case

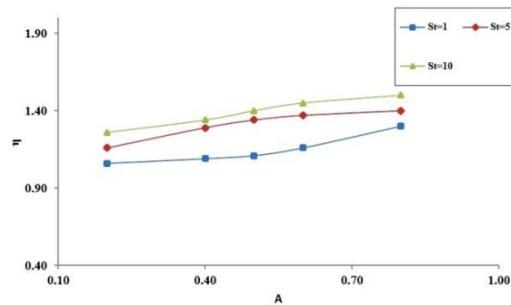


Figure 4: Comparison of enhancement ratio with varying amplitude at different  $St$  for  $Re = 10$

The conventional wavy shaped microchannel contributes to a considerable increase in the PD because of the narrow tightening section of the flow passages which stops flow transition and mixing at low  $Re$ . Figs. 6 & 7 shows non-dimensional pressure difference ( $\Delta p$ ) both for serpentine channel at  $Re \sim 10$  and  $Re \sim 100$ . It is observed that more PD at low  $Re$  whereas at high  $Re$  ( $\Delta p$ ) the PD is marginal and in both the cases it is less than the steady case. Figure 6 shows that the PD does not increase considerably at low  $Re$ , region at all  $St$  for serpentine channel and the gradient of the plot increases rapidly after 50% of amplitude whereas at high  $Re$  the gradual increase of PD is noticed at different  $St$  values. So it can be observed considering all the cases; the current wave shaped microchannel can enrich the heat transfer efficiency, also with a trivial enrichment in PD evaluated with steady case.

Figure 8 depicts the change of mean time-averaged friction coefficient along the longitudinal axis for raccoon and serpentine channel which reveals that there is periodic change of mean friction coefficient from higher value to lower value at particular  $Re$  within the range. Obviously, it is found that in the convergent region the cross section area decreases which causes more resistance to flow of fluid due to the more viscous shear stress and it contributes the increase in coefficient of friction at the narrow spaces. In the divergent area, opposing to the convergent area, the fluid experiences less resistance to flow with the increase in cross section area, which causes rapid decrease of coefficient of friction at the divergent section. The convergent and divergent section of raccoon channel are exposed to more surface area compare to serpentine channel and so the raccoon channel shows higher value of skin friction coefficient than the serpentine channel.

The friction factors versus  $Re$  for different  $St$  are shown in Figure 9. It can be monitored from the figure that in the low  $Re$  region friction is more. This is the flow region where the flow is viscous in nature causing high PD causes the enrichment in the friction factor. The PD is principally due to the surface friction effect. In the higher

Re region, with additional increase of Re number the friction factor decreases, that is the flow resistance decreases with higher flow velocity and low PD.

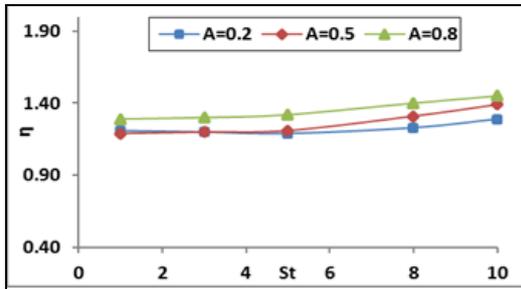


Figure 5: Comparison of enhancement ratio with varying amplitude at different St for Re=100

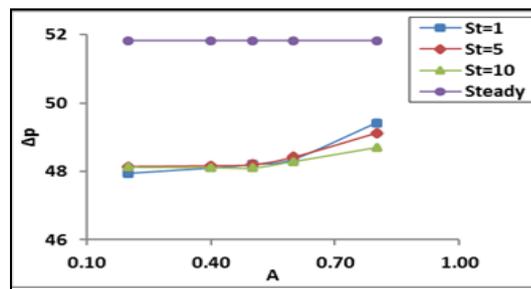


Figure 6: Variation of dimensionless pressure difference with amplitude at different St and steady case at Re

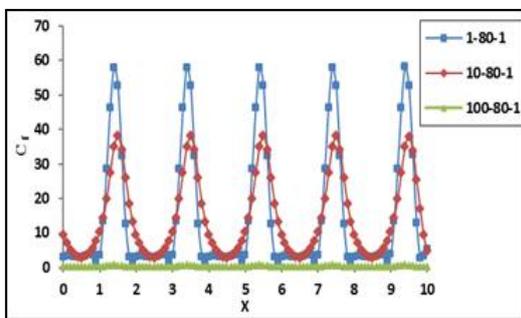


Figure 8: Variation of time averaged skin friction coefficient along the length at different Re and A=0.8, St=1

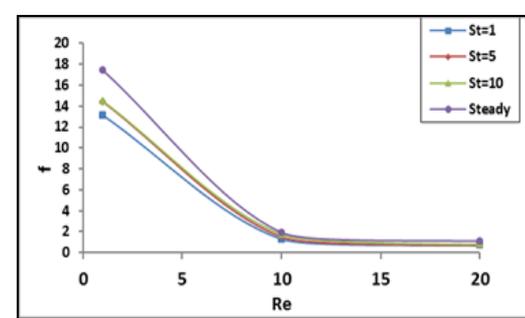


Figure 9: Variation of friction factor with Re under different St and Steady case

#### 4. Conclusion

The unsteady laminar fluid flow developing concurrently and heat transfer within a 2-D wavy shaped microchannel, due to sinusoidal variation in the velocity component at inlet is delivered. The analysis of wavy microchannel shows that at lower Reynolds number very little augmentation is seen at low frequency and high amplitude but at higher Reynolds number a considerable efficiency of heat removal was observed. From results, it is observed that the heat transfer enrichment is highest at an optimal value of  $St = 5$  and having a amplitude more than 50% that of for lower Reynolds number. At upper Reynolds number, an uninterrupted quick enrichment of heat transfer was found. The better heat transfer was attributed to better assimilation of the fluid layer at wavy surface with the core fluid flow. Another important observation is that the flow develops fully by thermally with smaller entry area because of the enhanced mixing process. The assimilation of a sinusoidal shifting velocity component into steady flow in a wavy profile micro channel confirms to be excellent and vigorous way of improving the heat transfer. The study demonstrates that significant enrichment in thermal performance can be attained by introduction of a pulsating component in the developing flow regime although staying in the laminar system.

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