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Application Study of Peridynamics Theory in Failure Analysis of Brittle Material

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Based on the motion equation in integral form, Peridynamics (PD) is the non-local continuum mechanics theory, filling the gap of traditional damage mechanics theory to some extent in terms of theoretical framework and algorithm, and developing into the new theory for material failure research. This paper, adopting the bond-based PD theory, uses C language programming, and make numeral simulation and analysis of the failure process of the two typical brittle materials (concrete and glass), which shows the whole process of the material failure, including crack sprouting, propagation and failure, and proves that the bond-based PD theory can be applied in the failure analysis of brittle material.

1. Introduction

Based on the motion equation in integral form, PD is the non-local continuum mechanics theory, mainly for solving the discontinuity caused by material failure, such as cracks. It was proposed by American scholar Sling in 2000, who firstly put forward the PD theory for the failure analysis of prototype microelastic brittle material (PMB) (Silling, 2000); this theory as the development foundation is the hot research topic now, and also the main theoretical basis for this paper. In 2007, the PD theory for the brittle material and plastic material was also raised (Meggiolaro et al., 2005), being favoured by lots of scholars despite its undeveloped theoretical framework for different material performance. The PD theory can remedy the weakness of mechanical theory (damage mechanics and fracture mechanics) for traditional material failure, e.g. in terms of motion equation, adopt the motion equation in integral form to solve the singularity problem when using the traditional differential motion equation; in algorithm, apply the new meshless algorithm and solve the issues with mesh generation and re-construction. In some degree, this PD theory can replace the traditional mechanics theory for studying material failure (Bobaru and Hu, 2012; Madenci and Oterkus, 2014).
In this paper, the brittle material is supposed as PMB approximately, and use the bond-based BD theory to describe the property of this material, having micro modulus, similar to the constant of elastic coefficient; when the bond elongation reaches critical point, it means that the bond break, unrecoverable; the bond won’t fail when being compressed. The material failure indicates the transformation process of the material from continuity to discontinuity, which can be well solved by BD theory: by analysing the crack propagation during the material failure, it can provide a reference for material optimization.

2. PD theory

This paper takes the bond-based PD theory (Ha et al., 2015; Lee et al., 2016; Qian et al., 2017) as its theoretical basis, which actually belongs to the constitutive relation based on one material point within the material and another material point within the range R. This constitutive relation means the interaction force between the material points, to be shown with the constitutive force function f, and use the motion equation meeting the Newton’s second law in the integral form:

$$\rho\ddot{\mathbf{u}}(\mathbf{x}_i,t)=\int_R \mathbf{f}(\mathbf{u}(x_j,t)-\mathbf{u}(x_i,t),x_j-x_i)dV_j+b(x_i,t)\quad \forall j\in R \tag{1}$$

In Formula, ρ stands for the density of material point *i*, x_i and x_j for location of *i* and *j*, $u(x_i, t)$ and $u(x_j, t)$ for the displacement of material points *i* and *j*; $u(x_i, t)$ means the secondary derivation of $u(x_j, t)$ for time, i.e.

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acceleration speed, *b(x_i, t)*means the external density on material point *i*, R the near field and the radium in this field is denoted with *δ* shown in formula(2) and Figure 1; the bold-type letter means vector. Use *ξ* and *η* as the relative location and relative displacement, with the relation in Formula (3) and Figure 2.

$$R = R(\mathbf{x}_i, \delta) = \{\mathbf{x}_j \in \mathrm{R} : \left|\mathbf{x}_j - \mathbf{x}_i\right| \le \delta\} \tag{2}$$

$$\boldsymbol{\xi} = \mathbf{x}_j - \mathbf{x}_i \qquad \boldsymbol{\eta} = \mathbf{u}(\mathbf{x}_j, \mathrm{t}) - \mathbf{u}(\mathbf{x}_i, \mathrm{t}) \tag{3}$$

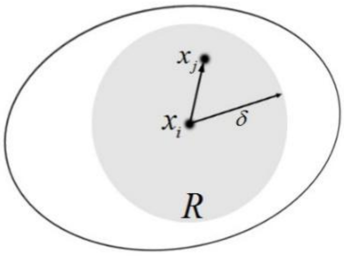


Figure 1: Horizon

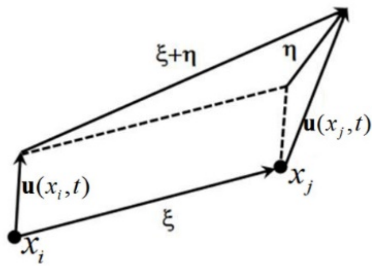


Figure 2: Relative position and displacement

The bond-based BD theory is used for PMB failure. In this theory, the interrelation between the different material points is called as Bond with the poisson ratio of the material 1/4, but in literature (Silling et al., 2007), the poisson ratio had no great impact on the results when changed to other values. The constitutive force function for the PMB model in the bond-based PD theory is denoted as (Silling, 2007; Silling and Askari, 2005):

$$\mathbf{f} = cs\mu(\boldsymbol{\xi})\frac{\boldsymbol{\eta}+\boldsymbol{\xi}}{|\boldsymbol{\eta}+\boldsymbol{\xi}|} \tag{4}$$

Where *c* is micro modular function about |*ξ*|, *δ*, and can be written as c(|*ξ*|, *δ*)

$$c(|\boldsymbol{\xi}|, \delta) = \begin{cases} \dfrac{6E}{\pi\delta^4(1-2\nu)} & |\xi| \le \delta \\ 0 & others \end{cases} \tag{5}$$

v means poisson ratio, and *s* the elongation of bond

$$s = \frac{|\boldsymbol{\eta}+\boldsymbol{\xi}| - |\boldsymbol{\xi}|}{|\boldsymbol{\xi}|} \tag{6}$$

μ(ξ) indicated if crack or not, with 1 (normal) or 0 (fracture)

$$\mu(\boldsymbol{\xi}) = \begin{cases} 1, & s < s_0 \\ 0, & other \end{cases} \tag{7}$$

In formula, s_0 means the critical elongation, and according to literature (Silling et al., 2007) can be expressed as

$$s_0 = \sqrt{\frac{10G_0}{\pi c \delta^5}} \tag{8}$$

Where G_0 means the fracture energy of the material.

3. Calculation method

Figure 1 indicates that the near-field of one certain point in the material is made of countless material points with certain property information (material property parameters, displacement and acceleration etc.). For convenience of calculation, disperse the material into small cubic space lattices with limited number, one size and simple arrangement shown in Figure 3, which are called as point-cell (its length of side is Δx), constituting the structure of analysis object. The dispersion processing method doesn't need to separate such complicated mesh as infinite element method, and have the advantages of the meshless method (Bobaru and Zhang, 2015).

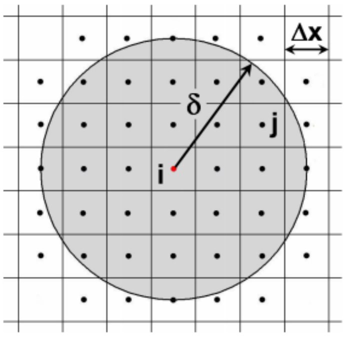


Figure 3: Discretized

According to the motion equation of PD theory, the integral computation can be denoted in the summation form, then the motion equation is simplified into summation form

$$\rho \ddot{\mathbf{u}}_i^n = \sum_{j=1}^{M} \mathbf{f}(\mathbf{u}_j^n - \mathbf{u}_i^n, \mathbf{x}_j - \mathbf{x}_i) V_j + \mathbf{b}(\mathbf{x}_i, t^n) \tag{9}$$

In formula 9, x_j is the volume of point cell in the near field. Considering the shape variance of point cells in the near field and easy computation, the volume of point cell j can be shown as

$$V_j = \begin{cases} (\Delta x)^3 & |\xi| \le \delta\text{-}0.5\Delta x \\ \dfrac{\delta + 0.5\Delta x - |\xi|}{\Delta x}(\Delta x)^3 & \delta\text{-}0.5\Delta x < |\xi| \le \delta + 0.5\Delta x \\ 0 & other \end{cases} \tag{10}$$

In formula 9, t stands for time, n for time steps, and Δt for step length of time. The time step length shall directly influence the convergence of the calculation results, therefore, to ensure the result convergence, according to literation (Yu et al., 2010):

$$\Delta t \le \beta_{safe} \frac{(|\xi|)_{\min}}{(c_k)_{\max}} \tag{11}$$

Where c_k represents the maximum wave velocity, $c_k = \sqrt{k/\rho}$, β_{safe} is safety factor, and take 0.8 generally.

4. Case study of brittle material failure

In this case, take the 50×50×3 (mm) glass sheet as analytic object with density of glass material 2440kg/m^3, elasticity modulus 72GPa, poisson ratio 0.22, fracture energy 135J/m^2. With the central surface area of the sheet glass applied with 5000N impact force by the rigid ball R=2mm, suppose that the structure was made of isotropous PMB materials, disperse the sheet glass into Δx=0.5 mm long point-cell; the radium in near-field δ is 1.5mm and total of point cells is 71407. To ensure the convergence, take the step of time 5×10^{-8}s.

According to the PD theory, the author firstly configures the boundary conditions, and then solves the displacement, speed, damage and other factors of each point unit. The calculation results are exported through C language programming. The flow chart of C language programming is show in Figure 4.

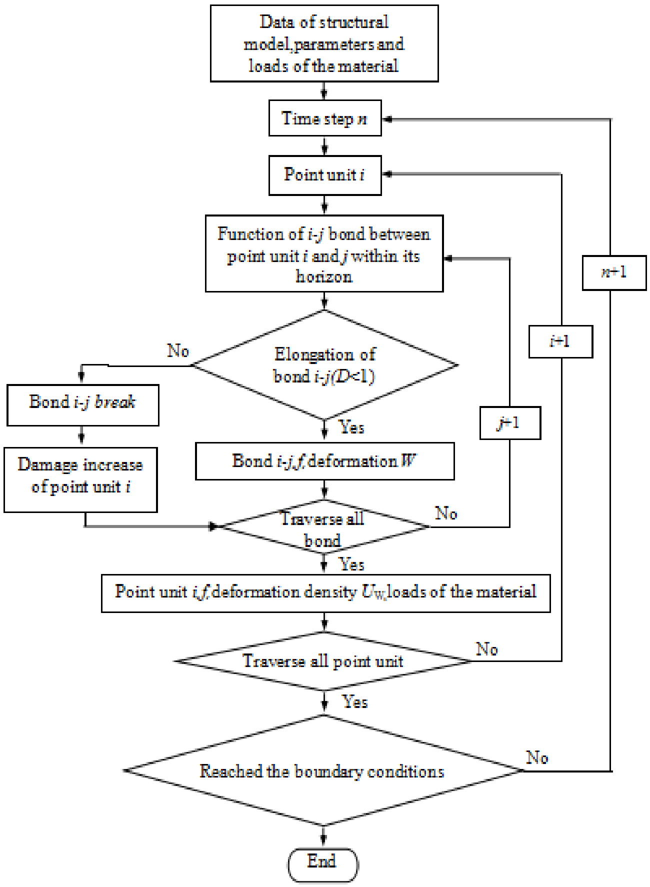


Figure 4: Flowchart of C language programming

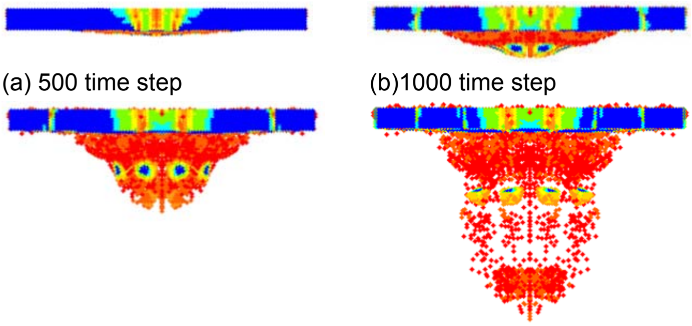
After BD theory modelling (Ansari et al., 2016) and numeral simulation by C language programming, the failure state of sheet glass under impact load was concluded as shown in Figure 5 and 6.

Figure 5(a)(b)(c)(d) shows the front view at the time step 500, 1000, 1500 and 2000 respectively when the sheet glass was shocked by rigid ball. In Figure 5(a), after 500-time step, mainly the area near the impact central point in the glass material has been damaged, but not serious; in figure 5(b), the impacted area got dispersed, making material heaved; in figure 5(c) and 5(d), the damaged area was gradually propagated by the impact force; when the rigid ball penetrated the sheet glass, the material started to fall off, basically consistent with the actual process.

Figure 6(a)(b)(c)(d) shows the top view at the time step 500, 1000, 1500 and 2000 respectively when the sheet glass was shocked by rigid ball. In figure 6(a), under impact, there sprouted the internal crack on the sheet glass, and the cracks got spread around; in figure 6(b), with time increasing, the impact force was slowly spread around the impact Centre, similar to wave transmission, expanding the crack area. In Figure 6(c)(d), the crack propagation tended to be stable, no more extending, and the work on the material by the impact force was balanced through the release of the propagation energy, i.e. complete the energy conversion, and the material is damaged.

By analysis of the failure dynamic process of impact load on sheet glass, the process when the rigid ball penetrated the glass material under impact load, and the surface crack propagation of the sheet glass in Figure 5 and 6, it is known to us that the area around the impact centre is the dangerous area for the material under impact load, therefore, to improve the material reliability, it is feasible to make optimization for the impact area.

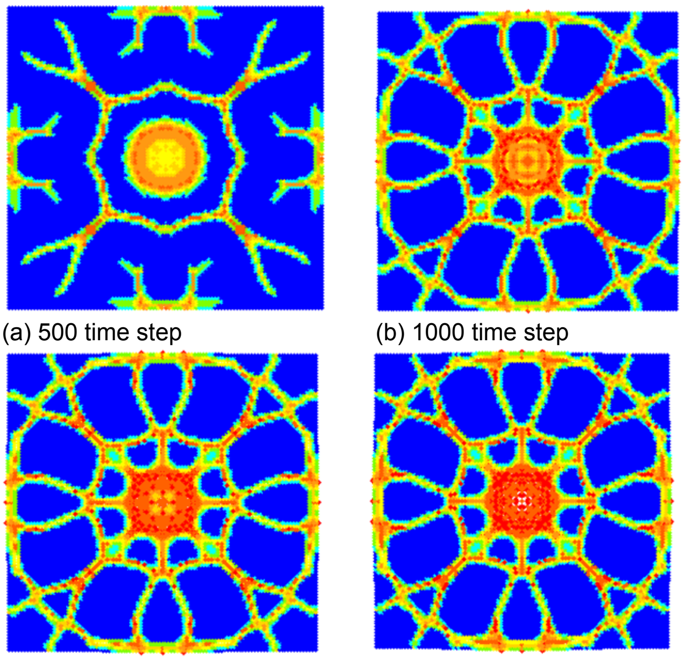
According to the two cases analysis, the results basically coincide with actual situation, also proving that the PD theory can meet the demands of material failure analysis, and providing the reference for material optimization.



(a) 500 time step (b)1000 time step

(c) 1500 time step (d) 2000 time step

Figure 5: lass flakes impact process (front view)



(a) 500 time step (b) 1000 time step

(c) 1500 time step (d) 2000 time step

Figure 6: Glass flakes crack (top view)

5. Conclusion

Based on the brittle material failure study applying the basic motion equation, constitutive force function and calculation method of BD theory, this paper makes numeral simulation and analysis of the failure process of two typical brittle materials, proving that the BD theory meets the demand of failure analysis and be superior to the traditional failure mechanics theory in terms of theoretical framework and algorithm; by further study, BD theory can replace the traditional failure mechanics theory.

In the case study, it is also found that there exist some defects about the bond-based BD theory mainly shown in the following: firstly, Lack of description of crack propagation mechanical response in the material failure process, similar to the intensity factor of crack propagation mechanics response parameter in fracture mechanics, J integral etc.; secondly, the BD algorithm efficiency and accuracy is contradictory relation, needing further optimization for the algorithm; thirdly, there is no description in constitutive relation about the traditional mechanics theoretical parameters such as stress-strain, and no inheritance from the research outcomes of traditional mechanics.

Acknowledgments

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