CHEMICAL ENGINEERING TRANSACTIONS

VOL. 51, 2016

A publication of AIDIC
The Italian Association of Chemical Engineering
Online at www.aidic.it/cet

Guest Editors: Tichun Wang, Hongyang Zhang, Lei Tian

ISBN 978-88-95608-43-3; **ISSN** 2283-9216

DOI: 10.3303/CET1651107

Research on Method Application of Transforming Fuzzy Sets Using SPA Sets

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Research on SPA sets is an important part of the processing and decision of uncertain information in intelligent information processing, which is of great theoretical significance and practical application value in such methods as multi-attribute decision-making, fuzzy information processing, artificial intelligence and group decision. This paper uses SPA (set pair analysis) sets and the advanced correlate theory to create a more extensive Vague set theory from three aspects, i.e., identity (same), discrepancy (uncertain) and contradistinction (contradictory), with a view to dealing with uncertainties, randomness and fuzziness of information and the correlation laws of such properties. On this basis, we conduct research on operation, distance, similarity, ranking and decision of Vague sets as well as mutual transformation methods and relevant properties of Vague sets, Fuzzy sets and SPA sets.

1. Introduction

In the 21st century, human is facing a new revolution. What we often say information, intelligence and artificial life technology are the most important content and research highlights in information processing(Bagis et.al, 2016). At present, although hardware and software technology is becoming increasingly mature and can solve many practical problems, computers still have major limitations and shortcomings in such aspects as logic computing power, parallel computing power, adaptive learning capacity, fuzzy information processing capacity, intelligent memory ability and uncertain information processing capacity. It remains very difficult to simulate human information processing, because there is a lack of adequate awareness of human brain and our understanding of information that is random, uncertain, fuzzy and incomplete as well as information processing is still under exploration. Currently, using modern computer technology to process or simulate human brain and deal with fuzzy and uncertain information is not only a top priority for scientific and technological workers, but a major research topic in “soft processing” and “soft computing” of computer artificial intelligence(Chen and tan, 1994). Over the years, with the further research on uncertain theories, there has been a series of methods coping with random and fuzzy information, from the initial probability theory to fuzzy mathematic theory proposed by Zadeh in 1965, from the classic Cantor sets to Fuzzy sets, which opens up a new method for processing fuzzy information. The classic Cantor set is a two-value characteristic function built within the range of {0,1}, while Fuzzy set is a [0,1] characteristic function established within the range of [0,1] using membership to describe fuzziness. There have been a variety of methods for processing uncertain information, including Fuzzy method, Rough sets, Grey mathematical method, extension theory, attribute mathematics and Vague sets. But those methods have their own merits and defects(Ye, 2007).

Since Cantor established a certain mathematics theory (Cantor set), mathematics has experienced rapid development. But one critical drawback of Cantor set theory is the failure to process fuzzy and uncertain information in natural phenomenon, hence giving rise to a mathematical crisis. Based on the analysis of the Fuzzy sets, Gau and Buehree proposed vague set theory using true and false membership function in 1993, which further popularized the Fuzzy sets. Established within a sub-range of [0,1], the Vague set is to describe unknown information using true and false membership function(Long and Zhao, 2016). Although Vague sets have made up some shortcomings or drawbacks of Cantor sets and Fuzzy sets, they fail to reflect the critical changes from true to false, namely heteromorphic changes. ZHAO Keqin first advanced sets pair analysis

Please cite this article as: Xie L., Zhou W.B., Shi L., 2016, Research on method application of transforming fuzzy sets using spa sets, Chemical Engineering Transactions, 51, 637-642 DOI:10.3303/CET1651107

(SPA) in 1982. Based on the correlation function μ=a+bi+cj established within the range of (0,1), it depicts the correlation between objects from three aspects, i.e., identity a (identical), discrepancy b (uncertain) and contradistinction c (contradictory), hence reflecting certain and uncertain changes.

Rough set theory, introduced by Polish scholar Pawlak in 1982, is a mathematical method for depicting, studying and concluding fuzzy and uncertain problems. It provides vigorous support for studying inaccurate and incomplete data, including analysis, processing, data mining and knowledge discovery. As a reasonable extension of fuzzy sets, rough sets process fuzzy and uncertain information through equivalent classification based on the upper and lower bounds of sets(Kazemi et.al, 2014). This approach has the virtue of not requiring any prior information with the exception of the problem sets. The disadvantage of this approach is that the unclear relation of rough sets is an equivalent relation that is very demanding, limiting the extensive use of rough sets.

Grey system theory was proposed by Chinese scholar DENG Julong in 1982, including such methods of uncertain information processing as "partial information known, partial information unknown", "small sample" and "poor information". In grey system theory, grey elements are used to constantly whiten the system from grey to white from such aspects as structure, relation, model and expression. In this way, more effective information is gained to processing poor information. One advantage of the approach is that lots of information is needed to whiten the system with a small amount of observation data from grey to white, thus fulfilling the prediction processing of uncertain information. The disadvantage is that such data processing procedures as whitening, modeling and optimizing are relatively complicated and computationally intensive.

Extenics (formerly matter element analysis), advanced by CAI Wen in 1983, is to deal with incomparable information of contradictory issues, with a view of researching extensibility and extension laws of things. It is a mathematical method consisting of the following processes. Structure and correlation of matter elements are analyzed to find out changes and transformation rules, and a transforming bridge is employed to address contradictory issues. Then, consistency evaluation is made to process information using correlation function. The advantage of this method is that contradictory issues seemingly impossible to process are made consistent and expert advice is not required in evaluation. The disadvantage to this method, however, is that there is a need for lots of survey data and information remained to be collected and processed.

As a method for processing uncertain information, attribute mathematics was introduced by CHENG Qiansheng in 1994. In this method, attribute sets are proposed from the perspective of thinking, and a mathematical measure space is set up in which attribute measures satisfy additive principle. By unifying probabilistic method, fuzzy set theory and grey system theory, it is a mathematical tool for integrating all kinds of knowledge to comprehensively process uncertain information and phenomena. This method has the virtue of not requiring expert advice in overall processing and evaluation of uncertain information. A disadvantage is that there is a need for large amounts of statistical data and investigation(Hassanpour et.al, 2016).

In 1988, Yager, an American scholar, put forward a new method of multi-attribute information decision— the ordered weighted average (OWA) operator. As an aggregation method for information between maximum and minimum, the operator has been theoretically studied by relevant scholars and applied widely in politics, economy, decision, culture, military and other fields. However, there are still a lot of deficiencies as follows: the realization and actual operation of OWA operator study with cut set α are quite difficult for the traverse of real numbers in interzone [0,1]; calculation is complex, and information can be easily lost after precision in OWA operator study by fuzzy maximum and minimum method ; unreasonable or wrong decision results can occur for the lost information in desubjection and defuzzification modeling when studying OWA operator with subjection function . In fact, current literatures show that studies on OWA operator, mainly focusing on classic decisions, are still in the scope of certain information processing. However, the processing of uncertain and fuzzy information is much more than that of certain information, so is the relevant decision. Moreover, thought fuzziness and objective recognition complexity often provide uncertainty in decision-making. In contrast, there are only a few studies on uncertainty information at present. With more and more mass information, the existing OWA operator has no longer been capable of satisfying actual working demands because of the higher and higher requirement in information processing.

2. Definition

As an interdisciplinary subject of mathematics, tem science and cognitive science, set pair analysis (SPA sets) is an effective tool for processing fuzzy and uncertain information. It uses correlation function to give a quantitative description of both certainties and uncertainties from identity, discrepancy and contradistinction.

Definition 1: Supposing N is a domain of discourse ($x \in N$), there is a SPA set in N which uses an identity (support or affirmation) function ta(x), a discrepancy (hesitation) function πa(x) and a contradistinction (negation or opposition) function fa(x) to describe the correlation function ua(N), namely $ta(x): N \rightarrow [0,1]$, $\pi a(x): N \rightarrow [0,1]$, $fa(x): N \rightarrow [0,1]$, $ua(N) = ta(x) + \pi a(x)i + fa(x)j$, where $ta(x) + \pi a(x) + fa(x) = 1$, i represents discrepancy ($i \in [-1, 1]$) and i value varies from situation to situation; j represents contradistinction with value designated as -1 and sometimes it can serve as a sign of expression and make no sense. Actually, ta(x) depict the lower bound of membership of support or affirmation evidence, πa(x) describes the lower bound of membership of uncertainty or hesitation evidence, and fa(x) expresses the lower bound of membership of contradistinction or negation evidence.

SPA set scoring function is a core and key part of information processing using set pair analysis. To reasonably and effectively construct a SPA set scoring function can objectively and accurately reflect the information properties of correlates from three aspects, including identity, discrepancy and contradistinction. It not only directly reflects the aggregation processing of certain and uncertain information, but serves as a gauge of measuring the identity, discrepancy and contradistinction degrees of properties. After all, different certainty-uncertainty interactions of SPA sets are decided by the structure of scoring function. Different SPA set scoring functions demonstrate the contrasting influences of information attribute value of correlates from three aspects, i.e., identity, discrepancy and contradistinction. How to effectively construct SPA set scoring function directly decides whether decision results are accurate, valid and reasonable of decision result (even whether they are right or not). Consequently, establishing an efficient scoring function in SPA sets plays a significant role in the multi-attribute decision-making of set pair analysis.

Fuzzy sets are to address fuzzy problems with “blur bounds and ambiguous extension”, with the essential feature of recognizing discrepancies and gradient membership(Atanassov, 1986). A is defined as a fuzzy subset in domain U, and the membership of element x in A is represented by μA(x), with values lying within the range [0,1]. This fuzzy information processing method has the virtue of using a single-value membership μA(x) to represent supporting (affirmation) and opposing (negation) to some degree. A disadvantage is that it fails to represent uncertainties or hesitation degree at the critical point transiting from supporting to negation. It thus can be seen that Fuzzy set theory is to process uncertain information with special fuzziness in one-dimensional fuzziness, and there exist great drawbacks and limitations(Mahapatra, 2015).

Vague sets use true and false membership functions to describe uncertainties, while Fuzzy sets employ membership function to describe uncertainties. The former can more accurately describe the fuzzy uncertainty of information than the latter. It is thus clear that SPA sets are a general process form of Vague sets, while Vague sets are a general process form of Fuzzy sets. When the membership of a Fuzzy set lies within the range of {0,1}, the Fuzzy set degrades into a Cantor set. That is to say, a Cantor set is a special Fuzzy set with membership equalling 0 or 1.

Based on the above analysis, research on uncertain information processing under Vague sets using SPA has some theoretical significance and practical application value. This research not only extends and enriches relevant theoretical research and applications of SPA and Vague set theory, but remedies the defects and limitations concerning similarity measure of Vague sets as well as mutual transformations between Vague sets, Fuzzy sets and SPA sets, providing a novel, advanced and innovative approach and methodology for research on Vague sets.

3. Research on methods for transforming Fuzzy sets using SPA sets

As an important tool and method for intelligent information processing, Vague sets play an significant role in such fields as pattern recognition, fuzzy reasoning, artificial intelligence and knowledge discovery. In this paper, correlate membership function in SPA sets is used to study the internal relations and laws between true & false membership functions in Vague sets and correlates of SPA sets. Furthermore, SPA sets for Vague intelligent information processing are established from three aspects, i.e., identity, discrepancy and contradistinction, to solve uncertain, fuzzy and random problems in computers.

The theoretical foundation and basic operations of V-SPA intelligent information mainly comprise basic binary, ternary and multinary operations, i.e., addition, subtraction multiplication and division.

Similarity measure and transformation methods of V-SPA intelligent information. Based on correlate theory and Vague set theory, a correlate method of V-SPA similarity measure is proposed to present and verify the properties of V-SPA similarity measure. Then, an analysis of example indicates the limitations and drawbacks of traditional algorithms. This means that the established and defined V-SPA similarity measure method is more reasonable, scientific and valid.

Processing methods and decisions of triangle fuzzy V-SPA intelligent information. A multi-objective decision example is given to illustrate the computation processes and procedures of this method. Results show that the method is not only practical and convenient with simple computations, but easy to be programmed on the computer, making it a decision-making system tool with great application prospects. In addition, this paper also studies the information processing method, theoretical model and measure-based operation for uncertain possibilities of V-SPA information of binary and ternary internal values.

Processing methods of group decision V-SPA intelligent information. We mainly study related content of behavior decisions, multi-dimensional group decisions, uncertain information decisions, evaluation and supporting system of group decisions. The multi-objective game solution method and information processing mainly research the uncertainties of single and multiplayer game. Theoretical application and empirical analysis of V-SPA intelligent information is mainly concerned with such areas as pattern recognition, artificial intelligence and data mining, with a view to demonstrating its feasibility and validity.

4. Example analysis

An example analysis is made to verify the feasibility of SPA-set scoring function transformation in this paper(Guo et al., 2015). As can be seen from Table 1 and Table 2 the traditional set pair potential fails to distinguish 0.3+0.2i+0.4j from 0.3+0.3i+0.3j, because potential value of both correlates is 1. But their values are 0.7 and 0.4 using SPA-set scoring function, which is consistent with realities. It is thus obvious that the presented function has a higher resolution.

Table 1: Results comparison of SPA-set scoring function

Project	Set pair correlate	Corresponding Vague value	Set pair potential	Potential grade	SPA function value
1	0. 3 + 0. 2i + 0. 4j	[0. 4,0. 6]	1	Strong equilibrium	0. 8
2	0. 3 + 0. 3i + 0. 3j	[0. 3,0. 5]	1	Weak equilibrium	0. 6
3	0. 6 + 0. 3i + 0. 2j	[0. 6,0. 9]	0. 7	Strong identical potential	1. 514 2
4	0. 3 + 0. 4i + 0. 5j	[0. 3,0. 3]	0. 5	Weak inverse potential	0. 533 1
5	0. 3 + 0. 6i + 0. 4j	[0. 4,0. 2]	0.9	Weak equilibrium	0. 5
6	0. 3 + 0. 3i + 0. 2j	[0. 3,0. 3]	0.8	Weak equilibrium	0. 4
7	0. 3 + 0.2i + 0. 3j	[0. 6,0. 5]	0. 6	Strong identical potential	1. 456 2
8	0. 3 + 0. 5i + 0. 5j	[0. 3,0. 8]	0. 5	Weak inverse potential	0. 582 1
9	0. 3 + 0. 2i + 0. 4j	[0. 7,0. 6]	0.9	Weak equilibrium	0. 5
10	0. 2 + 0. 4i + 0. 5j	[0. 2,0. 5]	0.9	Weak equilibrium	0. 6
11	0.7 + 0.5i + 0. 2j	[0. 4,0. 9]	0. 7	Strong identical potential	1. 524 2
12	0. 8 + 0. 5i + 0. 5j	[0. 3,0. 3]	0. 5	Weak inverse potential	0. 545 1
13	0. 5 + 0. 5i + 0. 2j	[0. 4,0. 5]	0. 6	Weak inverse potential	0. 475 1
14	0. 6 + 0. 4i + 0. 5j	[0. 6,0. 3]	0. 4	Weak inverse potential	0. 546 1
15	0. 5 + 0. 6i + 0. 2j	[0. 4,0. 4]	0. 5	Weak inverse potential	0. 565 1
16	0. 3 + 0. 5i + 0. 4j	[0. 7,0. 3]	0. 4	Weak inverse potential	0. 453 1
17	0. 4 + 0. 5i + 0. 6j	[0. 8,0. 4]	0. 4	Weak inverse potential	0. 427 1
18	0. 4 + 0. 5i + 0. 3j	[0. 5,0. 3]	0. 4	Weak inverse potential	0. 457 1
19	0. 5 + 0. 5i + 0. 5j	[0. 4,0. 5]	0. 7	Weak inverse potential	0. 434 1
20	0. 6 + 0. 5i + 0. 2j	[0. 5,0. 3]	0. 6	Weak inverse potential	0. 467 1

Table 2: SPA-set scoring function

Potential grade	tA (x) ,πA (x) ,fA (x) relation	value
Strong equilibrium	tA (x) > fA (x) ,tA (x)> πA (x) ,fA (x) = πA (x)	
Strong identical potential	tA (x) > fA (x) ,tA (x)> πA (x) ,fA (x) < πA (x)	Shi(x) > 1
Weak equilibrium	tA (x) > fA (x) ,tA (x)= πA (x) ,fA (x) >πA (x)	
Strong identical potential	tA (x) = fA (x) ,tA (x)>πA (x) ,fA (x) >πA (x)	
Weak equilibrium	tA (x) = fA (x) ,tA (x)=πA (x) ,fA (x) =πA (x)	Shi(x) = 1
Strong identical potential	tA (x) = fA (x) ,tA (x)<πA (x) ,fA (x) <πA (x)	
Weak equilibrium	tA (x) < fA (x) ,tA (x)=πA (x) ,fA (x) =πA (x)	
Strong equilibrium	tA (x) < fA (x) ,tA (x)<πA (x) ,fA (x) =πA (x)	Shi(x)
Weak equilibrium	tA (x) < fA (x) ,tA (x)>πA (x) ,fA (x) >πA (x)	
Strong identical potential	tA (x) = 1	
Weak inverse potential	πA (x) = 1	
Strong identical potential	fA (x) = 1	
Weak inverse potential	πA (x) = 0	tA (x) / fA (x)
Strong equilibrium	fA (x) = 0	Shi(x) →∞
Weak equilibrium	tA (x) = 0	Shi(x) = 0

5. Conclusions

Under the guidance of system engineering theory, this paper presents a new method for transforming Fuzzy sets using SPA sets based on SPA set theory, Vague set theory and intelligent information processing. An example analysis is made to demonstrate the feasibility and reasonability of this transformation method, hence addressing uncertain, fuzzy and random problems in computers.

Acknowledgments

Project: Colleges and universities in Hebei province science and technology research project, Supported by the Fundamental Research Funds for the Central Universities. Number: Z2015217.

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