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Lateral Transshipment Model in the Same Echelon Storage Sites

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In case of one-product, one-period, normal distribution demand, emergency lateral transshipment instant arrival, four transshipment rules in the same echelon storage sites are discussed in this paper. They are one-time and full-sharing transshipment, one-time and partial-sharing transshipment, multiple-time and full-sharing transshipment, and multiple-time and partial-sharing transshipment. A total cost model is constituted on the factors of transshipment cost and shortage cost. Numerical experiments show that multiple-time transshipment can avoid the shortage risk of supply location; partial-sharing transshipment can drive down the shortage risk of supply location; decision of transshipment rule mostly depends on unit cost of transshipment, unit cost of shortage, and distance between the storage sites.

1. Introduction

Starting from the 1970s, researchers did a lot of theoretical study on the lateral transshipment in the same echelon based on the assumption: there is no safety storage in every storage site, and transshipment will be carried from the available site to the shortage site in one certain rule. Lateral transshipment is based on the theory of risk pooling effect or shared inventory in order to realize centralized management and customer service. Lateral transshipment is an effective tool to adjust the balance between demand and inventory level. Due to the risk-pooling effect, the total cost will be decreased or the customer satisfaction level will be increased, when the increased cost of lateral transshipment is less than the cost of safety stock or emergent replenishment. However, the cost of lateral transshipment is highly dependent on its rules.

2. Research Review

A comprehensive review of current researches is listed as follows.

2.1 Rules and costs of transshipment

(Das, 1975) investigated the rules and costs of transshipment between two locations based on random customer demand and periodical inventory contral policy, and then discussed the applicable ordering strategy considering the availability of transshipment. (Karmarkar and Patel, 1977) studied the non-directional transshipment among several locations. (Kukreja and Schmidt, 2005) considering the change of demand and lead time, formulated a transshipment model with the assumption of full-sharing transshipment, lateral transshipment instant arrival, and transshipment cost relevant with transshipment frequency. (Yu and Liu, 2013) studied the transshipment between online stores and brick-and-mortar retailers, and concluded that the optimum level of inventory increases with the increase of transshipment cost. When the cost of transshipment is moderate, the inventory of online stores with transshipment is lower than that without transshipment; while the inventory of brick-and mortor retailers is reverse.

2.2 Dynamic transshipment

(Robinson, 1990) studied the dynamic and random storage problem when transshipment happens among several storage locations, and constructed a model to optimize the ordering and transhipping strategy in order to minimize the total expected cost. (Man, 2012) proposed a cause-effect loop diagram for transhipping relief

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material reservation, simulated with system dynamics, and discovered the implementation effect of the same inventory policy depends on the demand probability, unit transshipment cost and purchasing cost prior to the disaster. (Donmez and Turkay, 2013) constructed a mixed-integer linear programming model for the design of reverse logistics network that includes collection, sorting, export, recycling and disposal of waste batteries at a landfill area.

2.3 Transshipment in different inventory policies

(Xu et al., 2003) studied the transhipment problem assuming fixed ordering cost, independent (Q, R) inventory strategy and partial-sharing inventory, and subsequently analyzed the impact of transshipment on probability of non-shortage and demand satisfaction level. (Hu et al., 2005) by constructing the dynamic programming algorithm, minimized the cost of inventory and transshipment based on (s, S) inventory strategy, and studied how the transhipment policy is influenced by transhipment cost, holding cost and shortage cost. (Huo and Li, 2007) established a batch ordering model for transhipping spare parts among multiple locations in the same echelon, and calculated the probabilities of demand satisfaction, transshipment and shortage by configuring three factors: inventory level, required lead time and net inventory.

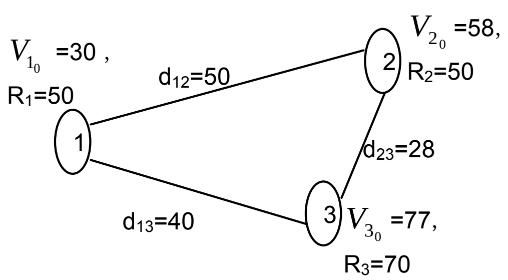
In summary, the current studies, first of all, defined the amount of transshipment relying either on empirical data or on one-time transshipment rule, which may lead to the shortage of supply location after transshipment, therefore this is not an optimized strategy in the long run. Secondly, to calculate the transshipment cost, only the quantity was taken into consideration, neglecting the impact of transshipment distance. Lastly, the risk of shortage after transshipment at the supply location was ignored in the current studies. Therefore it is practical to investigate the realistic transshipment rules and their applicability, and calculate the costs of various transshipment rules.

3. Model description and assumption

Two factors are considered when categorizing the transshipment rules:

3.1 Times of transshipment: one-time or multiple-time transshipment

One-time transshipment means the required quantity (reorder point minus inventory) of demand location (whose inventory is less than reorder point) is transported in one-time from the supply location (whose inventory is more than reorder point), which later on may lead to the shortage of the supply location. Multiple-time transshipment means only the surplus (inventory minus reorder point) is transported from the supply location to the demand location, which may require transhipments from multiple supply locations. An example is illustrated in Figure 1, where V_{j0} is the initial inventory; R_j is the reorder point; and d_{jj} is the distance between two locations.



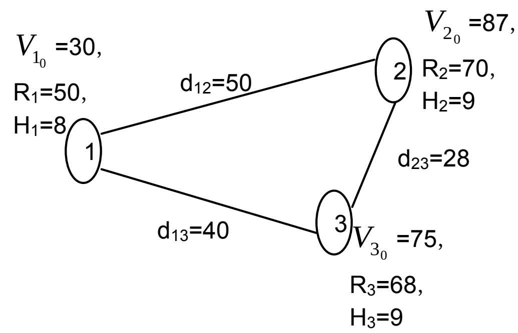


Figure 1: One-time and multiple-time transshipment *Figure 2: Full sharing and partial sharing transshipment*

As shown in Figure 1, for $V_{10}<R_1$, location 1 runs into an inventory shortage of 20; the surplus in location 2 is 8; and the surplus in location 3 is 7. The replenishment will always be transported from the nearest location.

Let's firstly apply one-time transshipment principle. The shortage of 20 in location 1 will be transported from location 3, which will lead to a shortage of 13 in location 3. Accordingly, the shortage in location 3 will be replenished from location 2, and eventually this will result in a shortage of 5 in location 2.

By following the multiple-time transshipment principle, the surplus of 7 in location 3 will firstly be transported to location 1, and another 8 from location 2 to location 1, and finally there will be a shortage of 5 in location 1.

3.2 Degree of sharing: full-sharing and partial-sharing.

As illustrated in Figure 2, full-sharing means transshipment occurs when $V_{j0}>R_j$, partial-sharing occurs only when $V_{j0}>R_j+H_j$, where H_j is the reserved safety inventory.

As shown in Figure 2, a shortage of 20 occurs in location 1. By the partial-sharing principle, only location 2 is qualified as the supply source. By the full-sharing principle, both location 2 and 3 are qualified as the supply source.

Two problems will be discussed in this paper. They are which is the most optimal transshipment rule in general and how the factors of unit cost of shortage, unit cost of transshipment and distance between storage sites impact on the transshipment rules. We assume family of the storage sites is J, and the demand of every storage site follows normal distribution.

3.3 Assumptions

We make the following assumptions:

a) transshipment is taken place among the same echelon storage sites, the coordinates of each storage locations are known;

b) single product, single period, the independent requirement at each storage site complies to normal distribution and (Q, R) inventory strategy;

c) assuming instant arrival of transshipment, distance of transshipment is calculated by coordinates of supply and demand sites;

d) holding cost during transshipment is included into transshipment cost;

e) total cost includes transshipment cost and shortage cost; and

f) the nearest supply location is the preference select.

4. Modeling the transshipment among locations at the same echelon

We begin with a few basic definitions. The notation for a series of parameters that are used in the model is presented in Table 1.

Table 1: Notations for key parameters and variables in the model

Notation	Definition	Notation	Definition
K	Safety factor of inventory, as the index of inventory availability, which is related to the service level	J	Family of the storage sites, where j is a certain site
C_1	Unit lateral transshipment cost (RMB/t.km)	C_2	Unit shortage cost (RMB/t)
u_j	Mean value of demand at location of j	σ_j	Standard deviation of the demand at location of j
L_j	Mean value of lead time at location of j (day)	σ_{Lj}	Standard deviation of lead time at location of j (day)
d_{jj}	Distance between locations	R_j	Reorder point at location of j
V_{jn}	Storage amount at location of jafter taking n iterations	V_{j0}	Initial storage amount at locaiton of j
Z_{jn}	Required transshipment amount at location of j after taking n iterations, is equal to the shortage at location of j after taking n iterations	H_j	Reserved storage for avoiding the risk of stockout at location of j
n	Iterative times		

We consider that shortage occurs in location of j, that is, $V_{jn}<R_j$, in the following four kinds of models.

In case of **one-time and full-sharing transshipment model**, we consider location of j'(j' $\in$j) satisfied by the conditions of $V_{jn}>R_j$ and $R_j=L_j\times u_j$ is exist. Thus supply location is selected by $d_{jj'}=\min\{d_{jk},k=1,2,\ldots j'\}$, and then the shortage amount in demand location of j, that is Z_{jn} can be presented as $Z_{jn}=R_j-V_{jn}$.

In case of **one-time and partial-sharing transshipment model**, we assume there exist location of j'(j' $\in$j) satisfied by the conditions of $V_{j'n}>R_{j'}+H_{j'}$ and $R_{j'}=L_{j'}\times u_{j'}$, select the supply location by $d_{jj'}=\min\{d_{jk}, k=1,2,\ldots j'\}$, and conclude the shortage amount in demand location of j, that is, $Z_{jn}=R_j-V_{jn}$.

In case of **multiple-time and full-sharing transshipment model**, assuming shortage is not allowed in supply location, that is, $Z_{jn}\le(v_{jn}-R_{j'})$, a demand location may be served by several supply locations, or a supply location may serve several demand locations. We consider that location of j' (j' $\in J$) satisfied by conditions of $V_{j'n}>R_{j'}$ and $R_{j'}=L_{j'}\times u_{j'}$ is exist, supply location is selected by $d_{jj'}=\min\{d_{jk}, k=1,2,\ldots j'\}$, and the formula of Z_{jn}, the shortage amount in demand location of j, is $Z_{jn}=\min[(R_j-V_{jn}),(V_{jn}-R_{j'})]$.

In case of **multiple-time and partial-sharing transshipment model**, location of j' (j' $\in J$) can be satisfied by conditions of $V_{j'n}>R_{j'}+H_j$ and $R_{j'}=L_{j'}\times u_{j'}$. We can select supply location by $d_{jj'}=\min\{d_{jk}, k=1, 2, \ldots j'\}$ and conclude the shortage amount in demand location of j, that is, $Z_{jn}=\min[(R_j-V_{jn}), (V_{jn}-R_{j'}-H_{j'})]$.

5. Model Solution

The four steps in the model solution are as follows.
Step 1, we calculate and sequence the initial shortage amount in location of j, that is, (R_j-V_{j0}); the location of maximum initial shortage amount is supplied proper transshipment amount from selected supply location according to the above transshipment rules. Thus, the first transshipment is taken place. And then we calculate cost of transshipment, that is, $C_{LT1}=C_1\times d_{jj}\times z_{j0}$, and the inventory amount of supply location and demand location after transshipment, that is, V_{j1};
Step 2, we calculate the shortage amount in location of j once again after the first transshipment, that is, $C_{LT2}\ldots C_{LTn}$, repeat step 1 until there is not demand location or supply location any more;
Step 3, we calculate the shortage cost in location of j after end of iterations, that is, $C_{q1}\ldots C_{qj}$, the formula is presented as follows:

$$C_{q_j}=C_2\int_{V_{j_n}}^{+\infty}(x-V_{j_n})f(x)d(x) \tag{1}$$

where $f(x)=\dfrac{1}{\sqrt{2\pi}\sqrt{L_j\sigma_j^{\ 2}+u_j^{\ 2}\sigma_{L_j}^2}}e^{-\frac{(x-L_ju_j)^2}{2(L_j\sigma_j^{\ 2}+u_j^{\ 2}\sigma_{L_j}^2)}}$;

Step4, we obtain the total cost, that is, TC, where

$$TC=C_{LT_1}+C_{LT_2}+\cdots+C_{LT_n}+C_{q_1}+C_{q_2}+\cdots C_{q_j} \tag{2}$$

6. Numerical examples

In our examples, for K=1.28, C_1=0.3/(t.km), C_2=15/t, and other data is as shown in Table 2.

Table 2 Coordinates of each location and variables of storage

locations	Coordinates(km)	V_{j0} (t)	H_j (t)	u_j (t)	σ_j (t)	L_j (day)	σ_{Lj} (day)
1	(-40, 10)	73	11	45	5	2	0.1
2	(-20, 70)	125	17	37	4	3	0.3
3	(-42, -32)	70	19	29	6	2	0.4
4	(-70, -43)	95	12	38	3	3	0.2
5	(-10, -60)	107	20	44	6	2	0.3
6	(-40, 32)	165	20	57	6	3	0.2
7	(-68, 10)	190	68	52	4	4	1
8	(12, 15)	50	9	35	5	1	0.1
9	(0, -40)	79	17	28	7	2	0.3
10	(-10, -10)	145	24	56	4	3	0.3

Programming and calculating in Matlab, we get the following results as shown in Table 3 and Table 4.

Table 3 Results of various transshipment rules

Transshipment rules	One-time and full-sharing transshipment	One-time and partial-sharing transshipment	Multiple-time and full-sharing transshipment	Multiple-time and partial-sharing transshipment
Cost(RMB)	T_{C1}=2,531	TC_2=2,026	TC_3=2,353	TC_4=1,949

Table 4 Sensitivity analysis of parameters

Parameters		Cost(RMB)				Ranking of rules
		T_{C1}	T_{C2}	T_{C3}	T_{C4}	
$d_{jj'}$	50%×$d_{jj'}$	1,762	1,630	1,673	1,890	T_{C2}, T_{C3}, T_{C1}, T_{C4}
	150%×$d_{jj'}$	3,300	2,421	3,033	2,007	T_{C4}, T_{C2}, T_{C3}, T_{C1}
C_1	C_1=0.15	1,762	1,630	1,673	1,890	T_{C2}, T_{C3}, T_{C1}, T_{C4}
	C_1=0.45	3,300	2,421	3,033	2,007	T_{C4}, T_{C2}, T_{C3}, T_{C1}
C_2	C_2=7.5	2,034	1,408	1,856	1,032	T_{C4}, T_{C2}, T_{C3}, T_{C1}
	C_2=22.5	3,028	2,643	2,850	2,864	T_{C2}, T_{C3}, T_{C4}, T_{C1}

Table 3 shows the ranking of lateral transshipment rules is T_{C4}, T_{C2}, T_{C3}, T_{C1}. In the case of one-time transshipment, the supply location may be new demand location after transshipping, which may cause circuitous transshipping and increasing transshipment cost. While full-sharing transshipment may give rise to the increasing shortage cost at supply location. Therefore, the transshipment rule integrated multiple-time transshipment with partial-sharing is preferable choice.

Table 4 presents in the case of one-time transshipment, supply location may be new demand location, which may lead to the increase of total transshipment distance and amount, therefore, multiple-time transshipment rule is better, especially when the distance between storage sites is longer or unit cost of transshipment is higher. Bigger unit cost of shortage may decrease the ratio of transshipment cost to total cost, so one-time and partial-sharing transshipment is superior.

7. Conclusion

Lateral transshipment is a new and effective way to regulate the balance between demand and supply. The following conclusions are reached after modeling and experimenting:

a) One-time transshipment may lead to the shortage of the supply location itself when its inventory surplus is less than the replenishing amount required at the demand location, which will cause subsequent secondary or multiple cross-haul transshipment and greatly increase the total transshipment cost. However, multiple-time transshipment will avoid this puzzlement;

b) Full-sharing transshipment will increase the risk of subsequent shortage of the supply location and hence the total cost of shortage, but the partial-sharing transshipment will effectively tackle this problem; and

c) Other factors, such as distance and cost, have impact on the ranking of various transshipment rules, so proper rules are highly dependent on the real scenario of each case.

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