

# The Failure-tree Analysis Based on Imprecise Probability and its Application on Tunnel Project

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Due to the inherent uncertainties in ground and groundwater conditions, tunnel projects often have to face potential risks of cost overrun or schedule delay. Risk analysis has become a required tool to identify and quantify risk, as well as visualize causes and effects, and the course (chain) of events. Various efforts have been made to risk assessment and analysis by using conventional methodologies with precise probabilities. However, because of limited information or experience in similar tunnel projects, available evidence in risk assessment and analysis usually relies on judgments from experienced engineers and experts. As a result, imprecision is involved in probability evaluations, which leading the results of risk assessment based on precise probability to the imprecise results. In this paper, a failure tree analysis method based on imprecise probability is established for the risk assessment of tunnel project, which making the results of risk assessment are more accurate and more reasonable.

## 1. Introduction

The tunnel construction project is faced with large risk because of its complex construction environment, complex geological conditions, large environmental risk, long construction period, and so on (Shi et al., 2015; Zhang et al., 2016). The risk analysis and assessment of the tunnel construction project will help to reduce the occurrence of engineering accidents.

Fault tree analysis is one of the commonly used quantitative and qualitative methods in the risk analysis method. Fault tree analysis is based on the causal logic between events, which focus on the relationship between risk and the reasons in the system. Based on logical deduction analysis, Fault tree analysis starts from a specific accident, and then analyzes the various possible causes of the accident, finally, identifies a variety of potential risk factors (Wu et al., 2015). According to the fault tree analysis, the risk factors of the system could be fully understanding, and the occurrence probability of risk or accident could be calculated, which could provide the plan of risk control or risk mitigation

When using the traditional fault tree analysis method, the occurrence probability of a risk or an accident is a specific value, which is called "precise probability". However, in practice, it is imprecise to use "precise probability" to describe the risk of an event due to the uncertainty of the event itself (Gierczak, 2014).

The uncertainty of events in the real world arise from the variance of the event itself and the ignorance about the subject matter. The variance of the event itself could be quantities assessed by using classic theories of precise probabilities, due to the variance have certain regularity (Mahmood et al., 2013; Wang and Hu, 2015). While the ignorance about the subject matter can be reduced if the amount of information is expanded the relationship between uncertainty, imprecision and randomness is shown in Figure 1. Note that the length of each part is only a schematic role, which has no special practical significance. Determine is the understanding of all the necessary information to aware of the consequences of the incident occurred clearly. The uncertainty is composed of imprecise and randomness. Randomness is an inherent property of an event itself, which usually expressed as a probability distribution or stochastic process of random variables; imprecision is the result of a gap between the available information and the complete information (Gustafson et al., 2013). Because of the limited information, assigning a precise value as the probability of an event is not practical.

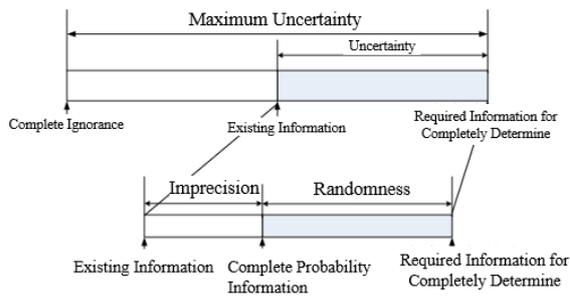


Figure 1: The Relationship of Uncertainty, Imprecision and Randomness

In the tunnel project, it is unrealistic to increase the amount of knowledge in relevant fields in a short time due to the various conditions. It means that assigning an exact numerical value as the probability of an event is impractical due to the inherent uncertainty.

**2. Fault tree analysis based on imprecise probability**

A fault tree analysis is a deductive analytical technique. It starts from a specified state of the system as the “Top event”, and includes all faults which could contribute the top event. At the bottom of the fault tree, the basic initiating faults, which could not be further developed, are called as “Basic events” linked by fault tree gates, including AND-, OR-, and Exclusive OR- gates. Sub-tree with OR-gate and Sub-tree with AND-gate are shown in Figure 2 (Song et al., 2015).

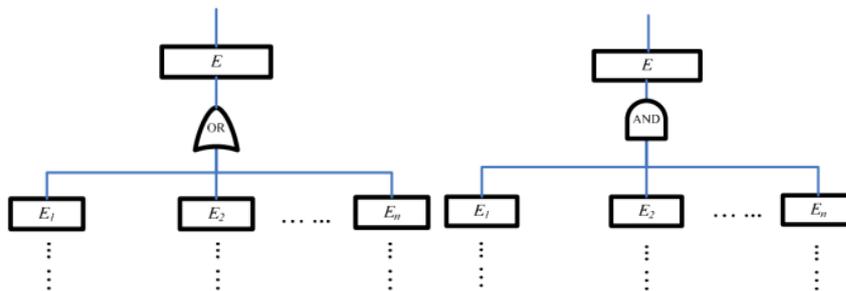


Figure 2: Sub-tree with OR-gate and Sub-tree with AND-gate

Any event in the fault tree has only two possible states: occurrence or not occurrence. The occurrence probabilities for the failure events are assumed to be given imprecisely, i.e. as interval probabilities  $[P_{low}, P_{upp}]$ . Let  $P$  be the joint probability distribution for sub-events  $E_1$  through  $E_n$ , and thus  $P$  is a matrix of dimensions  $2 \times 2 \times \dots \times 2$ , where the  $i$ -th index indicates the states of Event  $E_i$ ; let the subscript “1” denote the probability of occurrence, and “2” denote the probability of non-occurrence.

For the OR-gate, the occurrence probability for the failure event  $E$  is  $P(E)=1-P_{2,2,\dots,2}$ , where  $P_{2,2,\dots,2}$  is the probability of that none of the  $n$  events occurs.

For the AND-gate, the occurrence probability for the event  $E$  is  $P(E)=1-P_{1,1,\dots,1}$ , where  $P_{1,1,\dots,1}$  is the probability that all the  $n$  events occur

Let  $P_{i^{\wedge}j}$  be the joint probability distribution for sub-events  $E_i$  and  $E_j$ , which can be written in terms of the joint distribution  $P$  for  $E_1$  through  $E_n$ :

$$P_{i^{\wedge}j} = \sum_{\xi_1=1}^2 \dots \sum_{\xi_{i-1}=1}^2 \sum_{\xi_{i+1}=1}^2 \dots \sum_{\xi_{j-1}=1}^2 \sum_{\xi_{j+1}=1}^2 \dots \sum_{\xi_n=1}^2 P_{\xi_1, \xi_2, \dots, \xi_n}$$

According to the assumed interaction between  $E_i$  and  $E_j$ , the joint distribution  $P_{i^{\wedge}j}$  should fall in one of the following cases:

- (1) Unknown interaction:  $P_{i^{\wedge}j} \in \varphi_u$ ; (2) Epistemic irrelevance:  $P_{i^{\wedge}j} \in \varphi_e^{s(i)}$ ; (3) Epistemic independence:  $P_{i^{\wedge}j} \in \varphi_e$ ; (4) Strong independence:  $P_{i^{\wedge}j} \in \varphi_s$ ; (5) Uncertain correlation:  $P_{i^{\wedge}j} \in \varphi_c$ .

Take three sub-events  $E_1$ ,  $E_2$ , and  $E_3$  as an example. Assume epistemic independence between  $E_1$  and  $E_2$  and strong independence between  $E_2$  and  $E_3$ . Lower and upper  $P_{1,1,1}$  and  $P_{2,2,2}$  are obtained by solving the optimization problems below: minimize (maximize)  $P_{1,1,1}$  ( $P_{2,2,2}$ )  
 Subject to:

$$\mathbf{P}_{1^2} \in \Psi_E; \mathbf{P}_{2^3} \in \Psi_S; \mathbf{P} \geq 0; \sum_{i=1}^2 \sum_{j=1}^2 \sum_{l=1}^2 P_{i,j,l} = 1$$

Then lower and upper probabilities for event E are obtained by inserting lower and upper  $P_{1,1,1}$  and  $P_{2,2,2}$  into OR-gate or AND-gate, respectively.

### 3. Application of failure tree analysis based on imprecise probability on risk assessment of tunnel projects

#### 3.1 The application of failure tree analysis based on imprecise probability

Take a toll sub-river project as an example, where the failure of the project (Event E) is caused by two sub-events: technical failure (Event  $E_1$ ) or economical failure (Event  $E_2$ ), which are here assumed to be strongly independent. Technical failure may happen due to the occurrence of at least one of two epistemic ally independent events:

- (1) Total collapse: seawater fills the tunnel (Event  $E_{1,1}$ ), as a result of the occurrence of both too small rock cover (Event  $E_{1,1,1}$ ) and insufficient investigations (Event  $E_{1,1,2}$ ).  $E_{1,1,1}$  and  $E_{1,1,2}$  are here assumed to be linked by uncertain correlation with coefficient  $\rho = [0.6, 0.8]$ ;
- (2) The tunnel cannot be built (Event  $E_{1,2}$ ) because of difficult rock conditions (Event  $E_{1,2,1}$ ) and poor investigation (Event  $E_{1,2,2}$ ) occurring at the same time. Events  $E_{1,2,1}$  and  $E_{1,2,2}$  are assumed to be linked by unknown interaction.

The economical failure is triggered by two strongly independent events: either too small toll revenue (Event  $E_{2,1}$ ) or too high construction and maintenance costs (Event  $E_{2,2}$ ).

The structure of the fault tree is the same as Figure 3.

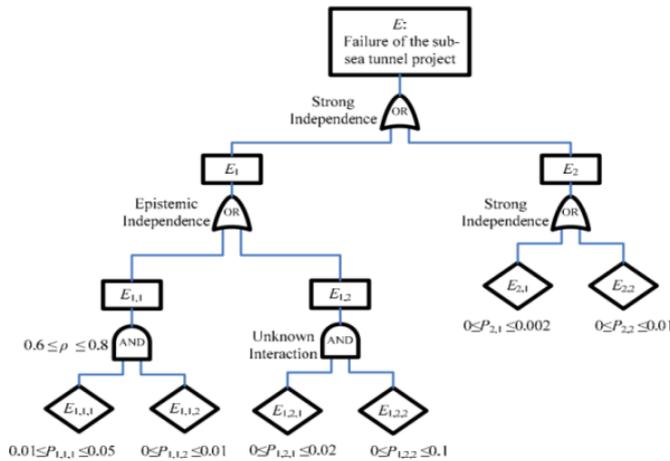


Figure 3: Fault tree analysis for the failure probability of sub-sea tunnel project with imprecise probabilities

First consider the sub-tree for Event  $E_{1,1}$  together with sub-events  $E_{1,1,1}$  and  $E_{1,1,2}$ , and determine the bounds on the occurrence probability of Event  $E_{1,1}$  by solving the followed optimization problems written in terms of the joint distribution P of  $E_{1,1,1}$  and  $E_{1,1,2}$ :

minimize (maximize)  $P_{1,1}$

Subject to:

$$0.01 < P_{1,1,1} < P_{1,1,2} < 0.05; 0 < P_{1,1,1} < P_{2,1} < 0.01; \sum_{i=1}^2 \sum_{j=1}^2 P_{ij} = 1; P_{ij} \geq 0$$

$$E(E_{1,1,1} E_{1,1,2}) \leq E(E_{1,1,1}) E(E_{1,1,2}) + 0.8 \sqrt{D_{1,1,1} D_{1,1,2}}; E(E_{1,1,1}) E(E_{1,1,2}) + 0.6 \sqrt{D_{1,1,1} D_{1,1,2}} \leq E(E_{1,1,1} E_{1,1,2})$$

where

$$E(E_{1,1})=(1 \ 0)\begin{pmatrix} P_{1,1} + P_{1,2} \\ P_{2,1} + P_{2,2} \end{pmatrix} = P_{1,1} + P_{1,2}; \quad E(E_{1,2})=(1 \ 0)\begin{pmatrix} P_{1,1} + P_{2,1} \\ P_{1,2} + P_{2,2} \end{pmatrix} = P_{1,1} + P_{2,1};$$

$$E(E_{1,1}E_{1,2}) = \mathbf{P} \odot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = P_{1,1}; \quad D_{1,1,1} = (P_{1,1} + P_{1,2})(P_{2,1} + P_{2,2}); \quad D_{1,1,2} = (P_{1,1} + P_{2,1})(P_{1,2} + P_{2,2})$$

The extreme values of the occurrence probability of  $E_{1,1}$  are found to be 0.01 and 0.0036. Detailed solutions are listed in Table 1.

Next, calculate the bounds on the probability of Event  $E_{1,2}$ , Solutions detailed in Table 2 show that the upper and lower occurrence probabilities for  $E_{1,2}$  are equal to 0.02 and 0, respectively.

The next step is to determine the extreme values of the occurrence probability of Event  $E_1$ . Replace the probabilities of  $E_{1,1}$  and  $E_{1,2}$  with intervals [0.0036, 0.01] and [0, 0.02], respectively. Since  $E_1$  is connected with  $E_{1,1}$  and  $E_{1,2}$  by an OR-gate, the optimization problems are: Minimize(Maximize) $1-P_{2,2}$

Subject to:

$$0.0036 < P_{1,1} < P_{1,2} < 0.01; \quad 0 < P_{1,1} < P_{2,1} < 0.02; \quad \mathbf{P} \in \Psi_E; \quad \sum_{i=1}^2 \sum_{j=1}^2 P_{i,j} = 1, \quad P_{i,j} \geq 0$$

where matrix P is the joint distribution of  $E_{1,1}$  and  $E_{1,2}$ .

The upper and lower probabilities for  $E_1$  are 0.0298 and 0.0036, respectively.

Table 1: Solutions for the optimization problems for the upper and lower probabilities of Event  $E_1$

	P: joint dist. of $E_{1,1,1}$ and $E_{1,1,2}$	$P(E_{1,1,1})$	$P(E_{1,1,2})$	$P(E_{1,1}) = P_{1,1}$
max	$\begin{pmatrix} 0.01 & 0.0055 \\ 0 & 0.9845 \end{pmatrix}$	0.0155	0.01	0.01
min	$\begin{pmatrix} 0.0036 & 0.0064 \\ 0 & 0.99 \end{pmatrix}$	0.01	0.0036	0.0036

Table 2: Solutions for the optimization problems for the upper and lower probabilities of Event  $E_{1,2}$

	P: joint dist. of $E_{1,2,1}$ and $E_{1,2,2}$	$P(E_{1,2,1})$	$P(E_{1,2,2})$	$P(E_{1,2}) = P_{1,2}$
max	$\begin{pmatrix} 0.02 & 0 \\ 0 & 0.98 \end{pmatrix}$	0.02	0.02	0.02
min	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	0	0	0

Table 3: Solutions for the optimization problems for the upper and lower probabilities of Event  $E_1$

	P: joint dist. of $E_{1,2,1}$ and $E_{1,2,2}$	$P(E_{1,1})$	$P(E_{1,2})$	$P(E_1)$
max	$\begin{pmatrix} 0.0002 & 0.0098 \\ 0.0198 & 0.9702 \end{pmatrix}$	0.01	0.02	0.0298
min	$\begin{pmatrix} 0 & 0.0036 \\ 0 & 0.9964 \end{pmatrix}$	0.0036	0	0.0036

As for the sub-tree of Event  $E_2$ , we determine the bounds on the occurrence probability by first multiplying all the extreme distributions of  $E_{2,1}$  by all extreme probability distributions of  $E_{2,2}$  and then by calculating  $P(E_2) = 1 - P_{2,2}$  on all the extreme joint distributions as shown in Table 4. The upper and lower occurrence probabilities for  $E_2$  are 0.012 and 0, respectively.

Finally, we come to the top event of the fault tree E, i.e. the failure of the sub-sea tunnel project. The extreme values of the occurrence probabilities are achieved by the same procedure as  $E_2$ . Table 5 lists all extreme points with their values of P(E), and the upper and lower occurrence probabilities for E are 0.041 and 0.004, respectively.

Table 4: Solutions for the optimization problems for the upper and lower probabilities of Event E2

Extreme points of $\Psi_{comb}$					$P(E_2)$
	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	
1	0.000	0.000	0.000	1.000	0.000
2	0.000	0.000	0.010	0.990	0.010
3	0.000	0.002	0.000	0.998	0.002
4	0.000	0.002	0.010	0.988	0.012

Table 5: Solutions for the optimization problems for the upper and lower probabilities of Event E

Extreme points of $\Psi_{comb}$					$P(E)$
	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	
1	0.000	0.004	0.000	0.996	0.004
2	0.000	0.030	0.000	0.970	0.030
3	0.000	0.004	0.012	0.984	0.016
4	0.000	0.029	0.012	0.959	0.041

From the results of the risk assessment, it can be found that the result of the event tree analysis method based on imprecise probability is a probability interval. In this case, the decision maker has the right to choose the more appropriate way to deal with the risk according to their own risk preference. Such a result is more reasonable than a single point obtained with the precise probability.

### 3.2 Empirical comparison

Wuhan Metro Line 2 Yangtze River Tunnel is a 1.27 km long and 8.1-meter diameter tunnel under crossing the Yangtze River. Because the tunnel started and ended in the downtown area, the major concerns were environmental concerns. A composed of experts in geotechnical engineering was set up at the beginning of the project to provide and ensure high quality technical solutions, where a risk analysis was conducted by using fault-trees to evaluate the environmental damage due to tunneling.

Figure 4 through Figure 7 show the fault-trees, where the top event is 'the lime trees are damaged due to the tunnelling activities'. It should be noted that all events are assumed to be independent to each other. Interaction noted in Figure4 through Figure 7, such as 'unknown interaction' etc., is applied only when imprecise probabilities is considered when using imprecise probability later. For the events at the bottom of the fault-trees, it is used precise probabilities, which are summarized in the columns under 'precise' in Table 6. Finally, the occurrence probability for the top event is equal to 0.105, which it is thought acceptable. However, the current status of the project showed that it is not a good estimation. The probability might be higher than 0.105 and thus it is not acceptable.

The two columns 'imprecise' heading in Table 6 list all imprecise probabilities which evaluate the uncertainty of the events at the bottom of the fault-trees, where the interval widths are equal to 0.1. As shown in Figure 4 through Figure 7, the interaction between events is assumed to be 'unknown interaction', 'independence', or 'uncertain correlation' with the correlation coefficient  $\rho \in [0.5, 0.8]$ .

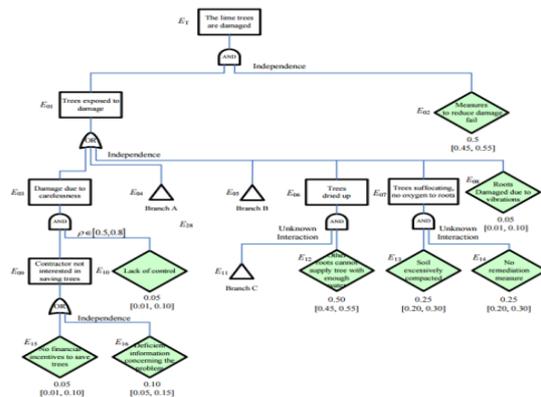


Figure 4: Fault tree for damage to trees due to tunnelling activities

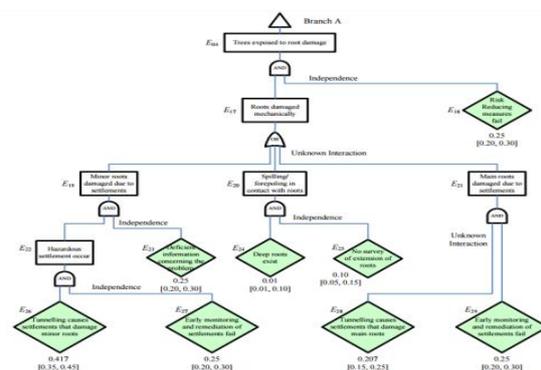


Figure 5: Fault tree for Branch A (roots damaged mechanically)

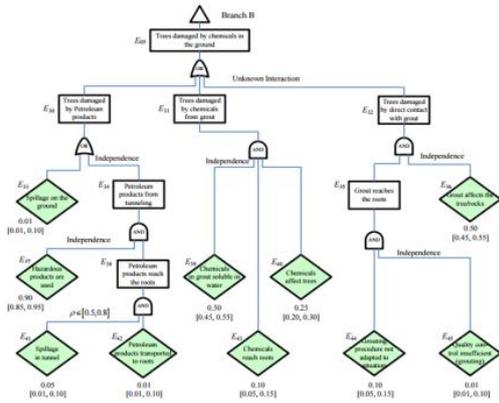


Figure 6: Fault tree for Branch B (Trees damaged by chemical in the ground)

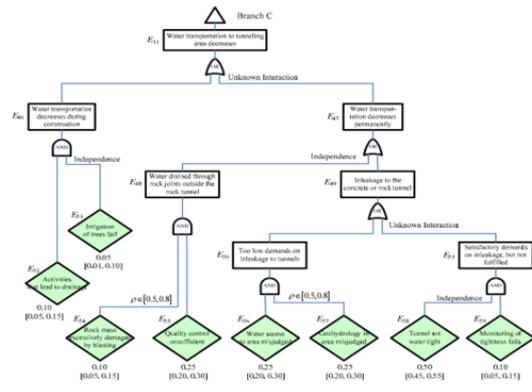


Figure 7: Fault tree for Branch C (water transportation to tunneling area decrease)

4. Conclusion

In this paper, the imprecise probability had been introduced into the process of risk assessment to deal with the problems caused by the uncertain information, and the Failure Event Tree Analysis based on imprecise probability had been also established. Finally, through an example of tunnel project, it is found that the event-tree analysis method based on imprecise probability could evaluate the risk of the tunnel project, which result is a probability interval. The result provides more choices for the decision-maker to choose the way to deal with risk, and the result is more reasonable.

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