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Novel Model for The Energy Minimisation of Natural Gas Transmission Networks

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Natural gas transmission through an extensive pipeline network is an energy consuming process required gas compression due to significant pressure drop. The main difficulties in optimising a natural gas transmission network are the nonconvexities of the pressure drop and the compressor constraints. To address the nonconvex issues, a novel model is proposed and demonstrated using an energy minimisation problem of natural gas transmission networks. The new model overcomes the nonconvexity of the pressure drop constraints by convexifying them into convex constraints, and addresses the nonconvex compressor constraints by rewriting them in to convex constraints and concave constraints. The model is formulated as a mixed-integer nonlinear programming (MINLP) model and tested by two different sizes networks. Computational results show the proposed model significantly reduces the solution time and improve solution quality.

1. Introduction

Natural gas is becoming one of the most important energy sources. It is estimated the global consumption of natural gas will increase by 69 % from 2012 to 2040 and will account for 24 % of the primal energy consumption (US Energy Information Administration, 2016). Much attention has been paid to natural gas production (Zhang et al., 2016), supply chain (Heckl et al., 2015), and power generation problems (Ali et al., 2016). To support the fast-growing natural gas industry, efficient and effective transmission networks are required to transport natural gas from the production region to the consumption region. In many cases, the transmission networks are extensive network systems which consist of pipelines, compression stations, and other components. Due to the long distance transportation and large gas volume, the transmission networks often consume large amount of energy for gas compression, which makes up the majority of the operating cost. It is estimated that a well-designed network reduces 20 % of the energy consumption for the compressor stations.(Schroeder, 1996) Therefore, it is economically attractive to optimise the transmission networks design to achieve higher energy efficiency.
However, after decades of construction and expansion, the transmission networks have developed into large and complicated systems, which lead to large model size. Moreover, the optimisation problem is exacerbated by the nonconvex pressure drop constraints of pipelines, which are the main elements of the networks. Combined with the nonconvex compressor constraints, the energy minimisation problem of natural gas transmission networks is regarded as a difficult problem featuring large model size and strong nonconvexity. To overcome these difficulties, in this paper, we propose a novel model for the energy minimisation problem of natural gas transmission networks.

2. Mathematical model

We present an MINLP model to minimise the energy consumption of natural gas transmission networks based on the definition as follows. A natural gas transmission network consists of nodes and arcs. The nodes include: the sources supplying natural gas to the networks at certain pressure; the users with certain gas demand and pressure requirements; and the junctions that has no supply or demand. The arcs include pipelines and

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compressor stations. The length and diameters of the pipelines are fixed. The compressors are assumed to be centrifugal type. For simplicity, it is also assumed each compressor station has one compressor. We also assume the networks are in steady state with constant gas supply and demand at the nodes. The network layout is fixed, and the length and the diameters of the pipelines are known.

The main feature of our model is the convex pressure drop constraints Eq(5) and Eq(6), which avoid the nonconvexity resulting from the traditional nonconvex formulation (Eq(22). Also, the nonconvex compressor constraint Eq(23) is reformulated into convex constraints and concave constraints Eq(13)-Eq(15), which are more computationally efficient.

$$min \sum_{i,j \in A_a} w_{i,j} \tag{1}$$

s.t.

$$\sum_{j|(i,j) \in A} m_{i,j} - \sum_{j|(j,i) \in A} m_{j,i} = S_i \quad i \in I \tag{2}$$

$$m_{i,j} = m^{+}_{i,j} - m^{-}_{i,j} \quad i,j \in A_p \tag{3}$$

$$\pi_i - \pi_j = \Delta\pi^{+}_{i,j} - \Delta\pi^{-}_{i,j} \quad i,j \in A_p \tag{4}$$

$$\Delta\pi^{+}_{i,j} \geq K_{i,j}\left(m^{+}_{i,j}\right)^2 \quad i,j \in A_p \tag{5}$$

$$\Delta\pi^{-}_{i,j} \geq K_{i,j}\left(m^{-}_{i,j}\right)^2 \quad i,j \in A_p \tag{6}$$

$$\Delta\pi^{+}_{i,j} \leq \Delta\Pi^{up}_{i,j} x_{i,j} \quad i,j \in A_p \tag{7}$$

$$\Delta\pi^{-}_{i,j} \leq -\Delta\Pi^{lo}_{i,j}(1-x_{i,j}) \quad i,j \in A_p \tag{8}$$

$$m^{-}_{i,j} \leq |M^{lo}_{i,j}|(1-x_{i,j}) \quad i,j \in A_p \tag{9}$$

$$m^{+}_{i,j} \leq M^{up}_{i,j} x_{i,j} \quad i,j \in A_p \tag{10}$$

$$\Delta\pi^{+}_{i,j} \leq K_{i,j} M^{up}_{i,j} m^{+}_{i,j} \quad i,j \in A_p \tag{11}$$

$$\Delta\pi^{-}_{i,j} \leq K_{i,j} |M^{lo}_{i,j}| m^{-}_{i,j} \quad i,j \in A_p \tag{12}$$

$$w_{i,j} \geq C_{i,j}\left[ms_{i,j}^{1-\frac{n}{2}} cr_{i,j}^{\frac{n}{2}} - m_{i,j}\right] \quad i,j \in A_a \tag{13}$$

$$\sqrt{cr_{i,j}\pi_i} \geq \sqrt{\pi_j} \quad i,j \in A_a \tag{14}$$

$$ms_{i,j}^{1-\frac{n}{2}} \geq m_{i,j} \quad i,j \in A_a \tag{15}$$

$$\pi_i \leq \pi_j \quad i,j \in A_a \tag{16}$$

$$CR^{lo}_{i,j} y_{i,j} + 1 - y_{i,j} \leq cr_{i,j} \leq CR^{up}_{i,j} y_{i,j} + 1 - y_{i,j} \quad i,j \in A_a \tag{17}$$

$$M^{lo}_{i,j} \leq m_{i,j} \leq M^{up}_{i,j} \quad i,j \in A \tag{18}$$

$$\Pi^{lo}_i \leq \pi_i \leq \Pi^{up}_i \quad i \in I \tag{19}$$

$$W^{lo}_{i,j} y_{i,j} \leq w_{i,j} \leq W^{up}_{i,j} y_{i,j} \quad i,j \in A_a \tag{20}$$

The objective is to minimise the energy consumption of the compressors Eq(1). The model subjects to the mass balance constraints Eq(2)-Eq(3), the pressure drop constraints Eq(4)-Eq(12), the compressor constraints Eq(13)-Eq(17), and the bounds of the variables Eq(18)-Eq(20).
The index i,j are the nodes of the network. I is the set of the nodes, A_a is the set of the active arcs (compressors), A_p is the set of the passive arcs (pipelines), and A is the set of the arcs ($A_p \cup A_a$).
For the variables, $w_{i,j}$ denotes the energy consumption of compressor i,j. $m_{i,j}$ denotes the mass flow rate of pipeline i,j. We define the flow rate is positive if the gas flows from node i to j, and negative from j to i. π_i is the square of the pressure at node i (π=p^2). $\Delta\pi^+_{i,j}$ and $\Delta\pi^-_{i,j}$ are the disaggregated variables representing the difference of the pressure square (π_i-π_j) in positive and negative direction (see Figure 1). $x_{i,j}$ is a binary variable to denote the flow direction. We define the flow direction is positive when $x_{i,j}$=1, and negative when $x_{i,j}$=0. $cr_{i,j}$ represents the square of compression ratio, and $ms_{i,j}$ is an auxiliary variable of $m_{i,j}$. $y_{i,j}$ is a binary variable to denote the operation of the compressors. The authors sefined define the compressor is operating when $y_{i,j}$=1, and is bypassed when $y_{i,j}$=0.

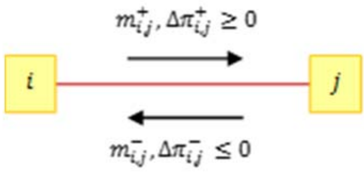

Figure 1: Flow direction of a pipeline.

In terms of the parameters, S_i denote the supply at node i. If S_i>0, the node is a gas source; If S_i<0, the node is a user; and if S_i=0, the node is a junction. $K_{i,j}$ the per length resistance of the pipeline calculated by:

$$K_{i,j}=\frac{K'L_{i,j}}{D^5_{i,j}}F_{i,j} \quad i,j \in A_p \tag{21}$$

where $F_{i,j}$ is friction coefficient, K' is a generic constant for fully turbulent gas flow (K'=1.234×$10^{-6}T_{avg}Z_{avg}$), $L_{i,j}$ is the length of the pipeline, and $D_{i,j}$ is the inner diameter of the pipeline.
Eq**(2)** indicates the difference of the outflow ($\sum_{j|(i,j)\in A} m_{i,j}$) and the inflow ($\sum_{j|(j,i)\in A} m_{j,i}$) of a node equals to the supply (S_i) of the node. Eq**(3)** and Eq**(4)** disaggregate the flow rate variables and square pressure difference variables. Eq**(5)** and Eq**(6)** define the relation of $\Delta\pi^+_{i,j}$ and $\Delta\pi^-_{i,j}$ with $m^+_{i,j}$ and $m^-_{i,j}$. Eq**(7)**-Eq**(10)** enforce $\Delta\pi^+_{i,j}$ and $m^+_{i,j}$ equal to 0 if the flow direction is negative, and $\Delta\pi^-_{i,j}$ and $m^-_{i,j}$ equal to 0 if the flow direction is positive. Eq**(10)** and Eq**(11)** are linear cuts to tighten the relaxation. Eq**(13)** calculates the energy consumption of the compressors. Eq**(14)** and Eq**(15)** define the variable $cr_{i,j}$. and $ms_{i,j}$, respectively. Eq**(16)** guarantees the discharged pressure of the compressors is no less than the suction pressure. Eq**(17)** ensures $cr_{i,j}$ is bounded when the compressor is operating, and force $cr_{i,j}$ equal to 1 when the compressor is bypassed.
Different from the typical pressure drop constraints written as equations (Eq.**(22)**), in the new model the pressure drop is formulated into convex inequality constraints (Eq**(5)** and Eq**(6)**), which avoids the nonconvexity resulted from nonlinear equations. Moreover, we replace the pressure variables p with new variables π to avoid the quadratic terms (i.e. p^2_i). The bilinear absolute value terms $m_{i,j}|m_{i,j}|$ are also eliminated by introducing binary variables $x_{i,j}$ to denote flow direction and disaggregated variables $\Delta\pi^+_{i,j}$, $\Delta\pi^-_{i,j}$, $m^+_{i,j}$, $m^-_{i,j}$.

$$p^2_i\text{-}p^2_j=K_{i,j}m_{i,j}|m_{i,j}| \quad i,j \in A_p \tag{22}$$

where p_i is the pressure at node i.
Similar to the pressure drop constraint, we rewrite the compressor constraint from equality constraints (Eq**(23)**) into inequality constraints (Eq**(13)**-Eq**(15)**) to avoid the nonconvexity. Also, new variables $cr_{i,j}$ and $ms_{i,j}$ are introduced to reformulate the nonconvex constraint functions $m_{i,j}(p_j/p_i)^n$ into concave functions $ms_{i,j}^{1-\frac{n}{2}}cr_{i,j}^{\frac{n}{2}}$.

$$w_{i,j}=C_{i,j}m_{i,j}\left[\left(\frac{p_j}{p_i}\right)^n\text{-}1\right] \quad i,j \in A_a \tag{23}$$

3. Examples

The proposed model is tested and compared with the nonconvex model by 2 transmission networks adopted from (Wu et al., 2000). The models are formulated on GAMS 24.5.3 (Brooke et al.,) on an Intel ® I5-2400 CPU @3.10 GHz, 8 GB RAM PC. The models are tested by deterministic MINLP solvers BARON (Tawarmalani and Sahinidis, 2005) and SCIP (Achterberg, 2009). The time limit is set to 7200 s with relative optimality tolerance of 10^{-6}.

3.1 Example 1

Example 1 is a small size network to verify the models. The network layout of Example 1 is presented in Table 3. The network has 10 nodes with 1 source (node 1), 5 users (nodes 5-7, 9, 10), 4 junctions (nodes 2-4, 8). The arcs include 6 pipelines and 3 compressors ((2,3), (3,4), (3,8)). All pipelines have the same length $L_{i,j}$ =3000 m, diameter $D_{i,j}$=0.762 m, and friction factor $F_{i,j}$=0.0085. Table 1 lists the parameters of the nodes. Since Example 1 is a tree-shaped network with fixed supply/demand, the flow rate is known for all of the arcs.

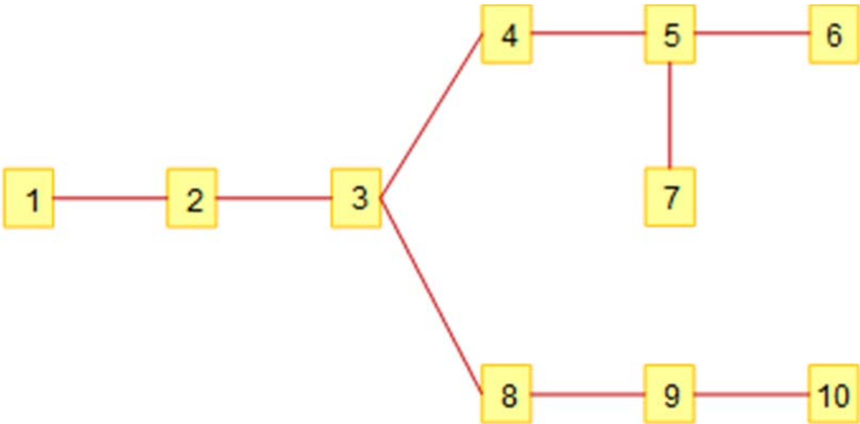

Figure 2: Network layout of Example 1

Table 1: Nodal parameters of Example 1

Node	P_i^{lo} (MPa)	P_i^{up} (MPa)	S_i (kg/s)	Node	P_i^{lo} (MPa)	P_i^{up} (MPa)	S_i (kg/s)
1	4.137	4.826	8	6	3.103	5.516	-1.5
2	4.137	5.516	0	7	3.103	5.516	-1.5
3	3.103	5.516	0	8	3.103	5.516	0
4	3.447	5.516	0	9	3.103	5.516	-1
5	3.103	5.516	-1	10	3.103	5.516	-3

Table 2 compares the results of the models of Example 1. The proposed model find the same optimal solution 0.6882 as the nonconvex model, and therefore, the new model is valid. Additionally, since the network is tree-shaped with fixed flow rate, only the discharged pressure of the compressors is the variable that will affect the objective value. Together with the small number of pipelines and compressors, both models reach the global optimal solution in less than 1 s.

Table 2: Model Performance of Example 1

Model	BARON		SCIP	
	Solution	CPU time (s)	Solution	CPU time (s)
Proposed	0.6882	0.02	0.6882	0.03
Nonconvex	0.6882	0.11	0.6882	0.19

3.2 Example 2

Example 2 is a medium size network to test the performance of the models. Figure 3 illustrates the network layout of Example 2. The network has 48 nodes, 43 pipelines, and 8 compressors ((2,9), (8,10), (12,13), (20,21), (21,22), (20,48), (24,46), (48,25)). The pressure limits of the nodes are [0.445 MPa, 10.445 MPa], except for node 1 ([6.305 MPa, 9.065 MPa]) and node 3 ([6.925 MPa, 8.030 MPa]). Parameters of the nodes and pipelines are listed in Table 3 and Table 4, respectively.

Table 3 Nodal parameters of Example 2

Node	S_i (kg/s)	Node	S_i (kg/s)	Node	S_i (kg/s)	Node	S_i (kg/s)
1	6	13	0	25	-5.5	37	0
2	0	14	0	26	0	38	-0.3
3	2	15	1	27	-0.5	39	-0.3
4	2	16	-0.5	28	0	40	-0.3
5	2	17	0	29	0	41	-0.3
6	2	18	1	30	-0.3	42	-0.4
7	2	19	0	31	-0.3	43	-0.4
8	0	20	4.5	32	0	44	-0.4
9	-4	21	0	33	-0.3	45	-1
10	0	22	0	34	-0.3	46	-2
11	-1	23	-2	35	-0.3	47	-1.8
12	0	24	0	36	-0.3	48	0

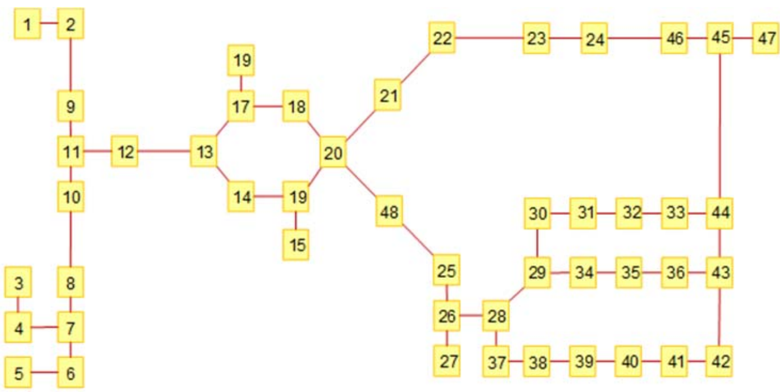

Figure 3: Network layout of Example 2

Table 4: Pipeline parameters of Example 2

Pipelines	$L_{i,j}$ (m)	$D_{i,j}$ (m)	$F_{i,j}$	Pipelines	$L_{i,j}$ (m)	$D_{i,j}$ (m)	$F_{i,j}$
(1,2)	10101.5	0.4572	0.0108	(30,31)	5050.7	0.3048	0.013
(3,4)	4517.5	0.4572	0.0108	(31,32)	4517.5	0.3048	0.013
(4,7)	5150.8	0.4572	0.0108	(32,33)	4517.5	0.3048	0.013
(5,6)	5150.8	0.3048	0.013	(33,44)	4517.5	0.3048	0.013
(6,7)	5150.8	0.4572	0.0108	(29,34)	5050.7	0.3048	0.013
(7,8)	5150.8	0.6096	0.009	(34,35)	4517.5	0.3048	0.013
(9,11)	10101.5	0.4572	0.0108	(35,36)	4517.5	0.3048	0.013
(10,11)	5150.8	0.6096	0.009	(36,43)	4517.5	0.3048	0.013
(11,12)	10101.5	0.9144	0.0085	(28,37)	5050.7	0.3048	0.013
(13,14)	10101.5	0.4572	0.0108	(37,38)	5050.7	0.3048	0.013
(14,19)	10101.5	0.4572	0.0108	(38,39)	5050.7	0.3048	0.013
(15,19)	10101.5	0.4572	0.0108	(39,40)	5050.7	0.3048	0.013
(19,20)	10101.5	0.4572	0.0108	(40,41)	5050.7	0.3048	0.013
(13,17)	10101.5	0.6096	0.0095	(41,42)	5050.7	0.3048	0.013
(17,16)	10101.5	0.4572	0.0108	(43,42)	4517.5	0.3048	0.013
(17,18)	10101.5	0.6096	0.0095	(44,43)	4517.5	0.3048	0.013
(18,20)	10101.5	0.6096	0.0095	(45,44)	8329.9	0.4572	0.0108
(25,26)	10101.5	0.4572	0.0108	(45,47)	5714.3	0.6096	0.009
(26,27)	7142.9	0.4572	0.0108	(46,45)	11517.5	0.6096	0.009
(26,28)	10101.5	0.4572	0.0108	(22,23)	11517.5	0.6096	0.009
(28,29)	5050.7	0.3048	0.013	(23,24)	11428.6	0.6096	0.009
(29,30)	4517.5	0.3048	0.013				

Example 2 is more complicated than Example 1 for its looped structure and the increase in the number of pipelines and compressors. Table 5 compares the performance of the proposed model and the nonconvex model. We observe that the proposed model reaches the global optimal solution within time limit. This is because the pressure drop constraints dominate in the model, and the convexification of the pressure drop constraints greatly reduces the nonconvexity of the model and the solution time.

Table 5: Model Performance of Example 2

Model	BARON Solution	CPU time (s)	SCIP Solution	CPU time (s)
Proposed	2.91105	47.89	2.91105	9.25
Nonconvex	3.73188	0.37	-	7200

4. Conclusions

A new MINLP model is presented for the energy minimisation of natural gas transmission networks. The model was tested by a small and a medium size network respectively for validating its feasibility and its efficiency for solution. By convexifying the pressure drop constraints and replacing the nonconvex compressor constraints by a set of concave and convex constraints, the proposed model works much efficient in finding good solutions.

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Reference

Achterberg T., 2009, SCIP: Solving Constraint Integer Programs, Mathematical Programming Computation, 1, 1-41.

Ali D.A., Gadalla M.A., Abdelaziz O.Y., Ashour F.H., 2016, Modelling of Coal-Biomass Blends Gasification and Power Plant Revamp Alternatives in Egypt's Natural Gas Sector, Chemical Engineering Transactions, 52, 49-54.

Brooke A., Kendrick D., Meeraus A., Raman R., 2003, GAMS: A Users Guide, GAMS Development Corporation, Washington, US.

Heckl I., Cabezas H., Friedler F., 2015, Designing Sustainable Supply Chains in the Energy-Water-Food Nexus by the P-Graph Methodology, Chemical Engineering Transactions, 45, 1351-1356.

Schroeder D., 1996, Hydraulic analysis in the natural gas industry. Advances in Industrial Engineering Applications and Practice I, 960-965.

Tawarmalani M., Sahinidis N.V., 2005, A Polyhedral Branch-and-Cut Approach to Global Optimization. Mathematical Programming, 103, 225-249.

US Energy Information Administration, 2016, International Energy Outlook 2016, US Department of Energy, Washington, US.

Wu S., Rios-Mercado R.Z., Boyd E.A., Scott L.R., 2000, Model Relaxations for the Fuel Cost Minimization of Steady-state Gas Pipeline Networks. Mathematical and Computer Modelling, 31, 197-220.