

The Longitudinal Flow of Oil and Petroleum Products in the Channels and Pipes. Part II

Yuriy A. Tolchinsky^a, Leonid L. Tovazhnyanskyy^a, Valery E. Ved^a, Petro O. Kapustenko^a, Evgeny V. Krasnokutskiy^a, Helen V. Ved^{a,*}, Marat I. Satayev^b, Abdilla A. Saipov^b

^aNational Technical University "Kharkiv Polytechnic Institute", 21 Frunze St., 61002 Kharkiv, Ukraine

^bM. Auezov South Kazakhstan State University, Department of life safety and environmental protection, Tauke Khan Avenue 5, 160012 Kazakhstan, Shymkent, 160012, Kazakhstan
valeriy.e.ved@gmail.com

A model associated fluid flow and heat transfer of longitudinal flows of oil in pipes and channels is considered. The model is an extended version of the Bingham flow on the case of flow with one or two solid cores. The fluid part is a two-phase medium, wherein the disperse phase may consist of solid particles, droplets and gas bubbles. The sliding on the borders of pipes and channels, thermal conductivity and viscosity of such systems and their impact on the characteristics of the solid core are discussed. The flow with a yield point dependence on the temperature is also considered.

1. Introduction

The flowing of oil petroleum products in channels and pipes represents by itself an example of a movement of multiphase mixture. During of such movement it is possible melting and hardening part of the flow area. It is possible the associated absorption and release of heat. The reason for the melting or solidification is the inflow or outflow of heat through the boundaries of the channels and pipes. The part of the flow which free from hardened substances participates in viscous flow, forming a flowing part of the movement. This fluid portion may at its phase composition to be a suspension, liquid or gas emulsion. To the suspension state calls a situation of incomplete curing, so that the solid core is formed. To liquid emulsion calls the situation of capture by the oil water or other fluid. Finally, as gas emulsion meets the situation of partial separation of the gas phase from the dissolved gases in oil. It follows that the motion of such mixture may be considered on the basis of Bingham model of the flow of a viscous fluid part. However, the model of such flow in the canonical form is insufficient because it cannot cover the entire range of related flow phenomena. There were formulated a longitudinal model of heat transfer and hydrodynamics for flow of single-phase fluid with one or two solid cores for laminar and turbulent flow regimes with Newtonian, power and the laws of turbulent viscosity in I part.

The heat transfer model represents a system of equations for temperatures of solid core (or cores) and temperatures of the fluid. The reciprocal relationship between models of heat transfer and hydrodynamics is realized through the cost, speed and heat transfer coefficients on the external and internal boundaries of the flow. All of these values are dependent on the values of the boundaries of the solid core (cores). These values as parameters are coefficient functions of the model equations of heat transfer. The Bingham model is generalized in two directions: one of them consists in introducing into the value of threshold yield dependence on shear rate; the other direction is to slip a solid core on the boundaries of the flow area. The problem of creating model of flow and heat transfer of oil and oil products, which takes into account their multi-phase, melting and solidification, is extremely actual. The current model is single-phase Newtonian and, in principle, cannot take into account specified features of flow and heat transfer.

2. Main part

The above hydrodynamic model allows a number of extensions that make it able to cover such observed in practice phenomenon as slipping through the borders, presence of solid, liquid and gas inclusions in the fluid part of the flow, viscosity and fluidity of the threshold shear rate, the viscosity and fluidity of threshold from temperature. Consider models of these phenomena in the order of their appearance, slide on the borders of a pipe or channel directly from the Bingham model and its generalizations to exponential and turbulent flow cannot be obtained because in the limiting case, when the conditions are met: $G_1^+ = h$ and $G_2^- = -h$, the adhesion of solid nuclei to borders so that the upper part of the upper reaches and the lower part of the bottom disappear. Fluid part is entirely located between the solid cores, which move with velocities of the upper and lower boundaries of the pipe or channel, respectively. In order to circumvent the difficulty specified it should assume the presence of a threshold slip at the boundaries of the pipe or channel, or very large, but finite value of viscosity in the solid core. The first of all the first assumption is considered. For simplicity this is done on the example of the upper core, which may come off from the top of the pipe or channel.

In this case, all formulas of the present work should be the value of W^+ in all the places of its occurrence, replace the velocity v^* , to be determined. The condition of sliding on the line $G^+=h$ can be written as:

$$f(\tau_0 - \tau^*) = \mu \frac{W^+ - v^*}{\delta} \quad (1)$$

where f – marking of functional dependence; τ^* – threshold slipping, PA; δ – the thickness of the thin fluid layer that separates the line $y = h$ and $y = G^+$, m. In the simplest possible case can use the first term in the expansion of f in powers of the difference $(\tau_0 - \tau^*)$. In this case, for values of v^* to be faithful to this expression:

$$v^* = W^+ - \frac{\delta}{\mu} (\tau_0 - \tau^*) \quad (2)$$

Determining the value of δ should be determined experimentally. However, it is possible do a qualitative evaluation based on the following considerations. The solidification process is started with some seed germ on which grown the solid phase. This process occurs in competition between nuclei. At this not the entire volume of oil hardens, but only part of it, having a certain fraction composition. The rest of the fractions remain fluid, but so that the whole system loses fluidity. In connection with such pattern in the system there is a natural scale with the dimension of length, namely δ , which determines the characteristic size of the solidified particles.

The second assumption, unlike the first, displays discussing phenomenon beyond the extended Bingham model. It consists in to assume that a solid core is not completely solid, but has, albeit small, but fluidity. In this case, the flow inside the core should solve the problem of viscous stoke flow in a pipe or channel width $(h - \Gamma^-)$ and the speed v^* of the borders to be determined. The solution to this simple problem in combination with the condition of continuity of shear stress at the boundary of the nuclei $y=h$ allows for the value of velocity v^* to obtain the following expression:

$$v^* = \frac{1 + \frac{h+\Gamma_1^-}{8W^+} \frac{\delta}{\mu} \frac{dP}{dx}}{1 + \frac{\mu_k}{\mu} \frac{\delta}{h-\Gamma_1^-}} \left/ \frac{1 - \frac{\mu_k}{\mu} \frac{\delta}{h-\Gamma_1^-} \frac{1}{W^+}}{1 + \frac{\mu_k}{\mu} \frac{\delta}{h-\Gamma_1^-}} \right.; \mu_k \gg \mu; \Gamma_1^+ = h; \quad (3)$$

in which μ_k – core viscosity, Pa·s.

Size μ_k and δ should be determined proceeding from the results of experiments. Similar reasoning applies to the lower core. In this case, in Eq. (3) should be replaced the value G_1^- on the size of G_2^+ , and the value of G_2^- should be equal to $(-h)$. In Eq. (3) the longitudinal pressure gradient remains the same and all formulas of the extended Bingham model. Accounting the dependence of threshold yield strength of shear rate can be performed without much difficulty, if it is demonstrated on the example of the Newtonian flow with a solid core inside. For this we need to refer to the Eq(12) for the quantities G^\pm from I part and to introduce the dependence on the shear rate in the amount τ_0 . Taking into account the fact that the first derivative of the speed of the fluid part of the boundary G^\pm vanishes it is possible to use the average values of this derivative on sections (G^+, h) and $(-h, G^-)$. For simplicity here a linear dependence of the threshold flow shear rate is proposed:

$$\tau_0 = \tau_{00} - \xi \left\langle \frac{\partial v^\pm}{\partial y} \right\rangle \quad (4)$$

where τ_{00} – constant, Pa; k is a constant Pa·s; ξ is a constant Pa·s; $\langle \dots \rangle$ – is the mean value of the derivative on these intervals. According to Eq(12) I from I part to G^\pm are obtained the following relations:

$$G^+ - G^- = \frac{\tau_1 + \tau_2}{dP/dx}; \quad G^\pm = \frac{\tau_1 + \tau_2}{2dP/dx} \pm \frac{\mu(W^+ - W^-)}{2\left(\frac{\tau_1 + \tau_2}{2} - h \cdot \frac{dP}{dx}\right)}; \quad (5)$$

$$\tau_1 = \tau_{00} - k \cdot \frac{dP}{dx} \cdot (h - G^+); \quad \tau_2 = \tau_{00} - k \cdot \frac{dP}{dx} \cdot (h + G^-).$$

It can be shown that equation system in Eq. (5) is reduced to two quadratic equations for the quantities G^\pm . For example, the same currents can easily consider the case of dependence of values of μ and τ on the temperature. In the approximation of thermal boundary layer and without making any assumptions about specific types of dependencies $\mu(T)$ and $\tau_0(T)$ it is possible to show that expression h in Eq(12) rom I part are transformed into the following system of equations:

$$G^+ - G^- = \frac{\tau_0(\psi^+ + \psi^-)}{dP/dx}; \quad \mu(T) = \mu_0\varphi(T); \quad \tau_0 = \tau_{00}\psi(T);$$

$$\frac{1}{2\mu_0} \cdot \frac{dP}{dx} \cdot \frac{(h - G^+)^2}{\varphi^+} - \frac{1}{2\mu_0} \cdot \frac{dP}{dx} \cdot \frac{(h + G^-)^2}{\varphi^-} + \frac{\tau_{00}}{\mu_0} \cdot (\psi^+ - \psi^-) \cdot \left(\frac{h - G^+}{\varphi^+} - \frac{h + G^-}{\varphi^-}\right) = W^+ - W^-; \quad (6)$$

$$\varphi^+ = \varphi[T(h)]; \quad \varphi^- = \varphi[T(-h)]; \quad \psi^+ = \psi[T^+(G^+)]; \quad \psi^- = \psi[T^-(G^-)].$$

It can be shown that for G^\pm this system of equations is also reduced to quadratic equations. The application of a hydrodynamic model for flows of suspensions, liquid emulsions and gas in the fluid part requires the ability to model the viscosity of mixtures. The difficulty is concluded in that universal laws do not exist. For Stokes flow regime of the fluid part of it is strictly known that the viscosity of suspensions and emulsions depends on the volume fraction of the dispersed phase; and in the limit of smallness of the latter obey the law of Einstein-Smoluchowskye and Hadamard-Rybczynskye. For moderate inertial currents from experimental data it follows that the velocity profile becomes more "filled" with the growth of the volume concentration. Finally, inertial and turbulent flows the viscosity of the mixtures depends on both volumetric and mass fraction of the dispersed phase. Thus, for the Stokes regime the Newtonian character of viscosity is retained, increasing the growth of volume fraction of the dispersed phase. The moderately inertial flow can be formally described by model of power law fluid with an exponent that depends on the volume fraction. Thus the following conditions on the viscosity characteristics are imposed:

$$\mu = \mu_0 f(\Phi); \quad f(\Phi \rightarrow 0) \rightarrow 1; \quad f(\Phi \rightarrow 0,5) \rightarrow \infty; \quad \text{for stock};$$

$$\mu = \mu_0 \left| \frac{\partial \vartheta}{\partial y} \right|^n; \quad n(\Phi \rightarrow 0) \rightarrow 0; \quad n(\Phi \rightarrow 0,5) \rightarrow 2-; \quad \text{for moderately inertial} \quad (7)$$

where Φ – volume fraction of dispersed phase, dimensionless.

The turbulent viscosity mixtures should be determined on the basis of experimental data. Case of gas emulsion requires separate consideration. The reason for this lies in the dependence of the solubility of the gas in the fluid pressure and temperature. In the hydrodynamic model presented in this manuscript in all formulas presents a longitudinal pressure gradient. If it is assumed that the region of the highest pressure at a given temperature there is a complete solubility, i.e., a gas as the dispersed phase is not, that by decreasing of pressure its isolation will be a occurred.

Based on Henry's law as the easiest for the Φ value the following expression is had:

$$\Phi = \frac{RT}{P \cdot M} \left(\frac{dH}{dT} \cdot \frac{dT}{dx} \cdot P + H \cdot \frac{dP}{dx} \right); \quad H(T) = e^{q/RT} \cdot H_0 \quad (8)$$

where R – y universal gas constant, J/grad·kmol; q – specific heat of gas bubbles formation, J/kmol; M – gas molar, kg/kmol; H_0 – constant, c·PA⁻¹. The Eq(8) should be considered as argument of the exponent n in the Eq(7). As the temperature T should be used above certain temperature T_\pm . There are presented the coefficients of heat emission in the model of heat change in the compulsory order. They include coefficient of thermal conductivity. For a homogeneous fluid it is a constant substance if its dependence is accounted from temperature. For the two-phase mixture conductivity is a function of the phases characteristics. Its definition is possible in the framework of the approach, rising to the Rayleigh and widely used in the description of particulate composites. Having in mind its application to liquid and gas emulsion, consider its essence. It consists of dividing the volume of the mixture in the unit cell so that each cell contains one representative of the dispersed phase and the surrounding continuous phase element. Such a cell allows for the splitting into sub-elements located along and across the local heat flux. These breakers meet the adiabatic and isothermal heat transfer (Figure 1).

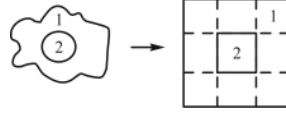


Figure 1: Elementary cell. The model of the elementary cell: horizontal dashed lines meet isothermal decomposition; vertical dashed lines meet the adiabatic splitting. 1 – solid phase; 2-disperse phase

For each type of transfer, the thermal conductivity has the following form:

$$\frac{\lambda_a}{\lambda_1} = \frac{\xi - (\xi - 1) \cdot (1 - \Phi^{2/3}) \cdot \Phi^{1/3}}{\xi - \Phi^{1/3} \cdot (\xi - 1)}, \quad \text{adiabatic splitting}$$

$$\frac{\lambda_i}{\lambda_1} = \frac{1 + (\xi - 1) \cdot \Phi^{2/3}}{1 + (\xi - 1) \cdot (1 - \Phi^{1/3}) \cdot \Phi^{2/3}}, \quad \text{isothermal splitting} \quad (9)$$

$$\xi = \lambda_2 / \lambda_1;$$

where λ_a , λ_i , λ_1 , λ_2 – coefficients of thermal conductivity of the mixture of solid and dispersed phases respectively, J/m·s·grad.

Combination for λ in Eq(9) with a weight of $1/2$ or weight, proportional to the angle of inclination of the heat flux vector to the vector orientation of the cell gives the value of the thermal conductivity of mixture. The result Eq(9) can be transferred to the case of a mixture with a liquid carrier phase, if the following reasoning is used. The coefficient of thermal conductivity characterizes the non-convection heat transfer. In this case, the carrier phase is stationary. In disperse phase in its individual elements, such as drops and gas bubbles possible micro-scale movement mechanism of natural convection. To describe heat transfer through these elements it is necessary to use the Rayleigh number Ra and Grashof Gr , defined on the scale of an element of the dispersed phase. Thus, natural convection enhances the transfer of heat in them so that the actual coefficient of thermal conductivity λ_1 should be modified according to the following rule:

$$\lambda_2 \rightarrow \lambda_2 \cdot \left(1 + \frac{\beta}{\nu^2} \nabla T g d^4\right), \quad (10)$$

Where the replacement of temperature difference for the scale element of dispersed phase through temperature gradient according to the rule: $\Delta T \cong \nabla T \cdot d$; β – coefficient of thermal expansion of fluid, grad⁻¹. In determining λ of the mixture should be considered as the carrier phase is stationary, despite the presence of the temperature distribution. In reality this is impossible. If it is assumed that the carrier phase is involved in the movement, it certainly makes the determination of thermal conductivity. The construction of thermal conductivity coefficient of liquid mixture is related to a linear combination of values λ_a and λ_i by Eq(9) with the substitution for λ_2 in Eq(10). The coefficient of thermal conductivity of mixture acquires certain dependence from temperature. In these formulas is the element size of the dispersed phase d . For emulsion droplets this value is given. For gas, this value should be based on the process of gas isolation and absorption. There are the following kinds of gas-isolation and gas-absorption: pressure and temperature changing. Pressure is decreased by moving in a pipe or channel, temperature is changed in accordance with Eq(1), Eq(3) I part. Out coming from the simple Henry's law, it is possible to calculate the volume fraction of gas phase emitted or absorbed from the following formula:

$$\Phi = \frac{d(HP_{\kappa})}{dx} \cdot \frac{RT}{MP_f}; \quad H = H_0 e^{-q/RT}, \quad (11)$$

where R – universal gas constant, J/kmol·grad; q – specific heat of gas isolation or absorption, J/kmol; M – molar of gas, kg/kmol; P_f and P_g – pressure in fluid and gas in accordingly, PA. To determine the diameter of bubbles should be a used such reasons. For definiteness below the case of gas isolation is considered. In any small volume flow of a mixture of length dx and cross-sectional area S_{\perp} occurs as the growth of bubbles, which were formed earlier in the section preceding this, and the birth of germinal bubbles. Assuming that the growth of bubbles occurs by the diffusion mechanism the equation for the diameter d can be written in the form:

$$\vartheta_f \cdot \frac{d(d)}{dx} = \beta_g \cdot (C^{\infty} - C_g); \quad C^{\infty} = H \cdot P_f \quad (12)$$

where β_g – coefficient of mass transfer at the interface, the dimension of which depends on the choice of dimension of concentration; C_g – gas concentration.

The process of nucleation is determined by the probability of formation of the embryo in the meta-stable liquid phase as a result of local overheating or local reduction of pressure. Using the expression for the critical size

d^* of germinal bubble: $d^* = 4\sigma/(P_f - P_g)$ and considering the work done by the surface tension σ at the interface boundary for the probability of local overheating and discharge you can obtain the following expression:

$$W_p \sim \exp\left\{-\frac{16\pi\sigma^3\vartheta_f^2}{3T(\vartheta_{\kappa} - \vartheta_f)^2 \cdot \delta \cdot P_{\kappa}}\right\}; \quad W_T \sim \exp\left\{-\frac{16\pi\sigma^3(\vartheta_{\kappa} - \vartheta_f)^2 \cdot T}{3q^2(\delta \cdot T_{\kappa})^2}\right\}, \quad (13)$$

where ϑ_g, ϑ_f – specific volumes of gas and liquid, respectively, $\text{m}^3/\text{molecule}$.

If $(\delta \cdot P_f)$ and $(\delta \cdot T_f)$ is integrated as changes of pressure and temperature on the length equal to the size of the bubble, $(\delta \cdot P_f)$ and $(\delta \cdot T_f)$ should be replaced by $(\nabla P \cdot d)$ and $(\nabla T \cdot d)$. The total number of embryos that may occur in the volume $(dx \cdot S_{\perp})$, can be estimated as $(dx \cdot S_{\perp})/d^{*3}$. for the number of embryos N^* of gas phase is obtained the following estimate:

$$N^* \sim \frac{dx \cdot S_{\perp}}{d^*} \cdot (\omega_p + \omega_T) \quad (14)$$

The total quantity of heat is released in the current mixture it can be considered spent for the nucleation and growth of bubbles. The expression for this balance is as follows:

$$N^* \cdot \omega \cdot q \cdot dx + \rho_g \cdot C_{Pg} \cdot d\Phi = C_p \cdot \rho \cdot S_{\perp} \cdot dT \quad (15)$$

where $d\Phi$ is determined from Eq(7).

It follows from Eqs. (7)-(15) that average bubble size can be calculated by taking the weighted average diameter by the number of bubbles generated in the interval dx near the point with x -coordinate. This weighted average diameter $\langle d \rangle$ has the following form:

$$\langle d \rangle = \frac{\int_0^x N^*(u) \cdot d(u, x) \cdot du}{\int_0^x N^*(u) \cdot du} \quad (16)$$

This diameter should be substituted in Eq(10) to calculate the thermal conductivity of the mixture using Eq(9). The value of $d(u, x)$ is a solution of Eq(12) which provided that the growth of the bubble is considered in the time interval corresponding to the spatial interval (u, x) .

3. Discussion of the results

The main result of this study is the model of coupling the heat transfer and hydrodynamics of the oil flow as a multiphase mixture, which may include a solid core based on the generalization of the model of longitudinal heat transfer model and Bingham flow with Newtonian fluid part. The presented in this manuscript the three-temperature heat-transfer model based on the viscous-plasticity generalization of the Bingham flow model in a pipe or channel, allows, in principle, to find the longitudinal distribution of temperatures.

There are developed and discussed two models of slip at the boundaries in the present work. One is formal, which postulated a relationship between the threshold slip speed and slip. The specific form of this communication should be established experimentally. Another model includes a full waiver of the hardness kernel. This means that the core has a shear rate within a non-zero, though small in magnitude. This model requires determination of the viscosity kernel, which is a great value. In this case the core becomes small, the curvature of the velocity profile. Both models include such easily detectable parameter, the thickness of the fluid layer. In the event of agreement with the experimental data, this parameter can act as a free. If it is assumed that this thin border layer must withstand the pressure that exists inside the flow region for reasons of dimension the thickness of this layer should be in the order of magnitude is equal to the ratio of specific one of surface tension to the pressure. This circumstance makes the problem of flow in a fluid part depends not only on the pressure gradient and the pressure. It should be emphasized that in the hydrodynamic model in the present work the pressure gradient is considered as a given external parameter. The dependence of the thickness of the thin border layer of pressure does not allow considering the pressure gradient is specified. In this case, the hydrodynamic model should include an equation to determine the pressure along the pipe or channel at the ends of the channel boundary values of pressure. This task can be formulated and solved, but this solution is beyond the scope of this work. From the model with a thin border layer of fluid, it follows that for large values of the velocity shear threshold the yield stress must vanish. In this paper, as an illustration is considered the linear dependence of threshold yield strength of shear rate; and for simplicity and solvability of the equations to determine the boundaries of the nucleus is taken into account only the transverse component of the shear rate. The specific form of dependence for the threshold yield strength must be determined experimentally. Suspensions and emulsions flowing in stokes regime remain at low volumetric concentrations Newtonian properties, which leads to their flow with stable velocity profile. This concentration depends on the amplitude

profile. With the increasing of magnitude volume concentration of the Newtonian property of flow is maintained. For inertial flow, as shows experiment, the filling of the velocity profile increases, i.e. suspension or emulsion in own flowing like a solid body. In this case, the appropriate model is a power law fluid with exponent, depending on the value of the volume concentration. A quantitative estimate of the magnitude of separation or gas absorption the authors proceed from Henry's law. This law is valid at low concentrations, so that with the growth of the last observed deviation from it. This somewhat limits the results presented in the work. Such limitation can be easily overcome if this law is considered as the first term in the Taylor series decomposition of the total dependence of the amount of dissolved gas from the pressure in the system. The model for calculating the heat transfer coefficient for particulate composites with isolated inclusions is standard, but its distribution in the emulsion can cause issues. In this case, the authors proceeded from the following reasoning. The presence of liquid or gaseous inclusions in the unit cell of the solid phase requires adding to the flow of heat by the mechanism of heat conduction through the inclusion of flow from natural convection. In this case, the conductivity of the composite acquires a dependence on temperature gradient. The transition from solid composite with a liquid inclusion to the emulsion as the composite of liquid (gas) the inclusion of solid and liquid phase requires the conditions of non-flowing of last one. Therefore such condition in the presence of the temperature gradient in real life is impossible. However, this is only a technique that allows building a model of the thermal conductivity so that the solution of thermal problems for emulsions is the property of their fluidity is restored. The model of nucleation in meta-stable conditions when the reasons for the formation of the embryo of a new (gas) phase acts as a fluctuation of pressure and (or) temperature is interpreted in such way that these fluctuations are determined by the longitudinal gradients of pressure and temperature and size of the inclusions. The model of growth or reduction of gas bubbles is the standard diffusion model. The mass transfer coefficient depends on the values of Reynolds and Prandtl number. The Reynolds number is determined by the size of the inclusion and relative velocity of gas and liquid phases. For very small bubbles close to the size of the embryonic mass transfer coefficient should be replaced by the ratio of thermal conductivity to the size of the bubble.

4. Conclusions

The presented in the present work the model of rheological properties of suspensions and emulsions, if by this properties of the solid core, in principle, able to cover many aspects of the flow of oil and oil products in pipes or channels under the conditions of heat exchange with the boundaries is understood. The model is open in respect to improvements of various components so that in combination with experimental data can be useful in practical applications.

Acknowledgments

The support of Grant of Education and Science Ministry of the Republic Kazakhstan in for priority "Rational use of natural resources, processing of raw materials and products", for sub priority "Mining technologies" is sincerely acknowledged.

Reference

- Abramovich G.N., 1984. The theory of turbulent jets. Nauka. Glavnaya redaktsiya fiziko-matematicheskoy literatury, Moscow, Russian Federation. (in Russian).
- Aleksin V.A., 2007. Mathematical models of turbulent flows. Izdatel'stvo MGIU, Moscow, Russia. (in Russian).
- Betchelor Dzh., 1973, Introduction to the fluid dynamics. Mir, Moscow, Russian Federation. (in Russian).
- Dulnev G.N., Novikov V.N., 1991, Transport processes in heterogeneous media. Energiya, Moscow, Russian Federation. (in Russian).
- Landau L.D., Lifshits Ye.M., 1986, Theoretical physics. Vol. 6. Hydrodynamics. Nauka. Glavnaya redaktsiya fiziko-matematicheskoy literatury, Moscow, Soviet Union. (in Russian).
- Loynyatskiy L.G., 1973, Fluid and gas mechanics. Nauka. Glavnaya redaktsiya fiziko-matematicheskoy literatury, Moscow, Soviet Union. (in Russian).
- Sirignano W.A., 2005, Volume averaging for the analysis of turbulent spray flows. International Journal of Multiphase flow, 31, 6, 675-705.
- Tolchinsky Y. A., Ved' V. E., Zhantasov M. K., Satayev M. I., Saipov A. A., 2016, The longitudinal flow of oil and petroleum products in the channels and pipes, Chemical Engineering Transactions, 52, 265-270 DOI:10.3303/CET1652045
- Volkov K.N., Yemelyanov V.N., 2008, Gas flow with particles. Fizmatlit, Moscow, Russian Federation. (in Russian).
- Zaychik L.I., Varaksin A.Yu., 1999. Influence of track of large particles on the intensity of the turbulence of the carrier flow. Teplofizika vysokikh temperatur, 37, 4, 683-687. (in Russian).