Neural-Network-Based and Robust Model-Based Predictive Control of a Tubular Heat Exchanger

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The paper is devoted to advanced control of a tubular heat exchanger with focus to energy savings. The controlled tubular heat exchanger (HE) was used for petroleum pre-heating by hot water. The controlled output was the measured temperature of the petroleum in the output stream and the control input was the volumetric flow rate of hot water. Two advanced control strategies were investigated in the set-point tracking, the neural-network-based predictive control and the robust model-based predictive control with integral action and with soft constraints on control inputs. The advanced control of the heat exchanger was implemented in the MATLAB/Simulink simulation environment. Simulation results obtained using advanced controllers were compared with the results ensured by a conventional PID controller and they confirmed significant improvement of the control performance. Moreover, advanced controllers reduced energy consumption measured by the total consumption of hot fluid used for heating.

1. Introduction

Heat exchangers are frequently used in industry and effective control of them is very important as they are energy intensive processes. Control of heat exchangers is a complex problem due to the nonlinear and asymmetric behaviour, temperature dependent flow properties, unknown fluid properties, time-varying parameters, etc. From these reasons, the advanced control strategies can be more successful in the set-point tracking, disturbance rejection and energy savings in comparison with conventional control. A tubular heat exchanger is the simplest form of heat exchanger and consists of two coaxial tubes carrying the hot and cold fluids. Two advanced control strategies are used for control of the tubular heat exchanger, i.e. the neural-network-based predictive control and the robust model-based predictive control. The neural-network-based predictive control (NNPC) is one of typical and straightforward applications of artificial neural networks (ANNs) to nonlinear control (Huang and Lewis, 2003) in recent years. An ANN has several advantages but one of the most recognized of these is the fact that it can actually learn from observing data sets. ANN takes data samples to obtain solutions, which save both, time and money. ANNs are considered fairly simple mathematical models to enhance existing data analysis technologies. ANNs have been successfully used in many engineering applications such as dynamic control, system identification and performance prediction of thermal systems in heat transfer applications. Model predictive control using a neural network model for SISO systems has been attempted by some researchers and was outlined in Hunt et al. (1992). The control law was represented by a neural-network function approximator, which was trained to minimize a control-relevant cost function. The applications of ANN for thermal analysis of heat exchangers are reviewed in detail (Mohanraj et al., 2015). Wang et al. (2016) investigated the multirate networked industrial process control problem in double-layer architecture, and finally, a continuous stirred tank reactor system was given in the simulation part to demonstrate the effectiveness of the proposed method. Daosud et al. (2016) studied and applied the NNPC to control a batch extractive distillation column used for the separation of waste solvent mixture of acetone and methanol.

The second advanced control approach is the model-based predictive control (MPC) that has been widely implemented in complex constrained multivariable control problems including chemical and thermal processes (Zhang et al., 2014). Robust MPC that considers the model parametric uncertainty and bounded disturbance...
has been studied extensively in past two decades, see e.g. Ding and Pan (2014). The receding horizon control strategy requires the evaluation of optimal control action in each control step. Linear matrix inequalities (LMIs) represent a powerful technique to handle uncertain systems, because they enable to convert the control design problem into the convex optimization in the form of semidefinite programming (SDP). The state feedback control law can be obtained as a solution of the SDP (Bakošová and Oravec, 2014). Various alternative robust MPC strategies were proposed in Oravec and Bakošová (2015a). The controlled tubular heat exchanger (HE) was used for petroleum pre-heating by hot water. The controlled output is the measured temperature of the petroleum in the output stream and the control input is the volumetric flow rate of hot water. The problem of set-point tracking was solved using the NNPC and robust MPC strategies, and obtained simulation results were compared with conventional PID control using various quality criteria. A new alternative robust MPC with integral action and with soft constraints on control inputs has been developed and used. Advanced and conventional control strategies were compared also according to energy savings measured by the hot water consumption used for control of the HE.

2. Tubular heat exchanger

A tubular heat exchanger is the simplest form of heat exchanger and consists of two coaxial tubes carrying the hot and cold fluids. The co-current tubular heat exchanger was considered in the research, where petroleum is heated by hot water passed through a copper tube (Vasičkaninová and Bakošová, 2015), (Figure 1).

![Figure 1: a) The heat exchanger scheme. b) The steady-state temperature profile for co-current flow.](image)

Among the input variables, the hot water flow rate, \( q_i(t) \), was selected as the control input. The controlled variable was the outlet petroleum temperature, \( T_{\text{out}} \). The mathematical model of the HE was derived under several simplifying assumptions. Coordinate \( z \) measures the distance of a modelled section from the inlet. The fluids move in a plug velocity profile and the petroleum, tube and water temperatures: \( T_1(z,t) \), \( T_2(z,t) \) and \( T_3(z,t) \), are functions of the axial coordinate, \( z \), and time, \( t \). The petroleum, water and tube material densities, \( \rho_i \), as well as the specific heat capacities, \( c_{pi} \), \( i = 1, 2, 3 \), are assumed to be constant.

The simplified nonlinear dynamic mathematical model of the co-current heat exchanger is described by three partial differential equations:

\[
\frac{d_1 \rho_1 c_{p1} \, \partial T_1(z,t)}{4h_1} + \frac{\rho_1 c_{p1} \, q_1}{\pi d_1} \frac{\partial T_1(z,t)}{\partial z} = -T_1(z,t) + T_2(z,t) \quad (1)
\]

\[
\frac{(d_3^3 - d_1^3) \rho_2 c_{p2} \, \partial T_2(z,t)}{4(d_1 h_1 + d_2 h_2)} = \frac{d_1 h_1}{d_1 h_1 + d_2 h_2} T_1(z,t) - T_2(z,t) + \frac{d_2 h_2}{d_1 h_1 + d_2 h_2} T_3(z,t) \quad (2)
\]

\[
\frac{(d_3^3 - d_2^3) \rho_3 c_{p3} \, \partial T_3(z,t)}{4d_2 h_2} = \frac{\rho_2 c_{p2} \, q_3(t)}{\pi d_2 h_2} \frac{\partial T_3(z,t)}{\partial z} = T_2(z,t) - T_3(z,t) \quad (3)
\]

Here, \( l \) is the length of the tube, \( d \) is the tube diameter, \( \rho \) the density, \( c_p \) the specific heat capacity, \( h \) the heat transfer coefficient, \( q \) the volumetric flow rate, \( 1 \) is petroleum, \( 2 \) is copper, \( 3 \) is hot water. Parameters and steady-state inputs of the heat exchanger are given in Vasičkaninová and Bakošová (2012).

For robust RMPC and PID controller design, the mathematical model is needed in the form of a linear discrete-time state-space model or a transfer function. As the HE is a nonlinear system with asymmetric dynamics, several step changes of the inlet mass flow-rate of the heating water were generated to identify the process. The Strejc method (Mikiš and Fikar, 2007) was used and the heat exchanger was identified in the form of the \( n^0 \) order plus time delay transfer function in Eq(4).
\[ G = \frac{K}{(s + 1)^p} e^{-Ds} = \frac{37.000}{(18s + 1)^{2}} e^{-2.4s} \]  

(4)

3. Control of the heat exchanger

Saving energy is a huge and costly problem for industry management and installing heat exchangers or heat exchanger networks should help to salvage as much energy as possible. But the HEs belong to the energy intensive processes and efficient control of them is very interesting for energy savings.

3.1 PID control of the heat exchanger

PID controller described by the transfer function

\[ C = k_p + \frac{k_i}{s} + k_d s \]  

(5)

with \( k_p \) the proportional gain, \( k_i \) the integral time and \( k_d \) the derivative time, was tuned using the Cohen-Coon method (Ogunnaike and Ray, 1994) for the model in the form Eq(4). The PID controller parameters obtained using the Cohen-Coon formulas are \( k_p = 1.7 \times 10^{-4}, k_i = 5.2 \times 10^{-6}, k_d = 8.5 \times 10^{-4} \).

3.2 Neural-network-based predictive control of the heat exchanger

Model-based predictive control (MPC) is a strategy that is widely used in process industry. MPC uses the system model to predict the system's future outputs based on the current value of the system output and future value of inputs. Using this information, it calculates the optimal value of the future control inputs with respect to a predefined cost function. A typical block diagram for MPC is shown in Figure 2.

![Figure 2: Structure of MPC.](image)

The objective of the predictive control strategy is to estimate the future output of the plant and to minimize a cost function (6) based on the error between the predicted output of the processes and the reference trajectory

\[ J = \sum_{i=N_1}^{N_2} (y_i(k + i) - \hat{y}(k + j))^2 + \mu \sum_{i=N_1}^{N_2} \Delta u^2(k + i - 1) \]  

(6)

where, \( N_1 \) is the control horizon, \( N_i \) is the minimum prediction horizon, \( N_e \) is the prediction horizon, \( i \) is the order of the predictor, \( y_i(k + i) \) is the future reference signal, \( \hat{y}(k + j) \) is the prediction of future outputs, \( u(k) \) is the control signal at time \( k, \Delta u(k + i - 1) = u(k + i - 1) - u(k + i - 2) \) is the control input change, \( \mu \) is the factor penalizing changes in the control signal. The cost function value is minimized in order to obtain the optimum control input that is applied to the controlled plant (Lazar and Pastravenu, 2002).

The neural-network-based predictive control (NNPC) is one of typical and straightforward applications of ANNs to robust MPC schemes. The first step in neural network predictive control is training the network. The Levenberg-Marquardt algorithm was chosen for network training. The training data were obtained from the controlled process represented by the non-linear model of the heat exchanger in Eq(1)-Eq(3) with the sampling interval 1 s. 1000 training samples were used. After the NN model was trained, the NNPC could be implemented for control of the HE. The parameters for NNPC were chosen as follows: the minimum prediction horizon \( N_i = 1 \), the maximum prediction horizon \( N_e = 6 \), the control horizon \( N_u = 2 \), the weight coefficient in the cost function \( \mu = 5 \), and the parameter for the reference trajectory calculation \( \alpha = 0.0001 \). For computing the control signals that optimize future plant performance, the minimization routine csrchbac was chosen. It is
one-dimensional minimization based on the backtracking method. The control input constraints were set: \( 1.5 \times 10^{-4} \leq q_{in} \leq 3.5 \times 10^{-4} \text{ m}^3 \text{ s}^{-1} \).

### 3.3 Alternative robust model predictive control with integral action

To investigate the advanced control of the tubular heat exchanger an alternative robust MPC approach was also implemented (Oravec and Bakošová, 2015a). For the robust controller design, the linear time-invariant state-space model in the discrete-time domain is needed, that is described in the form

\[
x(k + 1) = Ax(k) + Bu(k), \quad x(0) = x_0,
\]

\[
y(k) = Cx(k)
\]

where \( k \) represents the discrete time, \( x(k) \) is the vector of states, \( u(k) \) is the vector of control inputs, \( y(k) \) is the vector of the system outputs. The matrices \( A^{(v)}, B^{(v)}, C \) have appropriate dimensions. The model in Eq(7) is an uncertain system with the polytopic uncertainty. The matrices \( A^{(v)}, B^{(v)}, v = 1, \ldots, 4 \), describe the vertex systems of the uncertain system in Eq(7). The novelty of the approach is extension of the system in Eq(7) to implement the robust MPC with integral action that ensures the offset-free control. Then the robust static state-feedback control problem in the discrete-time domain can be formulated as follows: find a state-feedback control law (Oravec et al., 2015)

\[
u(k) = F_k x(k),
\]

for the system described by Eq(7), where the matrix \( F_k \) is the gain matrix of the static state-feedback robust controller in the \( k \)-th control step. Quality of the control performance is expressed by the quadratic cost function

\[
J = \sum_{k=0}^{n_k} \left( J_x(k) + J_u(k) \right) = \sum_{k=0}^{n_k} \left( x(k)^T W_k x(k) + u(k)^T W_u u(k) \right)
\]

where \( n_k \) is the total number of control steps. For design purposes the infinity control horizon is assumed, and \( W_k, W_u \) are real square symmetric positive definite weight matrices of the system states \( x(k) \) and the system inputs \( u(k) \), respectively. The aim is to design the controller \( F_k \) that ensures robust stability of all considered vertex systems and minimizes the quadratic criterion \( J \) in Eq(9). The control performance can be improved by taking into account symmetric constraints on the system outputs \( y(k) \) and inputs \( u(k) \) in the form

\[
\|y(t)\| \leq y_{\text{max}}, \quad \|u(t)\| \leq u_{\text{max}},
\]

Following conditions hold for the symmetric positively defined Lyapunov matrix \( P_k \) and the feedback controller \( F_k \)

\[
R_k = \gamma_k X_k^{-1}, \quad Y_k = F_k X_k, \quad \Rightarrow F_k = Y_k X_k^{-1},
\]

where \( \gamma_k \) is the auxiliary optimization parameter, \( X_k \) is the symmetric positive definite matrix, and \( Y_k \) is the auxiliary matrix. The robust stabilization problem can be solved as the convex optimization problem based on the following LMIs

\[
\min_{\gamma_k, X_k, Y_k} \gamma_k
\]

subject to

\[
\begin{bmatrix}
1 & x_k^T \\
* & x_k
\end{bmatrix} \geq \begin{bmatrix}
X_k & \begin{bmatrix}
(A^{(v)} X_k + B^{(v)} [E_j Y_k + E_j U_k]^T \\
X_k
\end{bmatrix}

\end{bmatrix} > 0,
\]

\[
\begin{bmatrix}
X_k & \begin{bmatrix}
(A^{(0)} X_k + B^{(0)} [E_j Y_k + E_j U_k]^T \\
X_k
\end{bmatrix} \\
* & * \\
* & *
\end{bmatrix} \begin{bmatrix}
\gamma_k I & 0 \\
0 & \gamma_k I
\end{bmatrix} \begin{bmatrix}
X_k & \begin{bmatrix}
\begin{bmatrix}
(E_j Y_k + E_j U_k)^T W_u
\end{bmatrix}

\end{bmatrix}

\end{bmatrix} \geq 0
\]

where \( \gamma_k \) is the auxiliary optimization parameter.
where \( v = 1, \ldots, n_v \). The symbol \( * \) denotes a symmetric structure of the matrix, and \( I, 0 \) are the identity and zero matrices of appropriate dimensions, respectively. The symmetric constraints on the control inputs and the controlled outputs in the form of Eq(10) are added to the optimization problem Eq(12) – Eq(14) as the LMI

\[
\begin{bmatrix}
  u_{\text{max}}^2 & U_k & X_k \\
  \cdot & X_k \\
  \cdot & \cdot
\end{bmatrix} \geq 0,
\begin{bmatrix}
  X_k \\
  \cdot
\end{bmatrix}
\left( A^{(v)}X_k + B^{(v)}\left[ E_j Y_k + E_j U_k \right] \right) C^T \geq 0
\]

where \( v = 1, \ldots, n_v, j = 1, \ldots, n_u \). The matrices \( E_j \) are the diagonal matrices with all variations of 1 and 0 on the principal diagonal and zeroes elsewhere; \( E_j^- \) are the complement matrices obtained as \( E_j^- = I - E_j \).

For the robust MPC with integral action the model in Eq(4) was transformed in the discrete-time system in Eq(7) using the sampling time \( t_s = 3 \) s. The \( \pm 5\% \) uncertainty on the system gain and the time constant was considered and four vertex systems were obtained for all combinations of minimum and maximum values of both parameters. The weights in the quadratic cost function \( J \) in Eq(9) were set as follows: \( W_x = 1 \times 10^4 \), \( W_u = 1 \). Integral action was designed also with the unit weighting. The robust MPC was designed using MUP toolbox (Oravec and Bakošová, 2015b), the optimization problem was formulated by the YALMIP toolbox (Löfberg, 2004) and solved by the solver MOSEK.

4. Results and discussion

Simulations of the advanced and the PID control were done using the model of the heat exchanger in Eq(1)-Eq(3) in MATLAB/Simulink R2014b using CPU i5 1.7 GHz and 6 GB RAM. The results of NNPC and PID control are compared in Figure 3a, where the trajectories of the petroleum temperature in the outlet stream are shown. The analytical quality criteria were also evaluated, see Table 1, where \( t_{\text{set}} \) represents the mean value of the settling time, ISE and IAE are integral of squared error and integral of absolute error and \( V_{\text{total}} \) is the total consumption of hot medium. NNPC assured the minimum values of all criteria. In comparison to PID control, the settling time was reduced in about 70 \%, the values of IAE and ISE decreased in 64 \% and 66 \%, respectively, the overall consumption of hot medium was reduced in approximately 32 \%. The disadvantage of this strategy was a small offset and oscillating control response. The results of robust MPC with integral action are presented in Figure 3b. The control trajectory achieved using robust MPC is smooth and without offset. In comparison with PID control, the settling time was reduced in about 33 \%, the values of IAE and ISE decreased in 56 \% and 49 \%, respectively, and the overall consumption of hot medium was reduced in approximately 5 \%.

![Figure 3](image)

**Figure 3:** a) Control trajectories ensured by NNPC (solid) and PID controller (dashed), reference (dash-dotted), tolerance (dotted). b) Control trajectories ensured by robust MPC (solid) and PID controller (dashed), reference (dash-dotted), and tolerance (dotted).

<table>
<thead>
<tr>
<th>Control strategy</th>
<th>( t_{\text{set}} ) [s]</th>
<th>IAE [-]</th>
<th>ISE [-]</th>
<th>( V_{\text{total}} ) [dm³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>27</td>
<td>143</td>
<td>199</td>
<td>129</td>
</tr>
<tr>
<td>Robust MPC</td>
<td>18</td>
<td>62</td>
<td>102</td>
<td>123</td>
</tr>
<tr>
<td>NNPC</td>
<td>8</td>
<td>51</td>
<td>67</td>
<td>87</td>
</tr>
</tbody>
</table>
5. Conclusions

Implementation of two advanced control strategies for the tubular heat exchanger led to the significant improvement of the closed-loop control performance compared to the PID control. The robust MPC with integral action assured the smooth and offset free control response. Closed-loop control performance achieved using NNPC was the best according to all followed quality criteria including the hot fluid consumption. The only disadvantage of the NNPC was small offset. Both advanced control strategies reduced energy consumption compared to conventional PID control. The further research will be focused on the implementation of advanced control strategies to the laboratory heat exchanger.

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References

Ding B., Pan H., 2014, Output feedback robust MPC with one free control move for the linear polytopic uncertain system with bounded disturbance, Automatica, 50, 2929-2935.