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# Cooperative Game Based Cost Analysis of Multi-Plant Heat Exchanger Network Integration in Chemical Industry Park

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Industrial energy saving is essential to the plant in the chemical industry park. Heat exchanger network of multi-plant can recover more potential energy. To ensure the network construction successfully, co-construct cost must be allocated reasonably. Cooperative game-theory based approach combines "Core" and "Risk-based Shapley value" with the network structure is put forward to give a contribute-equal cost imputation. A case study which includes three chemical plants is used to describe the approach and shows its practice value. The case study used Risk-based Shapley value illustrated plant which is confronted with high risk-loss will participate in the co-construction with lower cost, which means high risk and high yielded side by side.

# 1. Introduction

Industrial energy saving is aimed at achieving the operation of industrial production while minimizing energy use. With the development of industrial manufacturing technology, enterprises are gradually being located in the form of chemical industry park (CIP). As a result, factories in the CIP are becoming concentrated. Heat Exchanger Networks (HEN) synthesis, as one kind of effective energy recovery methods plays a very positive role in reducing energy use and increasing energy efficiency. Many papers related to synthesis of HEN have been published for a single plant (Escobar and Trierweiler, 2013), while only a minority of them concerns about integration of HEN for multi-plant in CIPs. Chen et al. (2016) was concerned with the energy hub approach of multi-plant heat integration, Cheng et al. (2014) use the Nash-equilibrium to solve multi-plant energy trading, and Tan et al. (2016) use cooperative game to solve multi-plant cost allocation problem. The high integration of factories in CIPs offers the opportunity for multi-plant to exchange heat, which means that more potential energy could be recycled, thus reducing the utility consumption. If HEN integration of multiplant is achieved, it could be a feasible way of energy saving in CIPs. However, the dilemma of multi-plant HEN integration is reflected not only in the optimization of pipeline network, but also in the allocation of coconstruction cost (Hiete et al., 2012). Considering the present complex and changeful economic situation, the analytical approach which combines game theory and the risk could provide a more objective and effective way to allocate the cost of this co-construction.

In this paper, an approach to allocating the co-construction cost of a HEN by combining "Core" and "Riskbased Shapley Value" is put forward. And a case study which includes three plants is used to introduce the approach.

# 2. Allocation based on cooperative game theory

# 2.1 Core – set of allocate solution

In cooperative game theory, the core is the set of feasible allocations that cannot be improved upon by a subset (a coalition) of the economy's consumers. It provides a reasonable way for all participants that are included in big coalition to gain their own part from the big coalition payoff.

Supposed that a set  $N = \{1, 2, 3, ..., n\}$  (of n players) and a function  $v(\cdot)$  that maps subsets of players to the real number:  $v: 2^N \to \mathbb{R}$  with  $v(\emptyset) = 0$ , while  $\emptyset$  denotes the empty set. The function  $v(\cdot)$  is called a characteristic function. If  $S \subseteq N$  is a coalition of players, then v(S) called the worth of coalition *S*, describing the total expected

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sum of payoffs that the members of *S* can obtain by cooperation. Defined  $x_{S,i}$  as the payoff of player *i* in coalition *S*,  $\sum_{i \in S} x_{S,i} = v(S)$ , then vector  $X_s = (x_{S,1}, x_{S,2}, ..., x_{S,n})$  is called a pre-imputation of coalition *S*.

Core C(v) means the payoff created by big coalition *N* have an imputation that meets the condition by Individual Rationality, Group Rationality, Coalitional Rationality and No Subsidy Principle.

Individual Rationality: pre-imputation vector  $X_N$  satisfies  $\forall i \in N, x_{N,i} \ge v(\{i\})$ .

Group Rationality: pre-imputation vector  $X_N$  satisfies  $\sum_{i \in N} x_{N,i} = v(N)$ .

Coalitional Rationality: pre-imputation vector  $X_N$  for any  $S \subseteq N$  satisfies  $\sum_{i \in S} x_{N,i} \ge v(S)$ .

No Subsidy Principle: pre-imputation vector  $X_N$  satisfies  $\forall i \in N, x_{N,i} \leq v(N) - v(N \setminus i)$ .

The  $v(N \setminus i)$  indicates the payoff created by coalition  $N \setminus i$ , that in contributed by all players in coalition N except the player *i*. No Subsidy Principle inequality shows that the payoff that the player *i* could gain must be lower than his marginal contribution.

Bondareva-Shapley theorem demonstrates that if the cooperative game is a balanced game, the Core is nonempty. Generally, in heat exchanger co-construction problem, the game structure is super-additive, balanced, and even convex. Note that, when the problem is concerned about cost allocation, the inequality sign of Individual Rationality, Coalitional Rationality, and No Subsidy Principle must be reversed.

#### 2.2 Shapley Value

In cooperative game theory, the Shapley value provides a measure of the "importance" of each player. If it is a cost problem, it can be used as a cost allocation algorithm. Shapley Value also supposed that big coalition N can be formed. When calculating the allocation of worth created by big coalition, the payoff of all other subcoalition S formed by players have been concerned, so that final imputation shows equity. Each player's allocation from the big coalition depends on player i's contribution for all potential sub-coalition which player i could participate. The difference between the Shapley Value and the Core is that the Shapley Value is a "onepoint solution", and if the Core is non-empty, it usually does be a high-dimensional feasible domain. While the Core is non-empty and Shapley Value is in the Core, then the Shapley is located in the barycentre of the Core. Usually, the Shapley Value is defined as every player's the average marginal contribution to all possible subcoalition. To a super-additive characteristic function $v(\cdot)$ , the Shapley value  $\Phi_i$  can be calculated in Eq(1)~(3):

$$\omega(S) = (|S|-1)!(n-|S|)!/n!$$
(1)

$$M(S) = v(S) - v(S \setminus i)$$
<sup>(2)</sup>

$$\Phi_{i} = \sum_{S \subseteq N, i \in S} \omega(S) M(S)$$
(3)

|S| is the number of players in coalition  $S, \omega(S)$  is the weight coefficient of coalition S in all coalition. M(S) is the marginal contribution of player i in coalition S.

To most cooperative game problems, the Shapley Value can be obtained by Eq(1)~(3). But for some problems, if a player exits from a coalition, it will bring extra loss to other players. Such as a three-plant (P1, P2 and P3) HEN synthesis problem,  $v(\{1,2\})$  is the payoff of a coalition between P1 and P2. If the HEN structure is built on three-plant cooperation, for some reasons P3 exits from the coalition (suddenly shutdowns),  $v(\{1,2,3\}\setminus\{3\})$  and  $v(\{1,2\})$  are different conception. For production continuity, P1 and P2 would cost more utility to compensate the exit of P3. To this kind of game problem, a risk-based Shapley Value is introduced. To analysis this situation, average marginal contribution of player should be further discussed (Grabisch and Xie, 2007).

#### 2.3 Average contribution of Shapley Value

Given player set *N*, there are totally *n*! different sequences to order the players. Record each sequence as  $\pi_{\sigma}$ , and record all sequences set as  $\pi(N), \pi_{\sigma} \in \pi(N)$ . Select one sequence from  $\pi(N)$  randomly, it may record as  $\pi_{\sigma} = (\pi_{\sigma}(1), \pi_{\sigma}(2), \dots, \pi_{\sigma}(n))$ , then the marginal contribution vector  $\mathbf{m}^{\sigma}(v)$  for this sequence  $\pi_{\sigma}$  is calculated as follows. This is  $\mathbf{m}^{\sigma}(v) = \{m_{\pi_{\sigma}(1)}^{\sigma}(v), m_{\pi_{\sigma}(2)}^{\sigma}(v), \dots, m_{\pi_{\sigma}(k)}^{\sigma}(v)\}$ , where  $m_{\pi_{\sigma}(1)}^{\sigma}(v) = v(\{\pi_{\sigma}(1)\})$ , and other elements in  $\mathbf{m}^{\sigma}(v)$  is calculated by Eq(4).

$$m_{\pi_{\sigma}(k)}^{\sigma}(v) = v\left(\left\{\pi_{\sigma}(1), \cdots, \pi_{\sigma}(k)\right\}\right) - v\left(\left\{\pi_{\sigma}(1), \cdots, \pi_{\sigma}(k-1)\right\}\right)$$

$$\tag{4}$$

Next, sort the sequence  $\mathbf{m}^{\sigma}(v)$  into nature order by player *i* which was originally formed in  $\pi_{\sigma}$ , and then the marginal contribution sequence  $\mathbf{m}^{\sigma}(v)$  can be sorted in a {1,2,3,...,n} positive order, and obtain sequence  $\mathbf{O}^{\sigma}(v)$ . e.g. When  $\pi_{\sigma} = \{3,1,4,2\}$  and  $\mathbf{m}^{\sigma}(v) = \{10,65,24,9\}$ , then  $\mathbf{O}^{\sigma}(v)$  should be  $\mathbf{O}^{\sigma}(v) = \{65,9,10,24\}$ .

After calculating all sequence  $\mathbf{m}^{\sigma}(v)$  in  $\pi(N)$ , and reordering into  $\mathbf{O}^{\sigma}(v)$ , the average marginal contribution can be calculated as Eq(5).

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$$\mathbf{\Phi}(v) = \frac{1}{n!} \sum_{\pi_{\sigma} \in \pi(N)} \mathbf{O}^{\sigma}(v)$$
(5)

This is the solution of Shapley Value Vector  $\Phi(v) = (\Phi_1, \Phi_2, \dots, \Phi_i, \dots, \Phi_n)$ , and the Shapley Value for each player *i* is  $\Phi_i$ , which describe the payoff for player *i*. In cost allocate problem, This refers to expenses. This result is equivalent to  $\Phi_i$  calculated by Eq(1)~(3), but it implies more concept details of "contribution-equality". However, it neglects the potential risks that the players bring to each other and encounter.

#### 2.4 Risk loss – Risk based Shapley Value

In actual cooperation, there may be the risk that participants drop out coalition halfway. The risk will cause certain results as follows: the sudden drop-out of some players in coalition brings "risk-loss" to other members of original coalition.

In reality, when coalition *S* has already been formed, if a player  $s \in S$  drop out of the coalition for some reason, then the worth of coalition  $\{S \setminus s\}$  will not reach the value of character function  $v(\{S \setminus s\})$ , this may lead to a situation that the coalition gain loss or cost more. Generally, if this circumstance takes place in probability, the coalition which lose some players would make other members suffer losses, so this is called "risk-loss".

Define i: S as the circumstances that the players  $\{S \setminus i\}$  in original coalition S all drop out except player i, which would make player i had to work alone. Also define w(i:S) as the payoff of player i in this situation. For each player in set  $N = \{1, 2, 3, ..., n\}$ ,  $\alpha_i \in [0, 1]$  is used to describe the risk possibility for player i dropping out of any possible coalition  $\{i\} \cup S$ ,  $S \subseteq \{N \setminus i\}$ . No matter which sub-coalition  $\{i\} \cup S$  from N, if player i takes part in it, he will drop out of this sub-coalition  $\{i\} \cup S$  at probability  $\alpha_i$ .

The probability  $\alpha_i$  has no preference, for any coalition which contain player *i*, thus the drop out probability is equal. In other words, this probability is not to depict the subjective intention of player *i*, but to describe the drop out probability from an objective view. In practice, the drop out are caused by Factory closures, poor management, capital chain broken and so on. Probability  $\alpha_i$  could be obtained by statistics similar factory annual collapse rate or calculate by professional risk assessment agency.

Given  $\forall S \subseteq \{N \setminus i\}$ , and supposed that the coalition  $\{i\} \cup S$  has been formed. If player *i* can take utopia payoff of marginal contribution, his payoff will be  $v(\{i\} \cup S) - v(S)$ . However, he is also taking the corresponding risks. Considering its most extreme situation, if for some reasons all other players drop out of the *S*, then finally the probability that player *i* works alone will be  $\prod_{j \in S} \alpha_j$ . Comparing this loss with the case of working alone, then the loss can be recorded in  $w(\{i\} \cup S)) - v(\{i\})$ . Considered the probability of this situation is  $\prod_{j \in S} \alpha_j$ , so that the expectation of this loss is calculated by Eq(6), and this means the "risk-loss" for player *i* in  $\{i\} \cup S, S \subseteq \{N \setminus i\}$ .

$$\left(\prod_{j\in S}\alpha_{j}\right)\cdot\left(w\left(\{i\}:\left(\{i\}\cup S\right)\right)-v\left(\{i\}\right)\right)$$
(6)

#### 2.5 Risk-based Shapley Value calculate

In order to consider the risk of Shapley value, the risk-loss can be combined with sorted marginal contribution in Eq(4). Obviously, player  $\pi_{\sigma}(k)$  obtains his payoff of marginal contribution with "marginal risk" from other members, and he also bring "risk" to others. If the worst risk i:S had happened, his payoff would become w(i:S), and his expectation of risk loss would be $w(\{i\}:(\{i\}\cup S))-v(\{i\})$ , so his Pure Expectation Payoff would become Eq(7).

$$\underbrace{\left(\nu(\{i\}\cup S)-\nu(S)\right)}_{\textcircled{0}} - \underbrace{\left(\prod_{j\in S}\alpha_{j}\right)}_{\textcircled{0}} \cdot \underbrace{\left(w(\{i\}:(\{i\}\cup S)\right)-\nu(\{i\})\right)}_{\textcircled{0}}$$
(7)

In this formula, ① refers to payoff of marginal contribution, ② refers to risk probability,③ refers to risk-loss. Next, combine the Eq(7) with the calculation of average marginal contribution Eq(4). In  $\sigma$  th sequence  $\pi_{\sigma}$ , pure expectation payoff vector of  $\pi_{\sigma}$  recorded as  $\mathbf{h}^{\sigma}(v) = \{h_{\pi_{\sigma}(1)}^{\sigma}(v), h_{\pi_{\sigma}(2)}^{\sigma}(v), \cdots, h_{\pi_{\sigma}(k)}^{\sigma}(v), \cdots, h_{\pi_{\sigma}(n)}^{\sigma}(v)\}$ , where  $h_{\pi_{\sigma}(1)}^{\sigma}(v) = v(\{\pi_{\sigma}(1)\})$  and other elements follow Eq(8).

$$h_{\pi_{\sigma}(k)}^{\sigma}(v) = v\left(\left\{\pi_{\sigma}(1), \dots, \pi_{\sigma}(k)\right\}\right) - v\left(\left\{\pi_{\sigma}(1), \dots, \pi_{\sigma}(k-1)\right\}\right) \\ - \left(\prod_{k=1}^{k-1} \alpha_{\pi_{\sigma}(k)}\right) \cdot \left(w\left(\left\{\pi_{\sigma}(k)\right\}: \left\{\pi_{\sigma}(1), \dots, \pi_{\sigma}(k)\right\}\right) - v\left(\left\{\pi_{\sigma}(k)\right\}\right)\right)$$
(8)

Next, sort the sequence  $\mathbf{h}^{\sigma}(v)$  into nature order by player *i* which was originally formed in  $\pi_{\sigma}$ , then the marginal contribution sequence  $\mathbf{h}^{\sigma}(v)$  can be sorted in a  $\{1, 2, 3, ..., n\}$  nature positive order, and obtain sequence  $\Theta^{\sigma}(v)$ . After calculating all sequences  $\mathbf{h}^{\sigma}(v)$  in  $\pi(N)$ , and reordering into  $\Theta^{\sigma}(v)$ , the average risk marginal contribution coefficient vector can be calculated in Eq(9).

$$\Psi(v) = \frac{1}{n!} \sum_{\pi_{\sigma} \in \pi(N)} \Theta^{\sigma}(v)$$
(9)

However, this solution doesn't meet the condition of Group Rationality, so the imputation of big coalition should be calculated by Eq(10). Based on the vectors calculated by Eq(9), the vectors are then be normalized and serve as coefficients of payoff of big coalition.

$$\Phi'(v) = \frac{\Psi(v)}{\Psi(v) \cdot \mathbf{1}_{n \times 1}} \cdot v(N)$$
(10)

This is the solution of Risk-based Shapley Value Vector  $\Phi'(v) = (\Phi'_1, \Phi'_2, \dots, \Phi'_n, \dots, \Phi'_n)$ , and the Risk base Shapley Value for each player *i* is  $\Phi'_i$ . Vector  $\mathbf{1}_{n\times i}$  in the denominator means n row and one column vector filled with all elements "1", and v(N) donates the payoff of big coalition. It can also be proved that this solution is equivalent with Shapley when risk does not exist or "risk-loss" equals zero.

## 3. Heat Exchanger Network synthesis

Heat exchanger network synthesis (HENS) has a far-reaching practical significance in the chemical engineering industry. In this paper, the HENs are optimized by applying the mathematical programming method proposed by Floudas et al. (1986). Calculating the Risk-based Shapley proposed in this paper needs synthesis all possible plant coalition. When taking the time-consuming algorithm to synthesis HEN, the solution of this complex problem will no longer be time-efficient.

Note that: in multi-plant HEN synthesis problem, it is tolerable to take any existing optimization algorithm. However, in case of synthesis of different multi-plant HEN coalition, the same kind of optimization algorithm must be used, so as to guarantee the fairness of solutions.

# 4. Simulation example for Chemical Industrial Park multi-plant heat exchanger synthesis

In order to introduce the process clearly, a case study which includes 3 chemical plants cited from Cheng et al. (2014) is used to explain the analysis. Table 1 shows the stream data. Utility stream could choose cold water (CW), Fuel and High-pressure steam (HP). Their inlet/outlet temperatures are 25 °C / 26 °C, 500 °C / 499 °C and 200 °C/199 °C separately. Their costs are 6.7(\$/kWy), 110(\$/kWy) and 79(\$/kWy) separately. Each heat exchanger cost calculates as  $10,000 + 670 \times \text{Area}^{0.83}$  (\$), Area calculate by Q/(U·LMDT), and the LMDT described by Chen (1987). Coefficient U equals 1 kW/(m<sup>2</sup>·°C). The drop out probability is  $\alpha_1 = \alpha_2 = \alpha_3 = \alpha = 0.01$ . Total cost calculates as: Utility cost + Heat exchanger cost. The rate of interest equals 0%, and all plants lifetime are 10 years.



Figure 1: Optimal structure of three-plant HEN synthesis

Optimization of multi-plant HEN synthesis is solved by GAMS software under the operation system of Win 7 Ultimate, and Figure 1 is the optimal structure of the example, and Table 2 show all possible coalition co-construction cost which is calculated considering the whole Multi-plant HEN lifetime in 10 y. From the data in Table 2, the three-plant HEN co-construct is the best choice. The advantages are that it recycles potential heat and reduces the use of utility as much as possible. Meanwhile, it satisfies  $v({1,2,3}) \le v({1})+v({2})+v({3})$ 

and  $v(\{1,2,3\}) \leq v(\{1\}) + v(\{2,3\})$ , which indicates that it is the cheapest way to exchange heat cooperatively with three-plant. In order to ensure the network construction successfully, co-construct cost must be allocated reasonably. Considering each plant has the risk to drop out of the multi-plant coalition. Here the risk-payoff is calculated by illustrating P1. Supposing that P1 and P2 have already been a coalition and then P2 quit halfway for some reason. To continue producing, the remedy for P1 is to use the utility in exchangers that are originally connected with P2, which is illustrated by the dashed line in Figure 2a. In this situation, the expense of P2 can be obtained by the following 3 principles: First, the utility cost is charged by P1. Second, the cost of heat exchanger that connected with P2 is half charged by P1. Third, the cost of the inner heat exchanger in P1 is charged totally by P1 itself. Using the calculation of this concept, risk-payoff of other plants in its possible subcoalition can also be obtained. The calculation results of all plants are shown in Table 3. With the results in Table 3 and Shapley Value in Table 4, Risk-based Shapley Value can be computed by Eq(7) to Eq(10). By comparing this with Shapley Value, it could be found that P2 bears less cost while P1 & P3 suffer more. The reason is that P2 is confronted with the largest risk-payoff relative to work separately from Table 3. In this three-plant case, it is not friendly to P2. As P1 or P2 or P3 they all know that co-construction is the better choice, with risk analysis, P2 can threaten P1 & P3 by work alone, unless P1 & P3 transfer of profits to avoid the collapse of the coalition. Finally, under the situation of risk probabilities equal to 0.01, the imputation of each plant is that: P1= 542,670.40, P2= 48,649.71, P3= 276,361.9. And this imputation is in the Core.

If the cost imputation is further considered with other risk probabilities, then the risk for all plant in this case is equal to  $\alpha$ . With the increase of  $\alpha$ , the imputation point of Risk-based Shapley Value is shown in Figure 2b. Figure 2b is a regular triangular representation. In a regular triangle, the sum of the distances from a point to its three edges is a constant. For this reason, it is suitable to illustrate this cost allocation problem because the total sum cost for the three-plant HEN construction is equal to 867,682. Note that the distance from the apex to its opposite base is also equal to this fixed value (867,682).

| Plant & | T <sub>in</sub> | Tout | Fcp     | Plant & | T <sub>in</sub> | Tout | Fcp     | Plant & | T <sub>in</sub> | Tout | Fcp     |
|---------|-----------------|------|---------|---------|-----------------|------|---------|---------|-----------------|------|---------|
| Stream  | (°C)            | (°C) | (kW/°C) | Stream  | (°C)            | (°C) | (kW/°C) | Stream  | (°C)            | (°C) | (kW/°C) |
| P1.H1   | 150             | 40   | 7.0     | P2.H1   | 200             | 70   | 5.5     | P3.H1   | 370             | 150  | 3.0     |
| P1.C1   | 60              | 140  | 9.0     | P2.C1   | 30              | 110  | 3.5     | P3.H2   | 200             | 40   | 5.5     |
| P1.C2   | 110             | 190  | 8.0     | P2.C2   | 140             | 190  | 7.5     | P3.C1   | 110             | 360  | 4.5     |

Table 1: Datasheet of three-plant process stream

280,500

280,500

280,500

371,300

319,950

0

{1,3}

{2,3}

{1,2,3}

| Coalition | HP (\$) | Fuel (\$) | CW (\$) | Fix cost (\$) | Area cost (\$) | Total cost (\$)       |
|-----------|---------|-----------|---------|---------------|----------------|-----------------------|
| {1}       | 632,000 | 0         | 14,070  | 40,000        | 24,434.02      | v({1}) = 710,504.02   |
| {2}       | 79,000  | 0         | 10,720  | 40,000        | 14,594.40      | v({2}) = 144,314.40   |
| {3}       | 0       | 280,500   | 44,890  | 50,000        | 20,801.35      | v({3}) = 396,191.35   |
| {1,2}     | 501,650 | 0         | 7,035   | 100,000       | 57,218.34      | v({1,2}) = 665,903.34 |

Table 2: Co-construction cost of HEN under different multi-plant coalition between three plants

| Table 3: Risk-payoff of eac | h plant (plaver | ) when it takes | part in all | possible sub-coalition |
|-----------------------------|-----------------|-----------------|-------------|------------------------|

36,850

48,910

36,515

| Plant                   | P1({1})                    | P2({2})                    | P3({3})                    |
|-------------------------|----------------------------|----------------------------|----------------------------|
| Characteristic function | v({1}) = 710,504.0         | v({2}) = 144,314.4         | v({3}) = 396,191.4         |
| Risk-payoff             | w({1}:{1,2}) = 897,526.3   | w({2}:{1,2}) = 621,092.2   | w({3}:{1,3}) = 414,631.2   |
| Risk-payoff             | w({1}:{1,3}) = 735,222.8   | w({2}:{2,3}) = 375,818.4   | w({3}:{2,3}) = 703,334.0   |
| Risk-payoff             | w({1}:{1,2,3}) = 863,798.5 | w({2}:{1,2,3}) = 614,878.2 | w({3}:{1,2,3}) = 415,130.7 |

120,000

80,000

150,000

58,394.12

39,847.02

80,717.02

 $v(\{1,3\}) = 867,044.12$ 

 $v({2,3}) = 449,257.02$ 

v({1,2,3}) = 867,682.02

| Table 4: Imputation of | each plant (player) | by Shapley Value | e and Risk-based | Shapley Va | alue |
|------------------------|---------------------|------------------|------------------|------------|------|
|------------------------|---------------------|------------------|------------------|------------|------|

| Plant                            | P1({1})    | P2({2})   | P3({3})    |
|----------------------------------|------------|-----------|------------|
| Shapley Value                    | 541,716.63 | 49,728.27 | 276,237.13 |
| Risk-based Shapley Value(α=0.01) | 542,670.40 | 48,649.71 | 276,361.90 |

The imputation in Figure 2b can be illustrated by point P, where the distance from point P to each edge represents the imputation of each plant. The boundary line of the Core can be drawn by the four principles of Core. Take P1 as an example, the imputation of P1 from three-plant coalition should not be higher than the

case when P1 work alone which is v{1}, and not be lower than P1's marginal contribution which is v{1,2,3}v{2,3}. The thick closed curve is the boundary of the Core in this case. The inner area of the Core is a feasible imputation. The thick Curve with "\*" is the Risk-based Shapley Value Curve. The Curve starts at Shapley Value point and ends outside the Core. The point "\*" refers to the Risk-based Shapley Value for each additional risk of 20 %. When  $\alpha$  = 0.2982, the Risk-based Shapley Value is just in the boundary of the Core, which implies that P2 can allocate the co-construction cost with minimum expense when the risk is up to 0.2982. If the risk is higher than this value, then the cooperation will collapse.



Figure 2: (a) The optimal result of P1&P2 HENS and (b) Multi-Plant Risk-based Shapley Value Curve

## 5. Conclusion

In this paper, the economic feasibility of the multi-plant HEN synthesis in CIP has been discussed. With a full combination of the cooperative game and risk analysis, a method was proposed to calculate co-construction cost allocation when risk exists. Simulation results revealed that this method considers the actual status of every plant and the effect brought by risk. This method can give a relatively fair co-construction cost allocation with huge practical value, so that it can ensure the multi-plant cooperation and the environment-friendly heat exchanger network could be taken into practise.

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