Research on Differentiated-Area Routing in Wireless Sensor Networks Based on the Vector Field

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The paper proposes a new scheme to analyze the differentiated-area routing in wireless sensor networks. In a single area of wireless sensor network, node message packets are generally in the same type, and are sent to one sink node (or multiple nodes) in an anycast way. However, in the differentiated area, message packets are divided into many types and the packets of the same type must be sent to a particular type of sink node. The paper firstly researches the single-area network routing, then proposes a set of partial differential equations similar to the Maxwell's equation of the static electric field, and finally extends the single network analysis method to the differentiated network and gets the dynamic routing relying on the rational changes of optimal cost function. Research results show that under certain conditions, the differentiated network routing problem can be decomposed into several single network problems, in which case the differentiated network problem can be analyzed through a series of independent single networks.

1. Introduction

At present, there have been many studies on the routing of wireless sensor networks (WSNs). Mo (2010) mentioned the SAS routing protocol which considers the energy consumption to construct a routing tree. HR (2012) introduced the minimum-cost forwarding algorithm of large-scale sensor networks. Shah (2011) and Midha (2014) describe similar routing scenes. Zhang (2010) describes the routing algorithm under the assumption that related information of node position is known. Kiani (2015) introduces how to analyze the routing using a mathematical method inspired by the electrostatic field. Tanwar (2015) further proposed to maximize network load processing using the minimum number of sensor node. Kong (2014) and Gao (2012) introduce the multipath routing algorithm based on the vector field. Jiang (2011) and Nguyen (2004) introduce the optimization problem based on the vector field and the method looking for the routing through the vector field.

The paper introduces the sensor network model of differentiated area. In the differentiated-area network model, the message packets generated on the source node are divided into various types. A significant difference between the differentiated-area vector filed model and the single-area vector field model is that for the former, the load vector in a point of WSN is not unique but divided into many load intensities according to its attributive classification.

2. Single-area WSN Routing

2.1 Vector field routing

Imagine a WSN containing area A and assume the source node is in the position of $(x,y) \in A$ and is generating messages at the rate of r (x, y) (bits/sec/m^2). The messages of source node get to the central node (sink node) $(x_0, y_0) \in A$ through the multi-hop routing. According to the assumption above, define the characteristic function $\rho(x, y)$ in area A as follows to represent the density of source message:

$$\rho(x, y) = r(x, y) - w_0 \delta(x - x_0) \delta(y - y_0)$$

(1)

In the equation, $\delta(x)$ is the Dirac function; w_0 is the sum of message sending rate and $w_0 = \int_A r(x,y)dxdy$.

It's clear that $\rho(x,y) = r(x,y)$ and $\int_A \rho(x,y)dxdy = 0$, except the sink node.

Under certain conditions, the vector field represents the communication load in network area. Use vector field $\vec{D}(x,y)$ to describe the density of messages flowing in network and to represent the number of messages to be transmitted on point (x, y) per second, including messages sent by other nodes to the node. The direction of $\vec{D}(x,y)$ represents the routing direction of the node. According to Jiang (2011), get the following equations:

$$\begin{cases} \vec{\nabla} \cdot \vec{D} = \rho(x,y) \\ \vec{\nabla} = \hat{i}\dfrac{\partial}{\partial x} + \hat{j}\dfrac{\partial}{\partial y} \end{cases} \tag{2}$$

x and y represent the coordinates in horizontal and vertical axes in Descartes rectangular coordinate system, respectively. $\hat{i}$ and $\hat{j}$ represent the unit normal vectors along x and y axes respectively. Formula (2) means the sum of messages passing through a closed curve in area A is the sum of messages sending from the source node in the closed curve. Formula (2)'s boundaries meet:

$$D_n(x,y) = 0, \ \forall (x,y) \in \partial A \tag{3}$$

where ∂A is A's boundary. $D_n(\chi)$ is the scalar of $\vec{D}$ along area A. The boundary condition comes from the fact that no message flows from the boundaries of network area. From the above, get the following formula:

$$\begin{cases} \vec{\nabla} \cdot \vec{D} = \rho(x,y) \\ D_n(x,y) = 0, \forall (x,y) \in \partial A \end{cases} \tag{4}$$

Find vector field $\vec{D}(x,y)$ satisfying formula (4). The vector field's streamline decides the set of paths from point (x, y) to the destination node. Whereas, if the paths staring from point (x, y) and ending at the destination node are given, $\vec{D}(x,y)$ satisfying formula (5) can be found, and its streamline matches with these paths.

Formula (4) doesn't give a uniform $\vec{D}(x,y)$, so the paper needs to add additional conditions to get the useful routing through the vector field. A direct method is to make $\vec{D}(x,y)$ distribute as uniformly as possible, and the routing obtained in this way can realize the high decentralization of network traffic. Besides, the method can reduce the congestion and conflicts of nodes and allow the network to have higher throughput. The spread of network load can be simulated with the following minimum cost function:

$$J(\vec{D}) = \int_A \left| \vec{D} - \vec{D}_{av} \right|^2 dxdy \tag{5}$$

where $\vec{D}_{av}$ is the mean of vector field $\vec{D}$ in set A and is defined as

$$\tag{6}$$

$$\vec{D}_{av} = \frac{1}{|A|} \int_A \vec{D}dxdy$$

The quadratic in formula (5) allows the load to distribute as uniformly as possible which can prevent the excessive load in local areas and the insufficient utilization of resources in other areas. It's interesting that the form of the cost function is very similar to the energy definition formula of the electrostatic field. The optimization problem above can be summarized as

$$\begin{cases} \min imize \ \ J(\vec{D}) = \int_A |\vec{D} - \vec{D}_{av}|^2 \ ds \\ where: \\ \vec{\nabla} \cdot \vec{D} = \rho \\ D_n(\chi) = 0, \ \ \chi \in the \ edge \ of \ A \end{cases} \tag{7}$$

The following Lemma 1 provides the key idea to solve the optimization problem:

Lemma 1: If $\overrightarrow{\text{D}^*}$ represents the optimal solution of equation (7), it must satisfy

$$\vec{\nabla} \times \overrightarrow{\text{D}^*} = 0 \tag{8}$$

where $\vec{\nabla} \times$ is a 2D-direction vector operator. For the vector field $\vec{\text{F}} = \begin{bmatrix} \text{F}_x & \text{F}_y \end{bmatrix}$, define the operation with the following equation:

$$\vec{\nabla} \times \vec{\text{F}} = \left(-\frac{\partial \text{F}_x}{\partial y} + \frac{\partial \text{F}_y}{\partial x} \right)\vec{\kappa} \tag{9}$$

where $\vec{\kappa}$ is the unit complex vector composed by $\vec{i}$ and $\vec{j}$, i.e., $\vec{\kappa} = \vec{i} \times \vec{j}$.

Base on the lemma above, one of the optimal solution $\overrightarrow{D^*}$'s spatial difference equation set can be written as

$$\vec{\nabla}\cdot\overrightarrow{D^*}=\rho \ , \ \vec{\nabla}\times\overrightarrow{D^*}=0 \tag{10}$$

The set of equations above is similar to the Maxwell's equations in the electrostatic field theory. In the spatial differential equation theory, it has been proved that the boundary condition of the equation set above can be given particularly through $\overrightarrow{D^*}$.

2.2 The analysis on WSN routing

Equation (7) is based on the assumption of the unified distribution of initial energy of node, but the assumption is obviously not realistic in practice. In WSNs, since all nodes try to send messages to the sink node, the neighbor nodes of sink node have a higher energy utilization rate compared with other nodes. Another problem should be taken into consideration is the randomness of the occurrence of trigger event. It may happen in sensor networks that some sensor nodes have a faster energy decrement than other nodes. Researchers should take this point into consideration and update the routing in order to reduce the utilization of residual energy nodes and thus prolong the life cycle of sensor networks.

Therefore, the more general form is shown in equation (11) in which a scalar weight function $K(x,y)$ in added as the coefficient of the scalar product defined by equation (7). The equation shows the positions owning high loads and expecting to reduce communication traffic to save node energies. The higher the value of $K(x,y)$ is, the less message flows point (x,y) expects to have. The lower the value of $K(x,y)$ is, the more message flows point (x,y) with enough energy can bear. Therefore, area routing can be changed by changing related value of $K(x,y)$.

$$\begin{cases} \min imize \ \ J(\overline{D})=\int_A K(x,y)\,|\,\overline{D}-\overline{D}_{av}\,|^2\, ds \\ where: \\ \vec{\nabla}\cdot\vec{D}=\rho \\ D_n(x,y)=0, \ \ (x,y)\in the \ edge \ of \ A \end{cases} \tag{11}$$

The following Lemma 2 gives the key to get the optimal solution.

Lemma 2: If $\overrightarrow{D^*}$ is the optimal solution of formula (11), then it satisfies the following formula:

$$\begin{cases} \vec{\nabla}\cdot\vec{E}(x,y)=0 \\ \vec{E}(x,y)=K(x,y)\vec{D}(x,y) \end{cases} \tag{12}$$

Basing on Lemma 2, a series of partial differential equations can be derived

$$\begin{cases} \vec{\nabla}\cdot\vec{D}(x,y)=\rho(x,y) \\ \vec{\nabla}\times\vec{E}(x,y)=0 \end{cases} \tag{13}$$

In the partial differential equation theory, $\vec{D}(x,y)$ and $\vec{E}(x,y)$ obtained from the equation above combined with boundary condition are uniform.

3. Differentiated WSN

3.1 The analysis on differentiated area routing

The paper extends the single network in the previous section to the differentiated-area WSN. In the differentiated area, message packets generated on the source node are divided into various types and the message packets of given types are sent to the sink node receiving given types of data. In this way, source nodes in different areas may send message packets to the same sink node or different sink nodes. Assume the sensor network area is divided into N types of set attributes. $r_i(x, y)$ is the density of the ith type of source node and (x_i0, y_i0) is the position of the ith type of sink node. The significant difference of differentiated-area vector field model and the single-area vector field model is that in former, the load vector in a point in WSN is not unique but divided into several intensities of load according to its attributive classification.

In the case similar to the single area, define the load vector field containing attribute set i in the differentiated area of sensor network as $\vec{D}^i(\chi)$, then equation (1) can be modified as

$$\rho^i(x,y)=r^i(x,y)-w^i_0\delta(x-x^i_0)\delta(y-y^i_0) \tag{14}$$

where $w^i_0=\int_A r^i(x,y)dxdy$. It must be pointed out that for certain set attributes of network, all nodes satisfy $\rho^i(x,y)=r^i(x,y)$ and $\int_A \rho^i(x,y)dxdy=0$, except the sink node of this type.

Similar to the case of single network, define that $\vec{D}^i(x,y)$ shows the size and direction of type i on point (x,y), then get the following equation visually:

$$\begin{cases} \vec{\nabla} \cdot \overrightarrow{D}^i(x,y) = \rho^i(x,y) \\ D_n^i(x,y) = 0, \forall (x,y) \in \partial A, 1 \le i \le N \end{cases} \quad (15)$$

3.2 Solve the differentiated-area WSN routing

Define the following quadratic cost function. First, modify the minimum cost differential equation of (15) as follows:

$$\begin{cases} \min imize \ J(\overline{D}) = \int_A K(\chi) \sum_{i=1}^{N} \sum_{j=1}^{N} h_{ij} \mid \overrightarrow{D}^i(\chi) - \overrightarrow{D}_{av}^j \mid ds \\ where: \\ \vec{\nabla} \cdot \overrightarrow{D}^i = \rho \\ D_n(\chi) = 0, \ \chi \in the \ edge \ of \ A \end{cases} \quad (16)$$

In equation (16), coefficient h_{ij} is a real-valued constant ($h_{ij} > 0$). Its definition is the extension of single-area optimization problem and assumes the single-area $h_{ii} > 0$. The optimal solution problem of the cost function above takes the transmission of different attributes of differentiated-area message packets into consideration and contain the intersection of attributes. If introduce a $N \times N$ matrix H to show the significance of h_{ij} in the vector field, the optimal solution problem above can be described in the following way:

$$\begin{cases} \min imize \ J(\overline{D}) = \int_A K(\chi) \sum_{i=1}^{N} \sum_{j=1}^{N} \underline{\overrightarrow{D}^i}^T(\chi) H \underline{\overrightarrow{D}^i}(\chi) \ ds \\ where: \\ \vec{\nabla} \cdot \overrightarrow{D}^i = \rho \\ D_n(\chi) = 0, \ \chi \in the \ edge \ of \ A \end{cases} \quad (17)$$

In formula (17), $\overrightarrow{D}^i(\chi) = [\underline{\overrightarrow{D}^i}^1(\chi), \underline{\overrightarrow{D}^i}^2(\chi), \underline{\overrightarrow{D}^i}^3(\chi), \cdots \underline{\overrightarrow{D}^i}^N(\chi)]^T$ is a column vector with the length of N and $\underline{\overrightarrow{D}^i}(\chi) = \overrightarrow{D}^i(\chi) - \overrightarrow{D}_{av}^i \quad (1 \le i \le N)$.

The expansion of $\underline{\overrightarrow{D}^i}^T(\chi) H \underline{\overrightarrow{D}^i}(\chi)$ includes both the scalar product of two vectors and the scalar-multiplication result of real number and vector, so the following equation can be obtained without considering the order of iterated multiplication:

$$\underline{\overrightarrow{D}^i}^T(\chi) H \underline{\overrightarrow{D}^i}(\chi) = \sum_{i=1}^{N} \sum_{j=1}^{N} h_{ij} \overrightarrow{D}^{i}(\chi) \cdot \overrightarrow{D}^{j}(\chi) \quad (18)$$

Lemma 3: If H is a positive definite matrix, then for the nonzero vector $\overrightarrow{D}$, $\underline{\overrightarrow{D}}^T(\chi) H \underline{\overrightarrow{D}}(\chi)$ is also positive definite.

The lemma above shows that the optimal solution problem of equation (18) is, actually, the positive definite matrix determination problem of $\underline{\overrightarrow{D}}^T(\chi) H \underline{\overrightarrow{D}}(\chi)$. It provides a key idea for the optimal solution problem of equation (17).

Proof:

Matrix H can be written as $H = R^T \Pi R$, where Π is a diagonal matrix containing positive eigenvalues and R is an orthogonal matrix.

Let $\lambda_1, \lambda_2, \lambda_3, \cdots \lambda_N$ represent the opposite diagonal eigenvalues in Π, then get

$$\underline{\overrightarrow{D}}^T(\chi) H \underline{\overrightarrow{D}}(\chi) = \underline{\overrightarrow{D}}^T(\chi) R^T \Pi R \underline{\overrightarrow{D}}(\chi) \quad (19)$$

Introduce a new variable symbol $\overrightarrow{\Lambda}$ to represent vector $R\overrightarrow{D}$, i.e. $\vec{\Lambda} = R\overrightarrow{D}$, and let $\overrightarrow{\Lambda_1}, \overrightarrow{\Lambda_2}, \overrightarrow{\Lambda_3}, \cdots \overrightarrow{\Lambda_N}$ represent $\overrightarrow{\Lambda}$'s eigenvalues, then get:

$$\underline{\overrightarrow{D}}^T(\chi) H \underline{\overrightarrow{D}}(\chi) = \sum_{i=1}^{N} \lambda_i \left| \overrightarrow{\Lambda_i} \right|^2 \quad (20)$$

Through vector operations, equation (15) can be directly rewritten as:

$$\begin{cases} \vec{\nabla} \cdot \overrightarrow{\Lambda}^i = \sum_{i=1}^{N} \vartheta_{ij} \rho^j \\ \Lambda_n^i(\chi) = 0, \ \chi \in the \ edge \ of \ A \end{cases} \quad (21)$$

where ϑ_{ij} represents matrix R's eigenvalue in line i and row j, and $\Lambda_n^i(\chi)$ is vector field $\overrightarrow{\Lambda}^i$'s scalar component. Then, equation (18)'s optimal solution problem can be written as the following equation:

$$\begin{cases} \min imize \ J(\overline{D}) = \int_A K(\chi) \sum_{i=1}^{N} \lambda_i \mid \overline{\Lambda_i} - \overline{\Lambda}_{av} \mid^2 ds \\ where: \\ \vec{\nabla} \cdot \overrightarrow{\Lambda}^i = \rho^j \\ \Lambda_n^i(\chi) = 0, \ \chi \in the \ edge \ of \ A \end{cases} \quad (22)$$

Solving the differential equation can help to find the ideal routing in the differentiated area model. The analysis results of single-area model show that the area optimal solution of each type is unique. Equation (16) can be expressed with several equations of (22), which means equation (16) also has the unique optimal solution.

From the analysis above, it can be found that differentiated-area WSN vector field theoretical model can be decomposed into several single-area load vector fields, which provides an effective theoretical basis to find the optimal solution of differentiated-area load vector field.

4. Simulation and Analysis

Considering the distribution of load vector field in differentiated-area sensor networks, the paper assumes that the messages generated in the source nodes are divided into five types and all messages are finally received by five sink nodes respectively according to message types. Let the coordinates of five sink nodes be (1.5, 2.5), (2, 4), (3, 3), (4, 1) and (4, 4), respectively. Moreover, the load is distributed evenly in the sensor network area; nodes in the network are in the same conditions; the initial energy is distributed uniformly. Decompose the differentiated-area load vector field into several single area problems and then solve the problems to get the load vector filed of the five sink nodes.

Set the Neumann boundary conditions in the boundaries of sensor network. Each sink node corresponds to a load vector field in the single area. Calculate the gradient value of each point in the sensor network area with the superposition principle, and draw up the points with equal load potentials in the area. With MATLAB, it's easy to show the changes of differentiated-area load gradient and the equipotential line in plane area visually. As shown in figure 2, the search of routing always proceeds in the fastest changing direction, low to high, of equipotential line value.
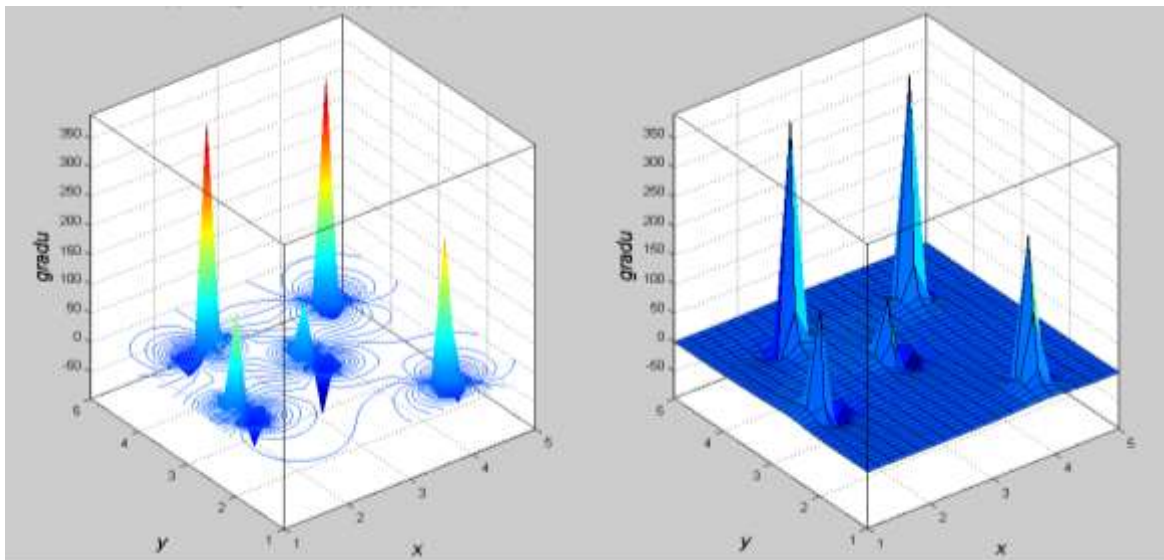

Figure 2: Gradient Expression and Equipotential Line of Multiple Sink Nodes

5. Conclusions

The paper introduces a routing method of WSN differentiated area based on the vector field. The paper defines a differentiated-area sensor network in which different types of messages have different routing modes. Different messages have different characteristics like the rate and the position of destination node. To solve the routing problem of differentiated-area WSNs, the paper defines a vector field to describe the routing and the load of each type of area, and builds a quadratic cost function to explore the load in each type of area and the interactions of areas. The research result shows that if the quadratic cost function has certain specific attributes, the differentiated-area network problem can be decomposed into several single-area network problems.

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