Robust Controller Design for a Heat Exchanger

Anna Vasičkaninová*, Monika Bakošová, Luboš Čirka, Martin Kalúz

Slovak University of Technology in Bratislava, Faculty of Chemical and Food Technology, Institute of Information Engineering, Automation, and Mathematics, Radiánskeho 9, 812 37 Bratislava, Slovak Republic
anna.vasickaninova@stuba.sk

The aim of the paper is to show the benefits of two advanced control strategies in heat exchanger control. Two robust control techniques were used for controller design: $H_2$ control and $\mu$-synthesis. $H_\infty$ optimal control is a frequency-domain optimization and design theory that was developed in response to the need for a design procedure that explicitly addresses questions of modeling errors. The $H_\infty$ robust controller design theory deals with defined uncertainties, like parametric uncertainty and performance specified as weight filters. The $H_\infty$ theory has some weaknesses; for instance, it does not consider the uncertainty structure. The $\mu$-synthesis theory is a further development of the $H_\infty$ control where the uncertainty structure is considered in the design. The designed controllers were verified in laboratory conditions on a real-time control of a laboratory heat exchanger.

1. Introduction

Heat exchangers belong to the often used equipment in the chemical and process industry and they are characterised by high energy demands. Many factors enter into the design of heat exchangers, including thermal analysis, weight, size, structural strength, pressure drop and cost. The heat exchanger modelling framework has been described and demonstrated in (Skaguen et al., 2013). The main purpose of (Daróczy et al., 2014) is to illustrate and analyse a heat exchanger arrangement problem in its most general form. In (Manassaldi et al., 2014) a mathematical model for the optimal design of air cooled heat exchangers is described.

Control of heat exchangers is a complex process due to the nonlinear behaviour and complexity caused by many phenomena such as leakage, friction, temperature dependent flow properties, contact resistance, unknown fluid properties. Various control strategies were developed for overcoming all mentioned problems. Simulation of a thermodynamic model of an earth-air heat exchanger was done and used along with a PID controller to estimate savings in energy consumption in (Diaz-Mendez et al., 2014). The robust model predictive control represents one of approaches that enable to design effective control algorithms for optimization of the control performance as well as to take process uncertainty into account (Bakošová and Oravec, 2013). The possibility to implement various robust model predictive control strategies for control of the heat exchanger network with uncertainty is demonstrated in (Oravec et al., 2015). In (Shi and You, 2015), a robust optimisation approach has been developed to handle the uncertainty in integrated planning, scheduling and dynamic optimisation for continuous manufacturing processes. In Veselý (2013), a survey of robust control design procedure is given. In Bell et al. (2015), a novel and robust solution approach is presented that can be used to predict the steady-state thermal heat transfer rate for counter flow heat exchangers. There exist various solutions also of the standard $H_\infty$ problem. The mixed $H_2/H_\infty$ performance criterion provides an interesting measure for the controller evaluation. The theoretic motivation for the mixed $H_2/H_\infty$ control problem has been discussed in Kwakernaak (2002). The goal of Zarabadipour et al. (2011) is to design a reduced order robust controller based on the balanced realization technique. The simulation results show that the reduced order controller design with applied $H_2/H_\infty$ has good results in frequency and time domain. In Ganji et al. (2013), different conventional and intelligent controllers are used that are implemented with a clear objective to control the outlet fluid temperature of the shell-and-tube heat exchanger system. For the dynamic
system with time varying characteristic and parametric uncertainties, a sliding mode controller is developed and an optimal \( H_\infty \) controller is designed based on \( \mu \)-synthesis with DK-iteration algorithm in Moradi et al. (2012). In De Souza et al. (2014) the problems of robust stability analysis and robust control of linear discrete-time periodic systems is investigated. The paper of Vasičkaninová and Bakošová (2015) deals with design and application of robust controllers for a shell-and-tube heat exchanger.

2. Controller design

2.1 \( H_\infty \) control

Various techniques are available for the design of \( H_\infty \)-controllers. Mixed sensitivity is the name given to the transfer function shaping problems in which the sensitivity function \( S = (I + GK)^{-1} \) is shaped along with one or more other closed-loop transfer functions such as \( KS \) or the complementary sensitivity function \( T = I - S \) in a typical one degree-of-freedom configuration, where \( G \) is the plant transfer function and \( K \) is the transfer function of the (sub-) optimal controller to be found (Skogestad and Postlethwaite, 2005). The shaping of multivariable transfer functions is based on the idea that a satisfactory definition of gain (range of gain) for a matrix transfer function is given by the singular values \( \sigma \) of the transfer function. Hence, the classical loop-shaping ideas of feedback design can be generalized to multivariable systems. In addition to the requirement that \( K \) stabilizes \( G \), the closed-loop objectives are as follows:

- For disturbance rejection make \( \bar{\sigma}(S) \) small,
- For noise attenuation make \( \bar{\sigma}(T) \) small,
- For reference tracking make \( \bar{\sigma}(T) = \sigma(T) = 1 \),
- For input usage (control energy) reduction make \( \bar{\sigma}(KS) \) small,
- For robust stability in the presence of an additive perturbation \( G_P = G + \Delta \), make \( \bar{\sigma}(KS) \) small,
- For robust stability in the presence of a multiplicative output perturbation \( G_P = (I + \Delta)G \), make \( \bar{\sigma}(T) \) small,

where \( \bar{\sigma}(A) \) is the maximum singular value of \( A \) and \( \sigma(A) \) is the minimum singular value of \( A \). It is known that the robust controller is designed to minimize the \( H_\infty \)-norm of the plant. Three weight functions are added to the control system for loop shaping (Bansal and Sharma, 2013). The classical feedback control system structure with weighting is shown in Figure 1.

![Figure 1: Control system for the synthesis of \( H_\infty \) controller](image)

The objective is to find the controller \( K(s) \) in the form of a rational function and to make the closed loop system stable satisfying expression \( \| N(G,K) \|_\infty < \gamma \) for a given \( \gamma > 0 \).

\[
\| N(G,K) \|_\infty = \begin{bmatrix}
W_1S \\
W_2KS \\
W_3T
\end{bmatrix}
\]

The user-defined weighting functions \( W_1, W_2, W_3 \) bound the largest singular values of the closed-loop transfer functions \( S \) (for performance), \( KS \) (to penalize large inputs) and \( T \) (for robustness and to avoid sensitivity to noise), respectively (Skogestad and Postlethwaite, 2005).

2.2 \( \mu \)-synthesis with DK-iteration

There is no analytical method to calculate a \( \mu \)-optimal controller. However, a numerical method for complex perturbations known as DK-iteration can be used (Balas et al., 1998). The generalized open-loop representation of the configuration in Figure 2 can be given by Eq(2) (Griffin and Fleming, 2003):
where the input and output vectors for this configuration contain the inputs and outputs relating to the input and output perturbations $u_\Delta$ and $y_\Delta$. Uncertainty at the plant input and performance requirements at the system output are described by weighting functions $W_u$ and $W_p$, respectively. $K$ is the transfer function of the controller and $G_d$ is disturbance model.

The closed-loop interconnection structure $N$, which incorporates $P$ and $K$ is given

$$N = \begin{bmatrix} -W_uKGS & -W_uKGdS \\ WpGS & WpGdS \end{bmatrix}$$

Figure 2: Closed-loop system with uncertainty and performance weighting

$\mu$-synthesis with DK-iteration is based on the upper bound:

$$\mu(N) \leq \min_D \| DND^{-1} \|$$

A scaling matrix $D$ is chosen such that it commutes with $\Delta$, the plant perturbation, i.e. $D\Delta = \Delta D$. The synthesis problem is then to find the controller such that

$$\min_K \left\{ \min_D \left( \| DN(K)D^{-1} \|_\infty \right) \right\}$$

by alternating the minimization with respect to $K$ and $D$ (keeping the other fixed). The iteration goes as follows (Skogestad and Postlethwaite, 1996):

- **$K$-step.** Synthesize an $H_\infty$-controller for the scaled problem with fixed $D(s)$.

  $$\min_K \left( \| DN(K)D^{-1} \|_\infty \right)$$

- **$D$-step.** Find $D(j\omega)$ to minimize at each frequency

  $$\sigma(D(j\omega)ND^{-1}(j\omega))$$

- **Fit the magnitude of each element of $D(j\omega)$ to a stable and minimum-phase transfer function $D(s)$.** Go to $K$-step.

Continue iteration until $\| DND^{-1} \|_\infty < 1$ or until the norm no longer decreases.

3. Description of the multifunction process control teaching system

The described approach was verified in a real-time control of the laboratory heat exchanger realized on the Multifunction Process Control Teaching System (MPCTS) the Armfield PCT 40 MPCTS (2007). The MPCTS is designed especially for teaching of a wide range of technological and chemical processes. The PCT40 is connected to the computer through data acquisition and control card, and each actuator (pumps, valves, heater, etc.) can be controlled directly from MATLAB software. The computer also displays the readings from various measurement sensors. The through-flow heat exchanger is a part of the MPCTS (Figure 3).
The heat exchanger (Figure 4) consists of a cylindrical vessel with fluid inlet and outlet, the heating element, internal cooling system and a set of sensors. The inputs are the heater power, handled by Solid State Relay, and the water flow rate $q_c$ in the cooling coil. The controlled variable is the temperature measured by the sensor T1. A constant amount of inlet and outlet water, which ensures a constant water level in the tank, is realized by equal adjustment of flow rate $q_c$ by means of two peristaltic pumps.

The objective of the work was to identify the controlled process, to determine the controller parameters and to evaluate the controller performance using these parameters. For the identification, a series of the positive and negative step changes of the input variable (opening of the valve) were generated, and the step responses of the outlet temperature were measured. According to these step changes, the nominal model was identified using the Strejc method (Mikleš and Fikar, 2007) in the form of the first order transfer function with the gain $Z = -0.53 \, ^\circ\mathrm{C} \, \%^{-1}$ and the time constant $\tau = 258 \, \mathrm{s}$.

The PI controller parameters were tuned for the model with the nominal values of the identified parameters using pole placement method. The enumerated PI controller parameters are $k_p = -8.8935$, $t_i = 164.9245 \, \mathrm{s}$ for poles $s_1, s_2 = -0.01$ (Mikleš and Fikar, 2007). In Figure 5 the control input opening of the valve and the PI control of the output temperature are shown.

In Figure 6 the control input and the $H_\infty$-control of the output temperature are shown. The $H_\infty$-controller for the heat exchanger was calculated in the form

$$K(s) = \frac{-12.73 \, s - 0.04468}{s^2 + 0.2489s + 0.0002}$$

(8)
The $\mu$-synthesis with $DK$-iteration controller for the heat exchanger was calculated in the form

$$K(s) = \frac{-87560s^3 - 127720s^2 - 891.6s - 1.56}{s^4 + 179.9s^3 + 261.8s^2 + 1.695s + 0.002735}$$  \hspace{1cm} (9)$$

In Figure 7 the control input and the $\mu$-synthesis control of the output temperature are shown.

The comparison of the proposed controllers was made using IAE integral performance index. The IAE values are given in Table 1. The smaller value of IAE is assured using the $\mu$-synthesis control, achievement of setpoint value is fast but the disadvantage is the higher vibration of the input and output variables.

<table>
<thead>
<tr>
<th>Controller</th>
<th>IAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI control</td>
<td>$2.8148 \times 10^5$</td>
</tr>
<tr>
<td>$H_\infty$ control</td>
<td>$2.8091 \times 10^5$</td>
</tr>
<tr>
<td>$\mu$-synthesis control</td>
<td>$2.8019 \times 10^5$</td>
</tr>
</tbody>
</table>

4. Conclusion

Two methods were used for robust controller design, $H_\infty$-controller and $\mu$-synthesis with $DK$-iteration controller. The presented approach was verified on the real laboratory system MPCTS - the Armfield PCT 40. The
control tests executed on the laboratory model gave satisfactory results. As the laboratory heat exchanger simulates an industrial process, it was proved that the investigated advanced control design methods could be successfully used to control industrial heat exchangers.

Acknowledgment

The authors gratefully acknowledge the contribution of the Scientific Grant Agency of the Slovak Republic under the grant 1/0112/16.

Reference


Bansal A., Sharma V., 2013, Design and Analysis of Robust H∞ Controller. Control Theory and Informatics, 3(2), 7-14, ISSN 2225-0492.


Shi H., You F., 2015, Robust optimisation for integrated planning, scheduling and dynamic optimisation of continuous processes under uncertainty, Chemical Engineering Transactions, 45, 859-864 doi:10.3303/CET1545144.


