

Chaos Control for Permanent Magnet Synchronous Motor with Disturbance

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This paper research how to chaos control in permanent magnet synchronous motor (PMSM) with disturbance base on adaptive back stepping of error compensation. This way can obtain smooth effect of chaos control and can remove oscillation in chaos control. Numerical simulations show the effectiveness of the theoretical analysis.

1. Introduction

With the development of chaos theory, there are many methods to control chaotic system (Ma et al., 2012a; 2012b, Winsor, 1995). There are linear and nonlinear generalized synchronization (Li et al., 2012), generalized synchronization (Wang and Meng, 2007), projective synchronization (Wang and He, 2008) and the back stepping nonlinear control, chaotic control of the coupled Logistic map (Wang and Wang, 2008), three methods of anti-synchronization of hyperchaotic chen system (Wang and Wang, 2007), hybrid control (Elmas and Ustun, 2008) and passivity control (Qi et al., 2005).

Many methods have been applied to control or suppress chaos in PMSM. For example (Kuo et al., 2007) raised controller base on fuzzy slide-mode to control chaotic PMSM. (Li et al., 2010) raised impulsive control method to control chaotic PMSM with uncertainties. (Chang, 2010) rose synchronous and control chaotic PMSM. (Yu et al., 2011) rose back stepping control way to control the chaotic PMSM system. (Chang et al., 2011) raised dither signal to control the chaotic PMSM system. However, these methods appear oscillation in chaos control which is not satisfying results.

In control chaos, PMSM appear oscillation due to unknown effect of error dynamics, PMSM oscillation show variables is not stable and control process is not stable, which effect control result. Adaptive back stepping methods is a kind of adaptive nonlinear control method and can make glably stability and good control results. For suppressing oscillation, we apply adaptive back stepping of error compensation to control chaotic PMSM. We add an error compensation item to every step virtual control design for compensate the effect of unknown error dynamics so that obtain more stable control process. This scheme can eliminate oscillation in course of chaos control. This scheme can achieve parameter identification. Finally, the simulation states the effectiveness of theoretical analysis.

This paper is organized as follows. In the next section, we analyse the dynamics analysis of PMSM system. In section 3, we introduce adaptive back stepping of error compensation. In section 4, the numerical simulations test the effectiveness of theoretical analysis. Finally, some conclusions are drawn in section 5.

2. PMSM system

The model of PMSM is showed as follows (Chang et al., 2011),

$$\begin{cases} \frac{di_d}{dt} = (u_d - R_1 + \omega L_q i_q) / L_d \\ \frac{di_q}{dt} = (u_q - R_1 i_q - \omega L_d i_d - \omega \psi_f) / L_q \\ \frac{d\omega}{dt} = [n_p \psi_f i_q + n_p (L_d - L_q) i_d i_q - T_L - \beta \omega] / J \end{cases} \quad (1)$$

where i_d , i_q and ω are variables, i_q is q -axis stator current, i_d is d -axis stator current, and ω is rotor angular speed. u_d is d -axis external voltage, u_q is q -axis external voltage, T_L are external torque; L_d is d -axis stator inductance, L_q is q -axis stator inductance. ψ_f is permanent magnet flux, R_1 is stator winding resistance, β is the viscous damping coefficient, J is rotor rotational inertia, n_p is the number of pole-pairs, R_1 , β , J , L_q , L_d , T_L are all positive. $X = \tilde{\lambda}, x, t = \tilde{\tau}, t$.

The system given by (1) can be convertible into non-dimensional zed form which can be expressed as follows:

$$\begin{cases} \dot{\tilde{i}}_d = -\frac{L_q}{L_d} \tilde{i}_d + \tilde{\omega} \tilde{i}_q + \tilde{u}_d \\ \dot{\tilde{i}}_q = -\tilde{i}_q - \omega \tilde{i}_d + \gamma \tilde{\omega} + \tilde{u}_q \\ \dot{\tilde{\omega}} = \sigma (\tilde{i}_q - \tilde{\omega}) + \xi \tilde{i}_d \tilde{i}_q - \tilde{T} \end{cases} \quad (2)$$

Where $\gamma = n_p \psi_f / R_1 \beta$, $\sigma = L_q \beta / R_1 J$, $u_q = n_p L_q \psi_f u_q / R_1^2 \beta$, $T_L = L_q^2 T_L / R_1^2 J$, $u_d = n_p L_q \psi_f u_d / R_1^2 \beta$, $\xi = L_q \beta^2 (L_d - L_q) / L_d J n_p \psi_f$, $n_p = 1$.

The system (2) is smooth air-gap when $L_d = L_q$. To show conveniently, assuming $i_d = \tilde{i}_d$, $i_q = \tilde{i}_q$, $\omega = \tilde{\omega}$, $u_d = \tilde{u}_d$, $u_q = \tilde{u}_q$. The system given by (2) can be simplified as follows:

$$\begin{cases} \dot{i}_d = -i_d + \omega i_d + u_d \\ \dot{i}_q = -i_q - \omega i_d + \gamma \omega + u_q \\ \dot{\omega} = \sigma (i_q - \omega) - T_L \end{cases} \quad (3)$$

At present, research the system given by (3) without external force which means PMSM no-load running or power disappeared suddenly, namely, $u_d = u_q = T_L = 0$. Then the system given by (3) can be expressed as follows:

$$\begin{cases} \dot{x}_1 = -x_1 + x_3 x_1 \\ \dot{x}_2 = -x_2 - x_3 x_1 + \gamma x_3 \\ \dot{x}_3 = \sigma (x_2 - x_3) \end{cases} \quad (4)$$

where x_1 stands for i_d , x_2 stands for i_q , x_3 stands for ω .

For the system given by (4)

$$\Delta V = \frac{\partial d\omega}{\partial \omega} + \frac{\partial di_q}{\partial i_q} + \frac{\partial di_d}{\partial i_d} = -(\sigma + 2) \quad (5)$$

Due to $\sigma > 0, \Delta V < 0$. So the system given by (4) is a dissipative system.

The system given by (4) have three equilibrium points: $(0, 0, 0)$, $(\gamma - 1, -(\gamma - 1)^{1/2}, -(\gamma - 1)^{1/2})$, $(\gamma - 1, (\gamma - 1)^{1/2}, (\gamma - 1)^{1/2})$ in theory.

3. Adaptive back stepping of error compensation

Transformations of system given by (4) variables as follows,

$$Y = A^* X, \mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \text{ so } \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix}.$$

System given by (4) variables are changed base on above transformations, so system with disturbance given by (4) can be expressed as follows,

$$\begin{cases} \dot{y}_1 = \sigma(y_2 - y_1) \\ \dot{y}_2 = \gamma y_1 - y_2 - y_1 y_3 \\ \dot{y}_3 = y_1 y_3 - y_3 + \xi \end{cases} \quad (6)$$

where ξ is disturbance term . In order to chaos control in system given by (6), the controller u is added to the third equation of system given by (6), system given by (6) with controlled can be expressed as follows,

$$\begin{cases} \dot{y}_1 = \sigma(y_2 - y_1) \\ \dot{y}_2 = \gamma y_1 - y_2 - y_1 y_3 \\ \dot{y}_3 = y_1 y_3 - y_3 + \xi + u \end{cases} \quad (7)$$

Theorem 1: The controller u satisfy,

$$\begin{aligned} u = & -e_1 e_3 - e_1 \alpha_2 + (e_3 + \alpha_2) + [(1 - p_2)^2 + p_1(1 - p_1)] \hat{\sigma} e_3 + \left\{ 1 + \hat{\sigma} [(1 - p_2)^2 + p_1(1 - p_1)] \right\} e_1 e_2 \\ & + \left\{ 1 + \hat{\sigma} [(1 - p_2)^2 + p_1(1 - p_1)] \right\} \left\{ \hat{\sigma} [(1 - p_2)^2 + p_1(1 - p_1)] (e_3 - 1) - \hat{\gamma} \right\} + (1 - p_2) p_3 e_3 \times \\ & \left\{ 1 + \hat{\sigma} [(1 - p_2)^2 + p_1(1 - p_1)] \right\} - \xi, \end{aligned} \quad (8)$$

And parameters adaptive law satisfy,

$$\begin{cases} \dot{\hat{\sigma}} = \frac{p_1(1 - p_1)}{1 - p_2} e_1 e_2 + (1 - p_2) e_1 e_2 \\ \dot{\hat{\gamma}} = \frac{1}{1 - p_2} e_1 e_2 \end{cases} \quad (9)$$

Then system given by (7) can realize adaptive back stepping control.

Proof: Defining three error variables:

$$\begin{cases} e_1 = y_1 - 0 \\ e_2 = y_2 - \alpha_1 \\ e_3 = y_3 - \alpha_2 \end{cases} \quad (10)$$

Where α_1 and α_2 are virtual control variable, adaptive back stepping of error compensation control include the following three steps.

Step 1: The time derivative of e_1 is,

$$\dot{e}_1 = \dot{y}_1 = \sigma(y_2 - y_1) = \sigma(\alpha_1 + e_2 - e_1) = -\sigma e_1 + \sigma e_2 + \sigma \alpha_1 \quad (11)$$

Define the Lyapunov function,

$$V_1 = \frac{1}{2} e_1^2 \quad (12)$$

Then the derivative of V_1 is

$$\dot{V}_1 = e_1 \dot{e}_1 = e_1 (-\sigma e_1 + \sigma e_2 + \sigma \alpha_1) = -\sigma e_1^2 + \sigma e_1 e_2 + \sigma \alpha_1 e_1 \quad (13)$$

The virtual control variable α_1 is defined as follows,

$$\alpha_1 = p_1 e_1 - p_2 e_2 \quad (14)$$

Where p_1 and p_2 are control parameters, $0 \leq p_1 < 1$ and $0 \leq p_2 < 1$. Substitute Eq. (14) in Eq. (13), we obtain,

$$\dot{V}_1 = -\sigma e_1^2 + \sigma e_1 e_2 + \sigma e_1 (p_1 e_1 - p_2 e_2) = -\sigma(1-p_1)e_1^2 + \sigma(1-p_2)e_1 e_2 \quad (15)$$

Step 2: The time derivative of e_2 , we obtain

$$\dot{e}_2 = \dot{y}_2 - \dot{\alpha}_1 = \gamma y_1 - y_2 - y_1 y_3 - p_1 \dot{e}_1 + p_2 \dot{e}_2 = \gamma e_1 - (e_2 + \alpha_1) - e_1(e_3 + \alpha_2) - p_1 \dot{e}_1 + p_2 \dot{e}_2, \quad (16)$$

substitute Eq. (11) in Eq. (16), we obtain

$$\begin{aligned} \dot{e}_2 &= \frac{1}{1-p_2} [(-\alpha_2 - p_1)e_1 + (-1 + p_2 + \sigma p_1 p_2 - \sigma p_1)e_2 - e_1 e_3 + \sigma p_1 e_1 - \sigma p_1^2 e_1 + \gamma e_1] \\ &= \frac{1}{1-p_2} [(-\alpha_2 - p_1)e_1 - (1-p_2)(\sigma p_1 + 1)e_2 - e_1 e_3 + p_1(1-p_1)\hat{\sigma}e_1 + \hat{\gamma}e_1 \\ &\quad - p_1(1-p_1)\tilde{\sigma}e_1 - \tilde{\gamma}e_1]. \end{aligned} \quad (17)$$

Where $\hat{\sigma}$ and $\hat{\gamma}$ are σ and γ estimates respectively. $\tilde{\sigma} = \hat{\sigma} - \sigma$, $\tilde{\gamma} = \hat{\gamma} - \gamma$, $\tilde{\sigma}$ and $\tilde{\gamma}$ are parameters estimation error.

Defining the Lyapunov function as follows,

$$V_2 = V_1 + (e_2^2 + \tilde{\sigma}^2 + \tilde{\gamma}^2)/2, \quad (18)$$

The time derivative of V_2 , we obtain

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + e_2 \dot{e}_2 + \tilde{\sigma} \dot{\tilde{\sigma}} + \tilde{\gamma} \dot{\tilde{\gamma}} \\ &= -\sigma(1-p_1)e_1^2 - \frac{e_1 e_2 e_3}{1-p_2} - (\sigma p_1 + 1)e_2^2 + \tilde{\sigma}[\dot{\tilde{\sigma}} - (1-p_2)e_1 e_2 - \frac{p_1(1-p_1)}{1-p_2}e_1 e_2] \\ &\quad + \tilde{\gamma}(\dot{\tilde{\gamma}} - \frac{e_1 e_2}{1-p_2}) - \frac{e_3}{1-p_2} \left\{ \alpha_2 - \hat{\sigma}[(1-p_2)^2 + p_1(1-p_1)] - \tilde{\gamma} \right\}, \end{aligned} \quad (19)$$

A virtual variable α_2 is defined as,

$$\alpha_2 = \hat{\sigma}[(1-p_2)^2 + p_1(1-p_1)]e_3 \quad (20)$$

Where $p_3 \in \mathbb{R}$, p_3 is a control parameter, substitute Eq. (9) and Eq. (20) in Eq. (19), we obtain,

$$\dot{V}_2 = -\sigma(1-p_1)e_1^2 - \frac{e_1 e_2 e_3}{1-p_2} - (\sigma p_1 + 1)e_2^2 - \frac{e_3}{1-p_2} \left\{ \hat{\sigma}[(1-p_2)^2 + p_1(1-p_1)](e_3 - 1) - \tilde{\gamma} \right\} \quad (21)$$

Step 3: The time derivative of e_3 , we obtain

$$\begin{aligned} \dot{e}_3 &= u + \xi + e_1(e_3 + \alpha_2) - (e_3 + \alpha_2) - \dot{\alpha}_2 \\ &= u + \xi + e_1 e_3 + e_1 \alpha_2 - (e_3 + \alpha_2) - [(1-p_2)^2 + p_1(1-p_1)]\hat{\sigma}e_3 \\ &\quad - [(1-p_2)^2 + p_1(1-p_1)]\hat{\sigma}\dot{e}_3. \end{aligned} \quad (22)$$

Eq. (22) is rewritten as Eq. (23),

$$\dot{e}_3 = \frac{u + \xi + e_1(e_3 + \alpha_2) - (e_3 + \alpha_2) - [(1-p_2)^2 + p_1(1-p_1)]\hat{\sigma}e_3}{1 + [(1-p_2)^2 + p_1(1-p_1)]\hat{\sigma}}.$$

Defining the Lyapunov function is,

$$V_3 = V_2 + \frac{1}{2(1-p_2)} e_3^2. \quad (23)$$

The derivative of V_3 , we get Eq.(24),

$$\begin{aligned} \dot{V}_3 &= \dot{V}_2 + \frac{1}{1-p_2} e_3 \dot{e}_3 \\ &= -\sigma(1-p_1)e_1^2 - (\sigma p_1 + 1)e_2^2 - \frac{e_1 e_2 e_3}{1-p_2} - \frac{e_3}{(1-p_2)} \left\{ \hat{\sigma}[(1-p_2)^2 + p_1(1-p_1)](e_3 - 1) - \hat{\gamma} \right\} \\ &\quad + \frac{e_3 \left\{ u + e_1 e_3 + e_1 \alpha_2 - (e_3 + \alpha_2) - [(1-p_2)^2 + p_1(1-p_1)] \hat{\sigma} e_3 \right\}}{(1-p_2) \left\{ 1 + \hat{\sigma}[(1-p_2)^2 + p_1(1-p_1)] \right\}}, \end{aligned} \tag{24}$$

where $p_3 > 0$.

Substitute Eq. (8) in Eq. (24), we obtain $\dot{V}_3 = -\sigma(1-P_1)e_1^2 - (\sigma P_1 + 1)e_2^2 - p_3 e_3^2$.

Since $V_3 \leq 0$, we have $e_1, e_2, e_3 \rightarrow 0$ as $t \rightarrow \infty$, p_1, p_2 and p_3 are chosen suitable numerical, $\alpha_1 \rightarrow 0$ and $\alpha_2 \rightarrow 0$ as $t \rightarrow \infty$, $(y_1, y_2, y_3) \rightarrow (0, 0, 0)$.

4. Numerical simulations

The initial conditions of system given by (7) are chosen as follows,

$$\gamma = 20, \sigma = 2, y_1(0) = 8, y_2(0) = 8, y_3(0) = 12, \hat{\gamma} = \hat{\sigma} = 10, p_1 = 0.5, p_2 = 0.2, p_3 = 1, \xi = \sin \theta.$$

The simulation results are illustrated in Figures 1, 2 and 3. Figure 1 shows that the system given by (7) without control states, y_1, y_2, y_3 , occur oscillation. And the system given by (7) without control is chaos. Figure 2 shows that the system given by (7) with control states, y_1, y_2, y_3 , tend to stable without occurring oscillation.

Figure 3 shows that the estimated values of parameters $\hat{\gamma}$ and $\hat{\sigma}$ converge to $\hat{\gamma} = -25$ and $\hat{\sigma} = 4$ as $t \rightarrow \infty$, respectively.

It can be observed that adaptive back stepping of error compensation can avoid oscillation in chaos control.

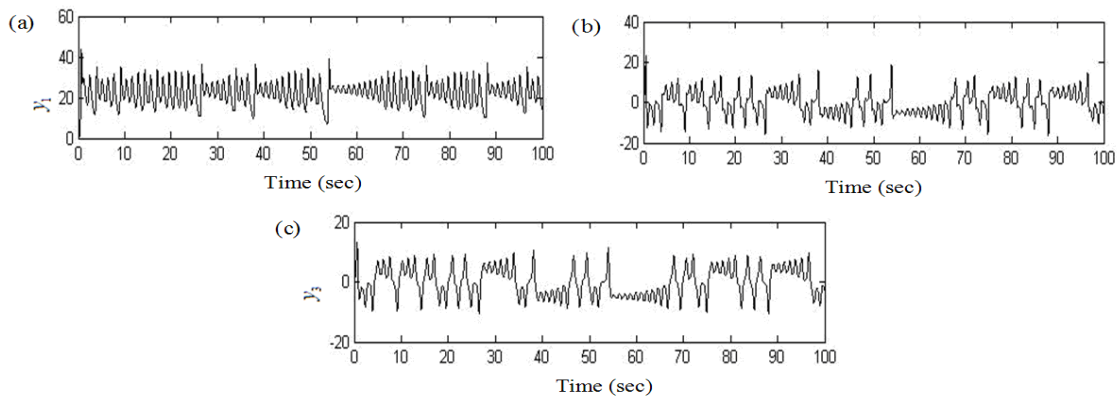


Figure 1: Trajectories of system given by (7) states without control. (a) Trajectory of state y_1 . (b) Trajectory of state y_2 . (c) Trajectory of state y_3 .

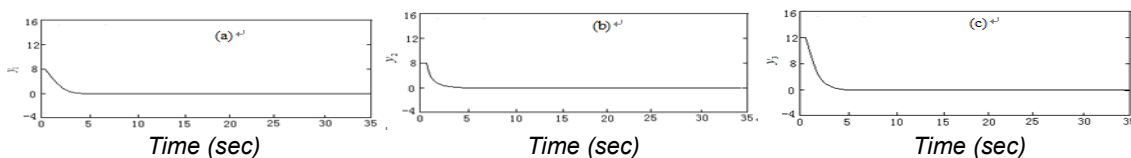


Figure 2: Trajectories of system given by (7) states with control (a) Trajectory of state y_1 (b) Trajectory of state y_2 (c) Trajectory of state y_3

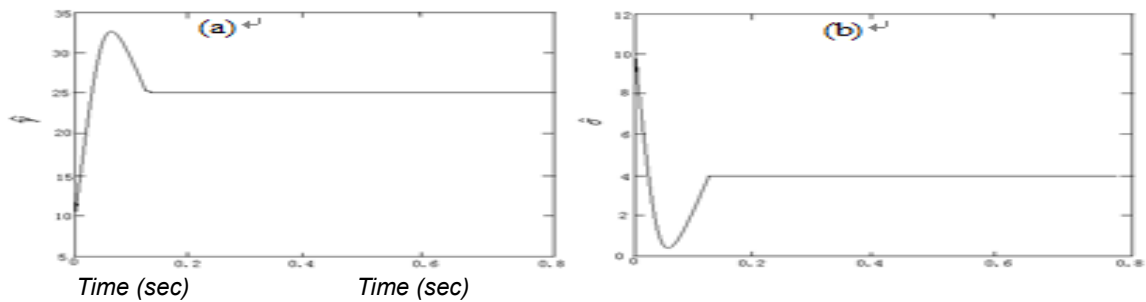


Figure 3: Identification results of $\hat{\gamma}$ and $\hat{\sigma}$ (a) Identification result of $\hat{\gamma}$ (b) Identification result of $\hat{\sigma}$

5. Conclusions

This paper come forward adaptive back stepping of error compensation to control chaotic PMSM. In order to control chaotic PMSM and avoid oscillation during chaos control. An error compensation item is developed to control chaotic PMSM, which can get smooth effect of control.

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