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P-Graph Approach to Allocation of Inoperability in Urban Infrastructure Systems

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Inoperability input-output modeling (IIM) was introduced as a methodology for determining ripple effects propagating through interdependent infrastructure systems as a result of disruptive events such as natural disasters. It is based on the dimensionless metric of inoperability which indicates degree of failure along a scale ranging from 0 to 1. Previous approaches have focused on calibration of interdependencies based on records of economic statistics; IIM has also been used mainly for identifying the vulnerability and criticality of system components. More recent work has demonstrated that the IIM framework can be the basis for optimal allocation of inoperabiltiy in order to minimize damage caused by disruptions. In this work, we propose a P-graph methodology derived from IIM. Interdependency coefficients are integrated within a P-graph model to enable limited capacity of infrastructure following a disruption to be optimally allocated. We demonstrate this methodology using a literature case study.

1. Introduction

Risk analysis has developed as an essential means of ensuring the safe operations of increasingly complex man-made systems. It is now recognized that systematic approaches are needed to understand both potential causes and consequences of adverse events. For example, in the case of urban infrastructure, risks may arise from port facilities (Kazaras et al., 2013) or road networks (Pastorino et al., 2014). According to Kaplan and Garrick (1981), proper risk analysis entails identifying:

- What can go wrong?
- What is the likelihood of each adverse event?
- What are the consequences of each adverse event?

The complexity of modern urban infrastructure systems makes the third risk analysis question especially crucial to decision-making. One of the methods developed recently to elucidate the potential of "ripple effects" to cascade through interdependent infrastructure systems is *inoperability input-output modelling* (IIM), which was first proposed by Haimes and Jiang (2001). The IIM framework is based on economic input-output (IO) analysis (Leontief, 1936), which describes linkages among economic sectors via linear equations. IO methods are sufficiently well-establishes as a tool for such purposes as economic analysis; fundamental principles as well as many of the potential applications can be found in modern textbooks (Miller and Blair, 2009). Conventional IO models measure physical or monetary flows of goods, but in principle, the same mathematical framework can be used to describe non-physical flows as well. For example, a recent work by Hester and Adams (2013) used an IO model to describe workplace influence patterns. Similarly, Haimes and Jiang (2001) proposed the dimensionless quantity known as *inoperability* as a normalized index of risk in infrastructure systems. Inoperability assumes a value of 0 for systems in normal state, fractional values for partially dysfunctional systems, and 1 for systems in a state of complete failure. Furthermore, it is assumed that inoperability exhibits linear behaviour, a property which has enabled it to be used as a quality index in source-sink systems solved via Mathematical Programming (Tan et al., 2011) or Pinch Analysis (Tan and Foo, 2013). Subsequent work refined the definition of

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inoperability as a fractional loss relative to a nominal desired state, which has enabled the calibration of IIM models from government economic statistics (Santos and Haimes, 2004). Alternative approaches for the calibration of IIM models have also been proposed, for example via fuzzy set theory and expert judgement (Setola et al., 2009). The methodology has been applied to the analysis of various disruptive events, such as intentional attacks (Santos and Haimes, 2004) and loss of key economic or natural resources (Khanna and Bakshi, 2009).
Beyond a purely descriptive application of impacts of adverse events, there have also been attempts to integrate IIM with optimization techniques to enable active management of ripple effects. An early linear vector optimization model was proposed by Kananen et al. (1990) using an IO framework. A brief description of an optimization-based extension of IIM was given by Haimes and Jiang (2001), and developed further in a subsequent paper (Jiang and Haimes, 2004). Tan et al. (2015a) proposed a linear programming model to evaluate the capacity of systems to absorb exogenous shocks. An optimization model for allocating capacity in urban systems under crisis conditions was developed by Holden et al. (2013). A recent approach based on fuzzy optimization for the economy-wide allocation of scarce resources was recently proposed by Tan et al. (2015b). In addition, it was recently shown by Aviso et al. (2015) that similar problems can be addressed using P-graph methodology; this is a graph theoretic approach whose mathematical foundations were proposed by Friedler et al. (1992a), and whose various applications have been surveyed in aa recent review (Lam, 2013). Analogous approaches for abnormal operations of polygeneration systems have also been proposed based on both mathematical programming (Kasivisvanathan et al., 2013) and P-graphs (Aviso et al., 2015).
In this paper, a P-graph based approach is proposed for the allocation of inoperability to interdependent urban infrastructure systems under an IIM framework. This hybrid methodology allows ripple effects caused to be managed to ensure that minimal overall damage is incurred for the system. The rest of the paper is organized as follows. Section 2 first gives a formal problem statement. The IIM framework is briefly described in Section 3, while the P-graph methodology is discussed in Section 4. Then, a case study based on the classic four-component example of Haimes and Jiang (2001) is shown in Section 5 to illustrate the methodology. Finally, concluding remarks are then given in Section 5.

2. Problem Statement

The formal problem statement is as follows:

- Given a system comprised of multiple infrastructure system components with known levels of mutual interdependency
- Given an adverse event that induces an initial inoperability in a subset of the said system components
- Given a function that describes the total system-level dysfunction (i.e., aggregated inoperability), taking into account both direct and indirect impacts of the adverse event
- The problem is to allocate reduced infrastructure capacity to give the minimum level of total system-level dysfunction

Mathematical programming and P-graph solution approaches are discussed in the succeeding sections.

3. IIM Methodology

The basic formulation of the IIM (Haimes and Jiang, 2001) is:

$$(\mathbf{I} - \mathbf{A}^*)\ \mathbf{q} = \mathbf{c}^* \qquad (1)$$

Where $\mathbf{I}$ is the identity matrix, $\mathbf{A}^*$ is the square interdependency matrix, $\mathbf{q}$ is the inoperability vector and $\mathbf{c}^*$ is the perturbation vector. Each element in $\mathbf{A}^*$ gives degrees of influence or interdependency of one component on another. For feasible systems, this equation may be rearranged to give:

$$\mathbf{q} = (\mathbf{I} - \mathbf{A}^*)^{-1}\ \mathbf{c}^* \qquad (2)$$

Eq(2) gives the total system inoperability, $\mathbf{q}$, as a function of the perturbation $\mathbf{c}^*$. The inverse $(\mathbf{I} - \mathbf{A}^*)^{-1}$ has amplification properties, such that each element of $\mathbf{q}$ is larger than the corresponding element in $\mathbf{c}^*$. The amplification simply means that total losses ($\mathbf{q}$) are greater than the direct impacts ($\mathbf{c}^*$) due to additional indirect effects resulting from system topology as described by $\mathbf{A}^*$.

Both Eq(1) and Eq(2) have zero degrees of freedom, thus allowing a unique solution to be identified for a given system. By introducing new assumptions, additional degrees of freedom may be obtained to allow for system optimization, resulting in the linear programming (LP) model:

minimize $\mathbf{w}^T\mathbf{q}$ (3a)

subject to:

$(\mathbf{I} - \mathbf{A}^*)\ \mathbf{q} = \mathbf{c}^*$ (3b)

$\mathbf{c}^*_L \leq \mathbf{c}^* \leq \mathbf{c}^*_U$ (3c)

$0 \leq q_j \leq 1 \quad \forall j$ (3d)

Where $\mathbf{w}$ is the weight vector that quantifies the importance of each infrastructure system component, and $\mathbf{c}^*_L$ and $\mathbf{c}^*_U$ are the lower and upper limits of perturbation $\mathbf{c}^*$. For the perturbed infrastructure $i = k$, we have $c^*_{k,L} = c^*_{k,U} = c^*_k$. For the infrastructure system components not subject to direct disruption ($i \neq k$), it is assumed that some $c^*_{i,L}$ can assume small negative values due to the presence of spare capacity. Thus, the spare capacity may be used to introduce negative perturbations in the system so as to offset anticipated losses from ripple effects. It should be noted that such negative perturbation levels have not been discussed in IIM literature, although in principle there can be physical basis for their existence. Hence, this model minimizes the total weighted inoperability of the system for a given perturbation from an adverse event using such interventions. Additional system constraints can be added to extend this into a mixed-integer linear programming (MILP) formulation, as described by Tan (2011).

4. P-Graph Methodology

P-graph methodology is a graph theoretic approach to solving process network synthesis (PNS) problems. Its axioms and theorems were first described by Friedler et al. (1992a), while the resulting implications for the development of solution algorithms were described in a subsequent paper (Friedler et al., 1992b). In particular, the rigorous approach to maximal structure generation is an important element to ensuring a comprehensive coverage of possible solutions (Friedler et al., 1993). Various applications covering a period of two decades are documented by Lam (2013). Meanwhile, recent applications to optimal adjustments to crisis conditions have been described for both economic (Aviso et al., 2015) and engineering (Tan et al., 2014) systems.

P-graph methodology is based on the description of streams and process units. Streams may be further classified as raw materials or products. The main components of the methodology are:

- Maximal structure generation (MSG) – the identification of all possible connections given a predefined set of processes.
- Solution structure generation (SSG) – the identification of feasible subsets of the maximal structure, each of which represents a candidate solution topology
- Accelerated branch and bound (ABB) – the identification of an optimal solution, where reduction of computing effort is achieved by restricting search to the feasible solution structures only

P-graph methodology may be implemented using the software PNS Draw and PNS Studio, both of which are available from the P-graph web site (P-graph, 2015), along with on-line tutorials.

5. Case Study

This case study is based on the four-component urban infrastructure system described in (Haimes and Jiang, 2001). The system is comprised of:

- Component 1 – Power infrastructure (e.g., power plant and transmission grid)
- Component 2 – Transportation infrastructure (e.g., road and rail networks)
- Component 3 – Health infrastructure (e.g., hospitals and clinics)
- Component 4 – Commercial infrastructure (e.g., grocery stores and retail outlets)

The coefficients of the interdependency matrix $\mathbf{A}^*$ are given in Table 1. The coefficients may be interpreted as in the following example. Complete failure of the power infrastructure (j = 1) will result in 40 % inoperability in the transportation infrastructure (i = 2) and 100 % inoperability of the health (i = 3) and commercial infrastructure (i = 4).

A storm is then assumed to render the transportation infrastructure partially inoperable, such that $\mathbf{c}^* = (0\ \ 0.2\ \ 0\ \ 0)^T$. However, it is assumed here that there is 10 % excess capacity in the power infrastructure which is normally left unutilized. Such spare capacity may be activated to offset the system disruptions. The maximal structure is shown in Figure 1. Solving the model via P-graph results in the solution shown in

Figure 2, where $c^*_1 = -0.1$. This solution implies that surplus capacity is activated in anticipation of system losses from ripple effects.

Table 1: Interdependency coefficients for the urban infrastructure system (Haimes and Jiang, 2001)

	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	0.9	0	0
$i = 2$	0.4	0	0	0
$i = 3$	1	0.8	0	0
$i = 4$	1	0.9	0	0

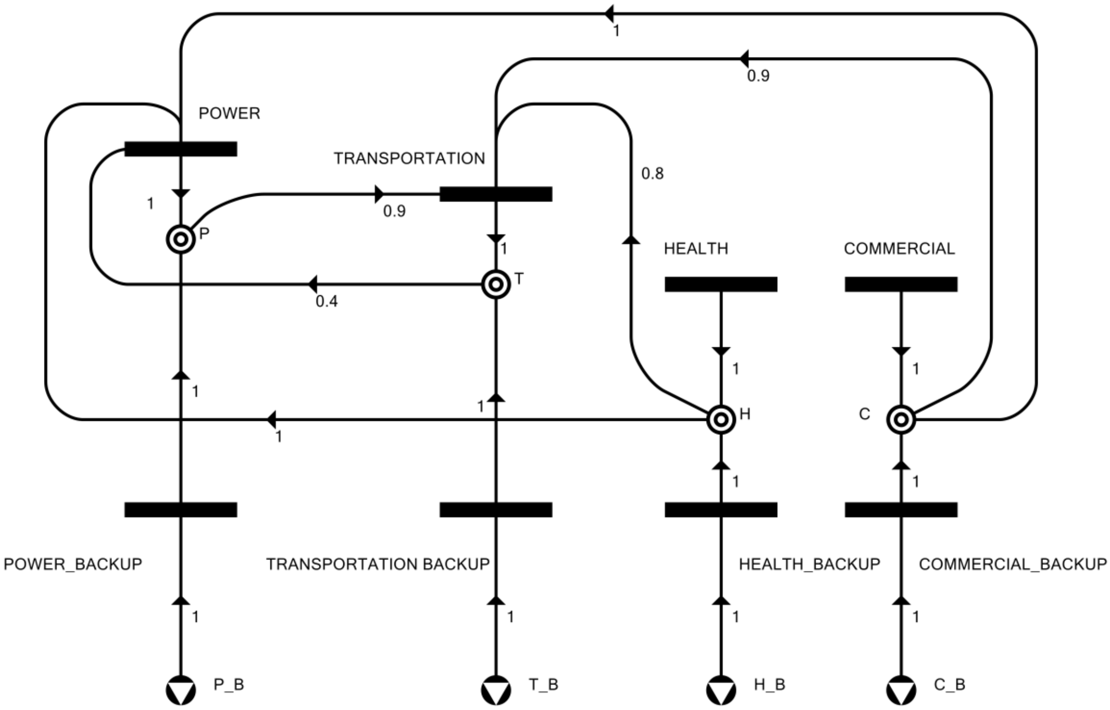

Figure 1: P-graph representation of maximal structure of infrastructure system

The values of perturbations and inoperability for the four infrastructure system components in the optimal solution are given in Table 2. Note that the same solution may be derived by solving the model defined by Eq(3a – d).

Table 2: Perturbation and inoperability of urban infrastructure system

	Perturbation	Inoperability
Power infrastructure	–0.1	0.125
Transportation infrastructure	+0.2	0.250
Health infrastructure	0	0.325
Commercial infrastructure	0	0.350

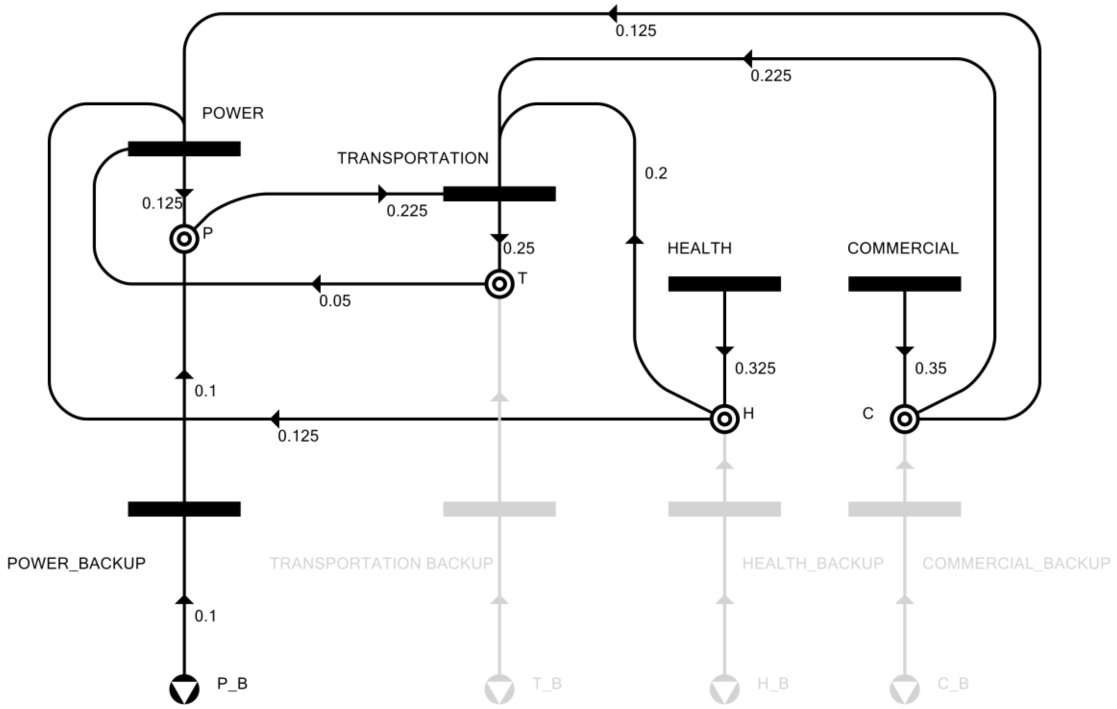

Figure 2: P-graph representation of optimal inoperability flows in infrastructure system

6. Conclusions

A P-graph based approach to the allocation of inoperability in urban infrastructure systems has been proposed in this work using the IIM framework. This hybrid methodology enables limited system capacity to be allocated in order to minimize system-wide losses incurred as ripple effects cascade through interdependent system components. A case study from literature involving interdependent urban infrastructure systems was solved to illustrate this approach. The solution achieved via P-graph methodology in this example identifies how excess capacity in electricity generation can be activated during the crisis to mitigate inoperability in other key system components. Similar principles can be readily extended to more complex urban infrastructure systems. Future work will address further research issues pertaining to calibration of the interdependency matrix and definition of priority weights assigned to each infrastructure system. This approach may further be extended for optimal planning of spare capacity.

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