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Study on the Effect of Residual Compressive Stress on Crack Propagation Based on XFEM

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Extended finite element method is developed in recent years. It is an effective numerical method to solve the discontinuous problem in conventional finite element framework. The jump function and the crack tip asymptotic displacement field function which can reflect the discontinuity of the crack surface are added in the conventional finite element displacement mode based on the idea of unit decomposition. It is effective to avoid the inconvenience caused by re-encrypting elements on the crack tip when do calculation of the fracture problem in conventional finite element method. In view of the characteristics of the residual compressive stress on the surface of the axle, the residual compressive stress in different directions was introduced before the crack growth. By controlling the initial residual stress, the thickness of the residual stress layer and the initial crack length, the stress intensity factors of the crack growth under different conditions are obtained. The results show that the residual stress, stress layer thickness and initial crack length have obvious influence on the crack growth.

1. Introduction

Crack propagation is one of the important factors that affect the life of structures or components. Crack propagation simulation is an important content in the field of structural analysis. A large number of fracture accidents show that the fracture of the component is due to various types of cracks. The existence and expansion of these cracks weaken the bearing capacity of structures, thus affecting the quality and safety of engineering structures. Therefore, the study of crack initiation and propagation is of great guiding significance for engineering design and construction. Based on the above objectives, the domestic and foreign scholars have carried out a lot of research work in the aspects of theory, experiment and numerical simulation（Huynh et. al, 2008)(Li et. al, 2010)(Yu, 2005). The extended finite element method is a new numerical method which does not need re-mesh to deal with discontinuous problems(Belytschko et. al, 2001)(Mose et. al, 1999)(Stolarska et.al, 2001). No internal details of structure are needed to be considered when the extended finite element computational grid are generated. It is only needs to be generated in accordance with the geometry of the structure. The existence of the crack, the hole and the inhomogeneity is manifested by using additional functions to enhance conventional displacement. Belytschko and Black(1999) proposed an extended finite element method solve the discontinuous problem in the conventional finite element framework. Moes et. al(1999) used the extended finite element method to simulate the bond crack growth with the stress intensity factor as the fracture criterion. Larsson and Fagerstrm(2005) based on the application of the bonded area in the shell model, the penetration crack growth of the thin wall structure is studied by using the discontinuous extended finite element method. Mariani and Perego(2003) used extended finite element method to simulate quasi static bond crack growth in brittle materials. Dolbow et. al(2000) used extended finite element method simulate crack growth under frictional contact. And this is the first time to simulate contact problem by using extended finite element method. Laborde et. Al(2005）and Chahine et .al(2006) studied the convergence problem of extended finite element diversity in crack area. And it is verified that accurate calculation results under crack propagation can be obtained using the extended finite element method. Chang and Cheng(2012) used finite element software ABAQUS to simulate the crack growth and the effect of reinforced particles on the crack growth of Al6061/Al2O3 composites was predicted by using the extended finite element method. Golewski et. al(2012) used extended finite element method to simulate crack growth under the composite material. Prosenjit et. al(2012) used extended finite element method to study the quasi-static crack growth of 7075 aluminum alloy. It shows that the extended finite element method can simulate the crack propagation problem.

2. Basic principle of extended finite element method

There is an arbitrary crack in the finite element mesh, the crack geometry is independent of the computational grid, as shown in figure 1. Define three node sets:
I is collection of all nodes in a discrete structure.
J is collection of nodes of crack completely through the element. Strengthen using a modified Heaviside step function $H(x)$. $H(x)$ is +1 when node is above the crack, -1 when node is below the crack.
K is field enhanced node set at crack tip. For the plane problem, K is node set in a circle at crack tip with radius r. For the space problem, K is node set around cylinder at crack tip with radius r.

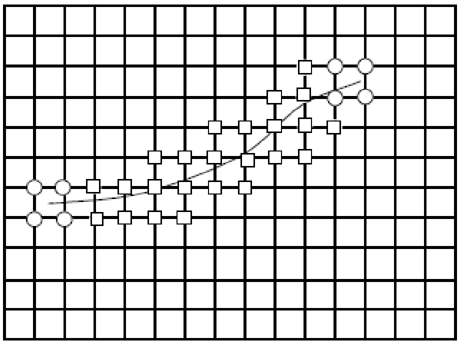

Figure 1: A mesh with any crack

The approximate form of the extended finite element displacement can be expressed by the following formula(Stazi et. al, 2003).

$$u^h(x) = \sum_{i \in I} N_i(x) u_i + \sum_{j \in J} \bar{N}_j(x) \left(H(x) - H(x_j) \right) a_j + \sum_{k \in K} \overline{N_k}(x) \sum_{l=1}^{4} (B_l(x) - B_l(x_k)) b_{lk} \quad (1)$$

$N_i(x)$ and $\bar{N}_j(x)$ are finite element shape function, u_i , a_j and b_{lk} are nodal displacements and nodal reinforcement variables. $N_i(x)$ and $\bar{N}_j(x)$ can be same or not same. $B_l(x)$ is base of the crack tip Westergaard field.

$$B \equiv [B_1 \;\; B_2 \;\; B_3 \;\; B_4] = [\sqrt{r}\sin\frac{\theta}{2} \;\; \sqrt{r}\cos\frac{\theta}{2} \;\; \sqrt{r}\sin\frac{\theta}{2}\cos\theta \;\; \sqrt{r}\cos\frac{\theta}{2}\cos\theta] \quad (2)$$

With the approximate form of displacement field. By using Bubnov-Galerkin method, the control equation of the extended finite element can be derived.

3. Finite element mode

According to the theory of linear elastic fracture mechanics(Tu, 2003). The stress intensity factor can be used to determine whether a structural crack can extend. Taking the most common, most basic and most dangerous i crack type as an example. The following formula is the general expression of stress intensity factor.

$$K_I = Y\sigma\sqrt{\pi a} \quad (3)$$

The stress intensity factor is proportional to the crack size a and the nominal stress σ of the component with crack. For the component with crack, when the criterion is satisfied with $K_I < K_{IC}$. Then the structure is safe. K_I is crack stress intensity factor. K_{IC} is fracture toughness of materials, and it is obtained through the fracture test generally. In the i type fracture problem, K_I is proportional to the square root of the original crack length and the nominal stress of the crack tip. For elastic-plastic fracture mechanics problems, COD and J integration methods are usually adopted(Li et. al,2005).

In order to study the influence of residual stress on crack propagation. On one side of the model, the residual stress is set by the subroutine. The controlled variable are size of initial residual stress and the thickness of residual stress layer. In addition, a crack is preset on the same side of the residual stress in the middle of the modle and controlled variable is the length of the initial crack. The effect of residual stress on crack growth was observed by changing the length of initial crack, the size of residual stress and the thickness of residual stress layer. The model is shown in Figure 2.

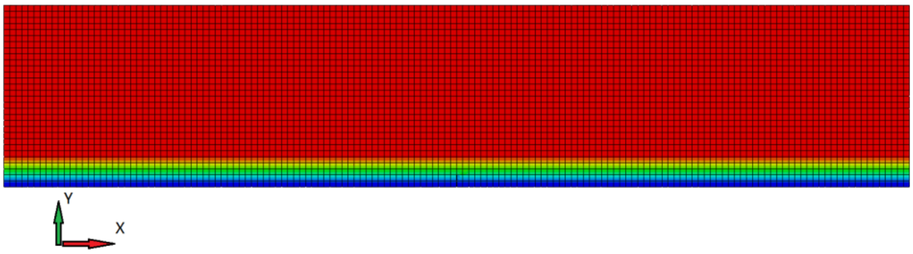

Figure 2: Model

The initial stress in the diagram is residual compressive stress in the direction of X axis, which is 200 MPa and the thickness is 5mm. The change of stress is linear transformation from 0mm to 5mm in the direction of Y axis. The initial crack length is 2mm. Full constraint is applied on the left side of the model. In the right side of the model, the tensile load is uniformly applied on the boundary.

4. Result analysis

Figure 3 shows the threshold value of crack growth under different initial crack lengths at different X-axial initial residual compressive stress. As can be seen from the graph, with the increase of the initial residual stress, the threshold K_{th} of the crack growth is gradually increased. And the threshold value of the initial crack of 2mm is higher than that of 1mm, but the increase is not obvious. That is because the nominal stress of 2mm initial crack is less than that of 2mm initial crack. It can be showed that the initial residual compressive stress of X-axial has a very significant inhibitory effect on the crack growth of Y-axial direction.

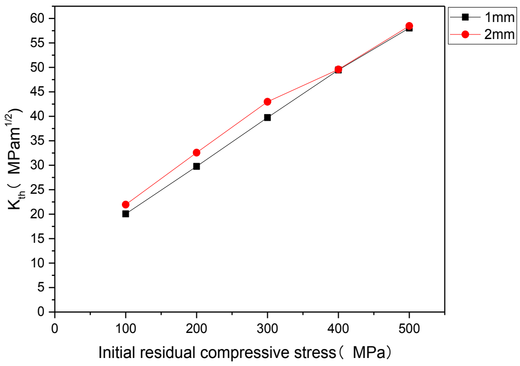

Figure 3: Threshold value of different initial crack length under different X-axial initial residual compressive stress

Threshold value and the nominal stress when crack extend under 200MPa X-axial initial residual compressive stress with different initial crack length are shown in figure 4. It can be seen from figures that the threshold of 2mm initial crack is the maximum. The threshold value is decreasing over 2mm. Although the threshold value of 1mm is less than the threshold value of 2mm crack, nominal stress in the initial crack of 1mm is larger than that of the nominal stress in the initial crack of 2mm. And it can be seen from the figure that the nominal stress decreases with the increasing of the initial crack. Model with longer initial crack is easier for crack to growth, and the change is very obvious.

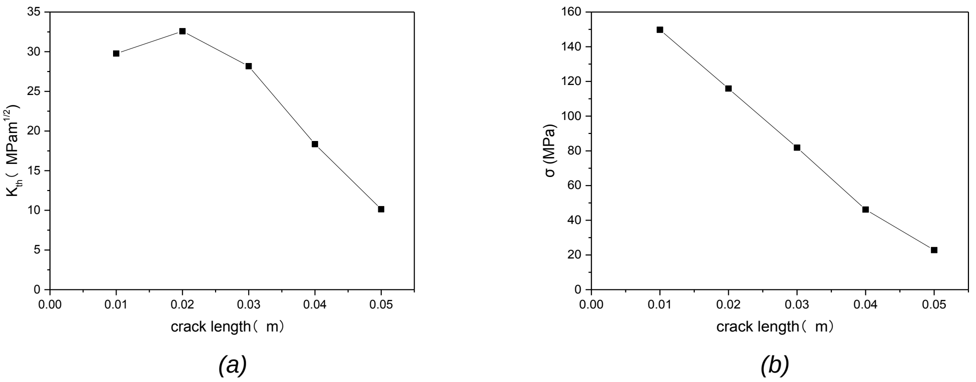

Figure 4: Threshold(a) and nominal stress(b) under 200Mpa X-axial initial residual compressive stress with different initial crack length

Figure 5 shows stress intensity factor in the whole fracture process under different initial crack with 200MPa X-axial and Y-axial direction initial residual compressive stress. As can be seen from the graph a, K_I of 4mm initial crack is larger than K_I of 5mm initial crack in the whole process. From the formula 3 we can see that the nominal stress of 4mm initial crack at each moment is larger than nominal stress of 5mm initial crack. It is proved that the initial crack of 4mm is more difficult to expand than the initial crack of 5mm. Because the X-axial residual compressive stress have strong inhibitory effect on crack propagation, the initial crack smaller than 4mm don't have enougth effective points during fracture process.
Stress intensity factor in the whole fracture process under different initial crack with 200MPa Y-axial direction initial residual compressive stress is shown in figure 5(a). It is shown that K_I of 1mm initial crack is bigger than that of other initial crack during whole fracture process. The smaller the initial crack is, the bigger the K_I in the fracture process. And it reflects the larger nominal stress, the more difficult the crack to extend. It can be proved that the initial crack length has a very significant effect on the crack growth. The smaller the crack is, the hard the crack to extend.

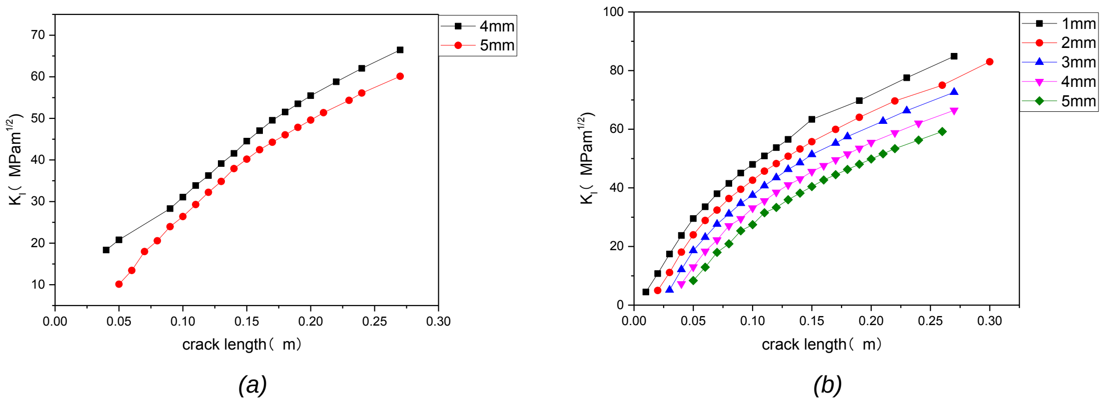

Figure 5: Stress intensity factor under different initial crack with 200MPa X-axial(a) and Y-axial(b) initial residual compressive stress

Figure 6 shows the stress intensity factor under different Y-axial residual compressive stress with 2mm initial crack.The K_I of 2mm crack under 100MPa initial stress is bigger than that of 200MPa and 300MPa when crack begins to extend. Thougth the initial crack is 2mm, the crack extend from 3mm directly under 400MPa and 500MPa. The K_I of 1mm initial crack is bigger than that of 2mm and 3mm at the beginning of crack extend. But the increase in the propagation process is smaller. It becomes the smallest before the model breaks. Thought K_I of 500MPa initial stress is smallest at the beginning, but as the crack expands, K_I becomes the biggest. As can be seen form above, Y-axial residual compressive stress has effect on Y-axial crack extension.

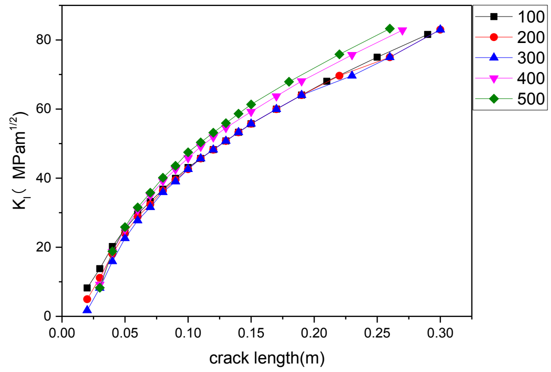

Figure 6: Stress intensity factor under different Y-axial residual compressive stress with 2mm initial crack

Crack growth threshold and nominal stress of 3mm initial crack under different thickness X-axial compressive residual stress layer are shown in figure 7.

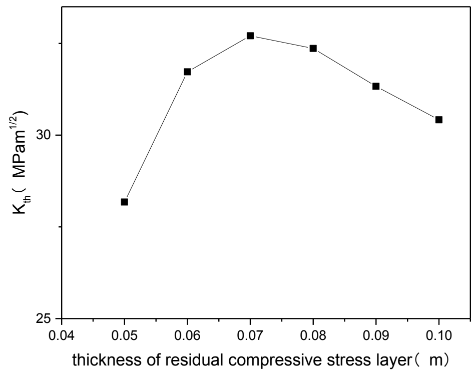

Figure 7: Threshold of 3mm initial crack under different thickness X-axial compressive residual stress layer

It can be seen from the figure that the value of K is maximum when the thickness of the residual stress layer is 7mm. The K value decreases with the increase of the thickness of the residual stress layer over 7mm. It can be explained that the thickness of residual stress layer has influence on crack propagation, but the thickness of residual stress layer is not as thick as possible.

5. Conclusions

1. With the increase of the initial residual compressive stress in the X direction, the threshold of the crack is gradually increased and the increasing trend is very obvious. The initial compressive stress of X-axial has a very significant inhibitory effect on the crack of Y-axial. The larger the Y-axial residual compressive stress is, the easier the crack growth is at the beginning. But as the crack length increases, the model with small Y-axial initial residual compressive stress is easier to extend.

2. The initial crack length has a significant influence on the crack growth in both X-axial and Y-axial direction. The shorter the initial crack is, the more difficult for the crack to extend. And it reflects the larger nominal stress. It can be proved that the initial crack length has a very significant effect on the crack growth. The smaller the crack is, the hard the crack to extend. K_I of smaller initial crack is larger than K_I of larger initial crack in the whole process. In the whole process, the model with longer initial crack length is more easily to extend.

3. The thickness of residual stress layer also has a significant effect on crack growth. When there is a X-axial initial residual compressive stress, the threshold increases with the increase of the thickness of the stress layer. When the stress layer thickness is 7mm, the threshold value reaches the maximum. The threshold value

decreases with the increase of stress layer thickness over 7mm. So 7mm residual stress layer thickness is the best.

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