



A Point Cloud Registration Algorithm for Free-form Surface

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To improve the searching efficiency of the precise position of workpiece with free-form surface, a new two-step point cloud registration method is proposed. It seeks to improve the preliminary registration algorithm and the iterative closest point (ICP) algorithm. Firstly, for preliminary registration, the co-planar 4-points sets are searched based on the constraints of distance and proportional relationship between points. Then the invariant of these co-planar 4-points sets are used for random sample consensus algorithm (RANSAC) to get initial position of the target point cloud. Secondly, re-sampling of source data, matching method based on double direction normal projection and elimination of unreliable point-pairs are used to improve the original ICP algorithm to optimize the preliminary registration result. The registration simulations of two simple workpieces were conducted. The results show that the improved algorithm has higher efficiency and better precision than the traditional ones.

1. Introduction

In the field of industrial manufacturing, some workpieces of free-form surface (such as titanium skin) without any obvious features should be made in a non-binding state with no fixtures. This can lead to unknown position and posture of the surface. Thereby it is unable to get the precise position for the subsequent work, like drilling, cutting and other processes. A traditional technique is to use the tools called "profile model" to fix the position manually, which leads to high manufacturing costs, inefficiency, poor flexibility and low yield. A registration process of two point clouds for free-form surfaces, where one point cloud is created from CAD model and the other is scanned from the corresponding workpiece, is proposed to achieve the precise position of the workpiece. With this method, there are no "profile model" tools and only the key is the algorithm which is used to get the precise alignment between the two point clouds.

Currently, there are usually two automatic registration techniques used in engineering: feature-based matching algorithm, for example sample-sphere matching algorithm based on normal (Meng and Zhang, 2011) and algorithm based on the boundary of point cloud (Wang and Zhang, 2012); ICP algorithm (Besl and Mckay, 1992).

In order to improve the accuracy and efficiency of the registration, this paper intends to design a new two-steps registration method: RANSAC algorithm based on co-planar 4-points sets for the preliminary registration and an improved ICP algorithm for accurate registration.

2. RANSAC algorithm based on co-planar 4-points sets for the preliminary registration

RANSAC algorithm (Fischler and Bolles, 1987; Chen and Hung, 1999) is an estimation algorithm with good robustness that can handle point cloud data and is widely used in the field of computer sciences. In this paper, for the preliminary registration, co-planar 4-points sets are used as the initial sample to improve RANSAC.

2.1 Co-planar 4-points sets

As shown in Figure 1, it is hypothesized that 4 non-collinear points from the target point cloud P is in the same plane. Then e as the intersection of ab and cd could be found. The length and the ratio as indicated in Eq(1) of the two lines are Euclidean invariant with rigid transformation (Sharp and Sang, 2002; Urfaloğlu and Mikulastik, 2006). Therefore the corresponding 4 co-planar points could be obtained in the reference point cloud Q based on the equal length of line segment and the same split ratio of the intersection.

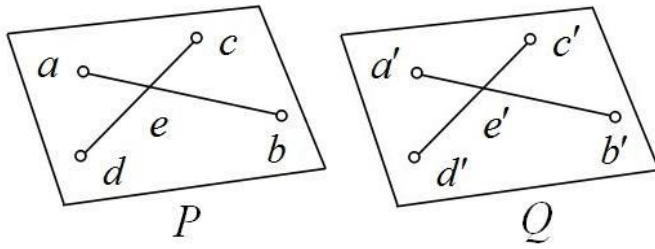


Figure 1: Co-planar 4-points

$$\begin{cases} d_1 = \|a-b\| = \|a'-b'\| \\ d_2 = \|c-d\| = \|c'-d'\| \\ r_1 = \frac{\|a-e\|}{\|a-b\|} = \frac{\|a'-e'\|}{\|a'-b'\|} \\ r_2 = \frac{\|c-e\|}{\|c-d\|} = \frac{\|c'-e'\|}{\|c'-d'\|} \end{cases} \quad (1)$$

2.2 Searching for the corresponding 4-points sets

The corresponding 4-points sets for preliminary registration can be searched as follows.

Firstly, in P , 4 non-collinear but co-planar points $\{a, b, c, d\}$ with distances long enough between each other are selected randomly in the overlapping region of the two point clouds based on the condition $0 < r_i < 1$ ($i = 1, 2$), otherwise the points should be sampled again. Then, d_1 , d_2 , r_1 and r_2 are calculated after the intersection e is obtained.

Secondly, all the two points with the distance d_1 in Q are stored in a point set S . And all the two points with the d_2 distance in Q are stored in a point set T .

Thirdly, the intersections e_1 and e_2 of every line segment in S are calculated as $e_1 = a' + r_1(b' - a')$ and $e_2 = a' + r_1(a' - b')$. Equally, the intersections e_3 and e_4 of each line segment in T are obtained as $e_3 = d' + r_2(c' - d')$ and $e_4 = d' + r_2(d' - c')$. If two intersections respectively from S and T satisfy $e_s = e_t$, the four end points of the two lines are the corresponding 4-points set $\{a', b', c', d'\}$, as illustrated in Figure 2.

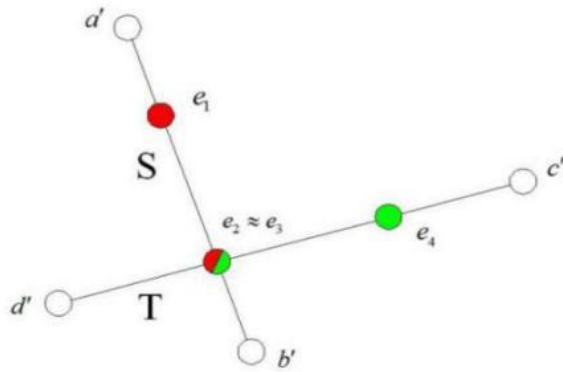


Figure 2: The corresponding 4-points set

2.3 RANSAC algorithm based on co-planar 4-points sets

RANSAC algorithm improved based on co-planar 4-points sets as follows is used for the preliminary registration.

- (1) Select 4 non-collinear points $\{a, b, c, d\}$ in a same plane randomly in P , and compute d_1 , d_2 , r_1 and r_2 ;
- (2) Find out the corresponding 4-points set $\{a', b', c', d'\}$ in Q with the method described in the section 2.2;
- (3) Take the two co-planar 4-points sets as estimating points to compute the Euclidean transformation matrix H_c between the two point clouds;
- (4) Calculate the degree of consistency between P and Q based on H_c . If the number of point-pairs that are matching with the error threshold δ (a given error threshold) is greater than n ($n = \lambda * N$, N is the point number

of P , and λ is the rate of the interior points of P obtained in the pre-processing of the point cloud), the transformation matrix H_c will be exported as a satisfactory one.

(5) Go back to step (1) and do iteration again. After K times random sampling and registration, transformation matrix H that has the best degree of consistency is selected as the final transformation matrix for preliminary registration;

$$K = \frac{\log(1-\sigma)}{\log(1-\lambda^4)} \quad (2)$$

Where σ is the probability that K times sampling contains at least a good sample.

3. The improved ICP algorithm for accurate registration

The position obtained by preliminary registration creates a favorable condition for the accurate registration. ICP approach is the most widely accepted algorithm for accurate registration. It has simple registration idea, high accuracy, robust and stable. But, all points in P should be took into iteration and it requires a larger overlap area and an original position as close as possible between the two point clouds, or the results may not necessarily be a global optimal solution but a local one (Senin and Colosimo, 2013; Zhou and Yong, 2011). So, there are more things need to be done for ICP.

3.1 Re-sampling of source data

The accuracy of the algorithm depends on the matching accuracy of point-pairs rather than the number of point-pairs. Thus one can use re-sampling way which extracts points from the target point cloud to reduce the number of points that need to be matched. There are several common methods for re-sampling: random sampling, uniformly distributed sampling, sampling based on feature space. It was shown that random sampling is better than the other two for the point cloud having no obvious features (Rusinkiewicz and Levoy, 2001). There are no distinctive features in workpieces with free-form surface. In each iteration, this paper takes random sampling as the re-sampling way.

3.2 Establishment of matching point-pair

Corresponding points in Q matching with re-sampled points should be found. This paper propose a double-direction normal projection matching method: The re-sampled point p_i from P is projected onto the fitting surface of Q along its normal vector. Firstly, when the projection intersection m point is exactly a point q_i of Q , the point m' can be obtained, which is the projection point of point q_i along its normal vector onto the point cloud P . If $\|p_i - m'\| \leq t$, a set of matched point-pair (p_i, q_i) could be built, for example (p_1, q_1) in Figure 3. If not, the point p_i should be deleted from the matched point-pairs. Secondly, when the projection intersection point denoted as m is not a point of cloud Q , as p_2 shown in Figure 3. The nearest point q_2 of the point m within the threshold range t is found in Q . Then the point m' , which is the projection point of the point q_2 along its normal vector onto the point cloud P can be obtained. If $\|p_2 - m'\| \leq t$, point-pair (p_2, q_2) could be used as matched point-pair, or the point p_2 should be deleted from the matched points. t is the mean value of all distances between the projection point m and its closest points.

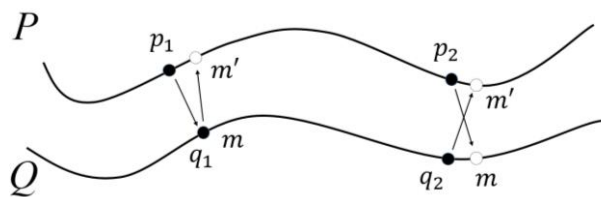


Figure 3 : Matching method based on double-direction normal projection

3.3 Elimination of unreliable point-pairs

Due to the strategy used to search for the putative correspondences, there are necessarily unreliable point-pairs in the matched point-pair sets established by the method proposed above. Commonly, there are two constrains to exclude unreliable points: the rigid constraints, the constraints based on geometric consistency. But they may diminish too much erroneous point-pairs or be time-consuming especially for the massive point clouds (Xie and Shang, 2010).

In this paper, a method using the distance and the point curvature features as constraints to eliminate unreliable point-pairs is applied as follows.

Firstly, all the distance between the matched points are calculated. If the distance satisfies Eq(3), then the

point-pair (p_i, q_i) would be removed from matched point set.

$$\left| \frac{2(d_i - D_m)}{d_{max} - d_{min}} \right| > 0.8 \quad (3)$$

Where d_i is the distance of the point-pair (p_i, q_i) , D_m is the mean distance of all point-pairs, d_{max} and d_{min} are the largest and smallest distance respectively.

Secondly, the matched point-pairs whose curvatures do not satisfy Eq(4), are eliminated from the remaining point-pairs after the first step.

$$\left\{ \begin{array}{l} \emptyset \min i = \frac{|k_1(p_i) - k_1(q_i)|}{|k_1(p_i) + k_1(q_i)|} \leq \varepsilon_1 \\ \emptyset \max i = \frac{|k_2(p_i) - k_2(q_i)|}{|k_2(p_i) + k_2(q_i)|} \leq \varepsilon_2 \\ \varepsilon_1 = \frac{\sum \emptyset \min i}{N} \\ \varepsilon_2 = \frac{\sum \emptyset \max i}{N} \end{array} \right. \quad (4)$$

Where $k_1(p_i)$ and $k_2(p_i)$ are the minimum and maximum values of the normal curvature at point p_i .

Only the remaining point-pairs after the two steps are considered into the subsequent alignment process.

3.4 The improved ICP algorithm

The steps of the improved ICP algorithm can be shown as follows.

- (1) Get the preliminary registration with RANSAC algorithm based on co-planar 4-points set;
- (2) Re-sample the target point cloud P . In this study, 20% points of P is left to keep its characteristics;
- (3) For each point p_i of the 20% points of P , search its matching point q_i' in Q based on the double-direction normal projection. And then eliminate erroneously matched point-pairs based on rigid distance constraint and curvature features constraint as described in section 3.3.
- (4) Estimate the transformation matrix $H_k = \begin{bmatrix} R_k & T_k \\ 0 & 1 \end{bmatrix}$ based on the remaining matched point-pairs by minimizing the objective function, where R is a 3×3 rotation matrix and T is a 3×1 translation matrix;
- (5) Update P with the equation $P' = H_k P = R_k P + T_k$;
- (6) If the difference of the registration error between two adjacent iterations meets the requirement as in Eq (5), the iteration ends and the registration is considered completed; if not, go back to step (2) and do iteration again.

$$\left| \frac{d_{k+1} - d_k}{d_k - d_{k-1}} \right| < \tau, \left(d_k = \frac{1}{N} \sum_{i=1}^N \|R_k P_{i-1} + T_k - P_{i-1}'\|^2 \right) \quad (5)$$

Where τ is a given threshold based on experience.

4. Simulation and analysis

Two group point clouds of free-form surface (where the point number of reference point cloud is more than that of target number) were used for simulation in the Matlab software with Intel Pentium Dual CPU and 2 GB memory. τ and δ were set as 0.1 and 0.5 respectively based on empirical experience. Figure 6-8 shows the results of the main registration steps.

Figure 4 shows the best co-planar 4-points sets found in the two free-form surfaces with RANSAC algorithm for preliminary registration. The rigid matrixes were finally calculated based on these two sets. And the preliminary registration results conducted using these matrixes is displayed in Figure 5. As indicated in Figure 5, the RANSAC algorithm based on co-planar 4-points proposed for preliminary registration brought the majority of the point data close to the registration requirements, but the fine registration is still in needs.

The accurate registration took the preliminary registration results as the new initial positions. And the results which fundamentally satisfied the registration accuracy are shown in Figure 6.

In addition, comparisons were made in this paper among "the traditional preliminary registration algorithm" based on features (Ko and Maekawa, 2003), "RANSAC algorithm based on co-planar 4-points" proposed for preliminary registration by this study, "the traditional preliminary registration algorithm + the traditional ICP algorithm" and "RANSAC algorithm based on co-planar 4-points + the improved ICP algorithm" both proposed by the paper. Table 1 and Table 2 list the registration uptime and errors of with these algorithms. As shown in the tables, there are almost no difference in registration accuracy between the traditional preliminary registration algorithm and the one proposed in this paper. But for traditional one, it is more time-consuming in terms of the calculation on the curvature of every point of the point cloud. It is also shown in tables that the registration speed and accuracy of the improved ICP algorithm are better than the traditional one.

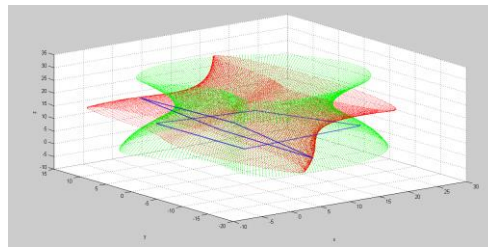
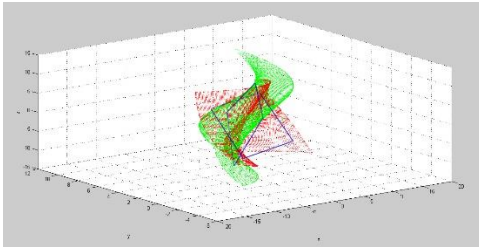


Figure 4: Co-planar 4-points sets

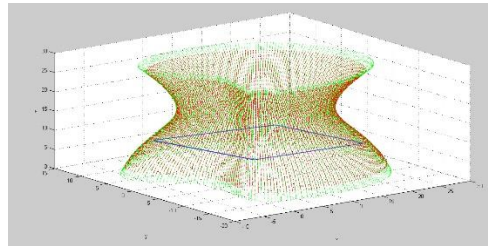
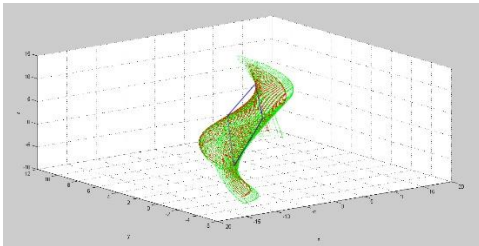


Figure 5: The results of preliminary registration

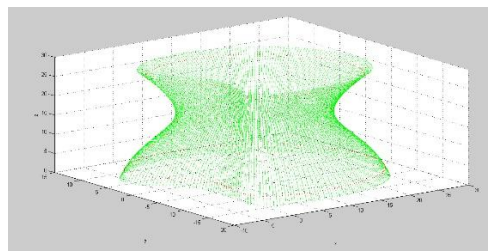
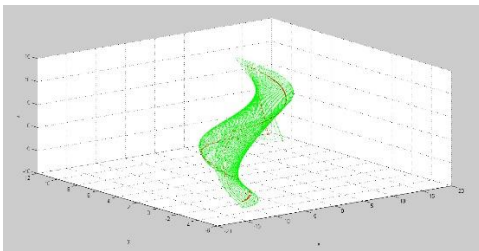


Figure 6: The results of accurate registration

Table 1: Registration Comparison of the first group of point cloud surfaces

| Algorithm | the point number of Q | the point number of P | Registration uptime (s) | Registration error (mm) |
|--|-------------------------|-------------------------|-------------------------|-------------------------|
| The traditional preliminary registration algorithm | | | 95.284 | 0.0945 |
| RANSAC algorithm based on co-planar 4-points | | | 45.250 | 0.0820 |
| traditional preliminary registration algorithm + the traditional ICP algorithm | 21780 | 18139 | 103.413 | 0.0078 |
| RANSAC algorithm based on co-planar 4-points + the improved ICP proposed | | | 60.128 | 0.0052 |

Table 2: Registration Comparison of the second group of point cloud surfaces

| Algorithm | the point number of Q | the point number of P | Registration uptime (s) | Registration error (mm) |
|---|-------------------------|-------------------------|-------------------------|-------------------------|
| The traditional preliminary registration algorithm | | | 205.613 | 0.0982 |
| RANSAC algorithm based on co-planar 4-points | | | 94.339 | 0.1056 |
| The traditional preliminary registration algorithm + the traditional ICP algorithm | 82800 | 36340 | 476.912 | 0.0128 |
| RANSAC algorithm based on co-planar 4-points + the improved ICP proposed by the paper | | | 228.108 | 0.0096 |

5. Conclusions

A RANSAC algorithm based on co-planar 4-points sets was proposed which could be employed before accurate registration to provide smaller registration error and accelerate the convergence speed. The

simulation results is in good agreement with the work (Papazov and Burschka, 2010). It was established that registration algorithm based on RANSAC framework has better robustness against noise, and could make good registration accuracy. Compared with the traditional ICP algorithm, the improved ICP algorithm proposed by the paper not only reduces the registration time which is important for the application in the production line, but also improves the accuracy of registration as well. The paper improved the key conditions about the study (Lu and Zheng, 2015) in detail, did new simulations and got more satisfying results. However, in order to provide a reliable position and orientation reference for flow-line production, the application system and the improvement of the algorithm require further study.

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