

# Optimal Sizing of a Photovoltaic-Thermal Hybrid System using Energy Ratio

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A photovoltaic-thermal hybrid system (PVT) is a renewable energy based cogeneration system as it simultaneously converts solar radiation to electricity and low temperature heat. For applications with limited space, such as building integrated systems, use of a PVT system may be better than installing separate collectors to produce electrical and thermal energy. Reduction of total space requirement for an integrated PVT system depends on the electrical and thermal energy requirements. The concept of energy ratio is introduced in this paper for selection of appropriate solar device to satisfy the thermal and electrical energy demands. A mathematical optimization problem is formulated to identify the range of energy ratio for which installation of PVT system is optimal.

## 1. Introduction

A photovoltaic-thermal hybrid system (PVT) is a renewable energy based cogeneration technology. It produces both electricity and heat by means of one integrated component, in which cells are applied on the thermal absorber. Dupeyrat et al. (2014) performed TRNSYS simulation to show that the integration of PV and thermal energy capture on a limited roof area provides higher electrical and exergy output. Many efforts are directed towards increasing thermal and electrical performances of PVT system: channel type with parabolic reflector (Garg and Agarwal, 1995), sheet and tube type (Zondag et al., 2002); booster reflectors (Tripanagnostopoulos et al., 2002), two absorber type (Bakker et al., 2002), polymer channel absorber (Sandnes and Rekstad, 2002), single-crystalline silicon with flat reflectors (Kostić et al., 2010), single glazed flat plate PVT with heat exchanger (Dupeyrat et al., 2011), unglazed box type prototype collector (Thakare et al., 2013), prototype PVT collector with square channel (Evola and Marletta, 2014). An integration of renewable energy in the form of photovoltaic power for running a heat and water system also presented by Martínez-Patiño et al. (2012) The performance improvement includes efforts to improve system performance by maintaining an optimum water flow rate (Kalogirou, 2001; Fudholi et al., 2014). The optimum flow rate result in increase the thermal and electrical performance of the PVT system (Chow, 2010). Aste et al. (2012) presented a methodology to calculate the best value of solar fraction ' $f$ ' for hybrid PVT systems under energetic end economic point of views.

The concept of energy ratio is introduced in this paper for selection of appropriate solar device to satisfy the thermal and electrical demands. A mathematical model is proposed to determine energy ratio ( $\alpha$ ) for choice of side by side PV and Thermal collector systems vis-a-vis PVT system. A simple methodology is developed to take decision about the solar device with the help of two dimensionless parameters i.e. area ratio ( $a_r$ ) and  $\alpha$ . The performance of PVT system over side by side PV and Thermal system is assessed for given solar insolation, ambient temperature and load profiles.

## 2. Mathematical model

A typical PVT based cogeneration system contains a PVT collector which is made up from a PV laminate. In this system, thermal energy is recovered which is otherwise lost from the PV laminate. It decreases the operational temperature of the PV laminate and increases the electrical efficiency. The thermal energy generated by the system is stored in storage tank and the thermal load is served at specific load temperature

where as electrical energy is stored in battery bank. The electrical load is supplied with the help of charge controller and inverter. As a consequence, in a limited space application, in comparison with separate PV and thermal system, an appropriate 'α' of PVT based cogeneration system can serve the both thermal and electrical demand efficiently. A PVT panel model proposed by Zondag et al. (2002) is adopted in this paper to explain the concept of 'α' of the PVT based renewable cogeneration system. A mathematical problem is formulated with the objective of minimizing the panel area with given thermal and electrical energy requirements. The optimization model formulation is mixed integer non linear programming (MINLP) problem for unity solar fraction PVT system with immediate water replenishment. The model divides in two parts as follows.

### 2.1 Thermal subsystem formulation

For PVT system, useful hourly thermal energy generation Eq(1) and electrical efficiency Eq(2) are the basic equations. In these,  $A_{pv}$  is the area of collector and  $\eta_{el}$  the electrical efficiency of the cells.  $F_r$  is the heat removal factor and  $T_{sti}$  and  $T_a$  are the hourly inlet and ambient temperatures respectively. The average solar radiation, incident on the PVT collector, in an hour and loss coefficient are given by  $I_T$  and  $U_L$ , respectively.  $\tau_a$  and  $\tau$  are the transmission absorption factor and transmission factor of glass. The mean plate temperature and reference temperature of cell are given by  $T_{pm}$  and  $T_{ref}$ . The symbol  $\eta_{ref}$  indicate reference cell efficiency at 25 °C and  $\beta$  is the cell efficiency temperature coefficient.

$$Q_u(t) = A_{pv} F_r ((\tau_a - \tau \eta_{el}) I_T(t) - U_L (T_{sti}(t) - T_a(t))) \quad (1)$$

$$\eta_{el}(t) = \eta_{ref} (1 - \beta (T_{pm}(t) - T_{ref})) \quad (2)$$

Selection of useful thermal output is carried out through binary variables  $x_1(t)$  and  $x_2(t)$ . The binary variable  $x_1(t)$  is used to separate out positive useful thermal gain. The same is then used to calculate hourly mean plate temperature ( $T_{pm}$ ) of the collector using Eq(3). The  $T_{pm}$  is a function of useful heat gain, heat transfer coefficient across cell and absorber ( $h_{ca}$ ),  $A_{pv}$  and mass flow rate ( $\dot{m}$ ). The hourly useful thermal output and electrical efficiency of the PVT collector are determined as the  $T_{pm}$  converged to constant value. Getting the value of electrical efficiency, the electrical output is determined by multiplying it with collector area and solar insolation.

$$T_{pm}(t) = x_1(t) \times \left( T_{sti}(t) + \frac{Q_u(t)}{(2 \times \dot{m} \times C_p)} + \frac{Q_u(t)}{(h_{ca} \times A_{pv})} \right) + x_2(t) \times \left( T_a(t) + \frac{(\tau_a - \tau \eta_{el}(t)) \times I_T(t)}{U_L} \right) \quad (3)$$

The demand met by solar energy ( $q_{ls}(t)$ ) to meet the load at a mass flow rate  $\dot{m}$  is assumed to be constant over a time step  $t$  and is calculated based on the initial temperature of the storage tank ( $T_{sti}$ ). The thermal energy supplied to load is calculated as,

$$q_{ls}(t) = \dot{m} C_p (T_{sti}(t) - T_a(t)) \quad (4)$$

The surface area of the cylindrical storage tank is assumed to be related to the storage volume ( $V_{st}$ ) by following relation with equal height to diameter ratio (Kulkarni et al., 2009).

$$A_{st} = 5.54 (V_{st})^{2/3} \quad (5)$$

The storage heat losses in the storage tank are estimated as:

$$Q_{stl}(t) = U_{st} A_{st} (T_{sti}(t) - T_a(t)) \quad (6)$$

The model predicts the hourly storage tank temperature using Eq(7).

$$T_{stf}(t) = \left\{ T_{sti}(t) + \frac{(Q_u^+(t) + q_{ls}(t) - q_{stl}(t))}{\rho \times C_p \times V_{st}} \right\} \quad (7)$$

In order to supply the thermal load at specific load temperature, load temperature constraint; and to avoid phase change of working fluid in a collector, saturation temperature constraint are imposed on the storage tank temperature respectively. The hourly storage temperature is obtained iteratively for a day assuming thermal steady state operation of storage tank.

### 2.2 Electrical subsystem formulation

The electrical energy generated by the PVT collector in an hour is calculated by Eq(8). The change in storage energy level can be represented as the difference between the electrical energy generated by the PVT

collector ( $P(t)$ ), the energy required by the load ( $D(t)$ ) and the excess energy ( $P_{du}(t)$ ) which is to be dumped. The energy balance on the electrical subsystem is accounted using Eq(9).

$$P(t) = \eta_{el}(t) \times A_{pv} \times I_T(t) \quad (8)$$

$$PS(t) = P(t) - D(t) - P_{du}(t) \quad (9)$$

The binary variables  $y1(t)$ ,  $y2(t)$  are used to select amount of energy transfer during charging or discharging respectively. The battery energy level for an hour is given by Eq(10).

$$Q_B(t) = Q_B(t-1) + y1(t) \times PS(t) \times CD1 + y2(t) \times PS(t) \times CD2 \quad (10)$$

where CD1 and CD2 are the charging and discharging efficiencies of the battery. During operation some of the excess energy needs to be dumped so that the battery energy level is kept to a minimum while meeting the load at all times. The daily excess energy dumped is considered as a non negative quantity.

The battery bank has to store larger energy than the hourly battery energy level. The constraint of maximum amount of energy that battery can store in a day is accounted by the variable BBC. The battery size (Br) is calculated using Eq(11) considering the depth of discharge of battery (DOD).

$$Br = BBC / DOD \quad (11)$$

### 2.3 Objective function

In order to satisfy given demands for a given time period, area of PVT collector has load temperature and storage related constraints. The solar useful thermal output, mean plate temperature and electrical output of a collector is decided by the area of PVT collector. The objective function described in Eqs(1), (3), (8) and (12) is to minimize area of PVT collector.

$$\text{Minimize } A_{pv} \quad (12)$$

The optimum solution is obtained using General Algebraic Modeling System (GAMS, 2007) software with BARON solver for the above MINLP formulation.

### 2.4 Computation of Energy ratio

Being solar renewable cogeneration, PVT generates both thermal and electrical energy. Hence, the daily thermal energy ( $Q_{TL}$ ) and electrical energy ( $Q_{EL}$ ) require to supply the thermal and electrical demands are need to be considered to compute the energy ratio. The energy ratio,  $\alpha$  is then defined as the ratio of the daily total thermal energy require to supply the thermal demand to the sum of total thermal and electrical energy require to meet the both thermal and electrical demands. The value of  $\alpha$  is of then calculated by obtaining the optimal area of PVT collector for the particular thermal and electrical demands with the mentioned thermal and electrical constraints by Eq(13).

$$\alpha = Q_{TL} / (Q_{TL} + Q_{EL}) \quad (13)$$

The value of  $\alpha$  lies between 0 and 1 for combine thermal and electrical demands where as it is 0 for only electrical demand and 1 for only thermal demand. In the analysis of the proposed energy ratio, a numerical optimization is carried out to determine minimum area of PVT collector and side by side PV and thermal system to meet the various thermal and electrical demands.

To determine the area require for individual PV and thermal system, the above explained model is adapted by considering the thermal and electrical subsystem formulation separately. The MINLP formulation constitutes by Eqs(8-11, 12) of electrical subsystem are used to determine area of PV system. The performance of the PV module depends on the PV cell temperature. The factors affecting cell temperature includes mainly ambient temperature, solar irradiance and wind speed etc. To account effect of cell temperature, mean plate temperature ( $T_{pm}$ ) in electrical efficiency Eq(2) is modified with the cell temperature ( $T_{cell}$ ) for PV system. Typically, over a range of environmental conditions, the nominal operating cell temperature (NOCT) is used to predict the cell temperature. According to the definition, NOCT is the temperature of the cells at a solar irradiance ( $G_{NOCT}$ ) of 800 W/m<sup>2</sup>, an ambient temperature ( $T_{a,NOCT}$ ) of 20 °C, and a wind speed of 1 m/s. As polycrystalline cell is considered for PVT and PV panel, the NOCT value of polycrystalline cell is taken as 44 °C (Davis et al., 2001). The performance of PV system is analyzed with the following cell temperature equation (Markvart, 2000).

$$T_{cell}(t) = T_a(t) + \left( \frac{NOCT - T_{a,NOCT}}{G_{NOCT}} \right) * I_T(t) \quad (14)$$

It is to be noted the behaviour of thermal system constitute non linear programming (NLP) mathematical formulation for separate thermal system. As only thermal output is generated by thermal system, the useful thermal output Eq(1) of PVT system is changed to Eq(15) to analyze the thermal system performance. The area require to meet the thermal demand by separate thermal system is determined using Eqs (3-7, 12 and 15).

$$Q_u(t) = A_{pV}((F_r \tau \alpha) I_T(t) - F_r U_L (T_{sti}(t) - T_a(t))) \quad (15)$$

The optimal area obtained in individual PV and thermal subsystem formulation is then added to get total area of side by side PV and thermal system.

### 2.5 Computation of Area ratio

The concept of non dimensional area, area ratio  $a_r$ , is introduced here to analyze area requirement for PVT system in comparison with separate PV and thermal system. The area ratio,  $a_r$  is defined as the ratio of the area of PVT to supply the both thermal and electrical demands to the sum of total area require by individual PV ( $A_{Photovoltaic}$ ) and Thermal ( $A_{Thermal}$ ) system to meet the thermal and electrical demand separately.

$$a_r = A_{pV} / (A_{Photovoltaic} + A_{Thermal}) \quad (16)$$

The value of  $a_r$  more than 1 infers demands can be met by separate PV and thermal system where as for  $a_r$  less than 1, PVT requires less area to serve the both demands.

## 3. Sizing of solar devices using energy ratio

Influence of energy ratio on sizing of solar devices is analyzed in this section. Two cases of electrical load dominant and thermal load dominant systems are presented to show the comparison of space requirement for PVT over side by side PV and thermal system. Keeping the same time-varying nature of load profile, energy ratio is obtained by varying total energy requirement. For the given insolation and ambient temperature, the energy ratio obtained in the range of 0 to 1. The size of the each solar device to satisfy thermal and electrical demands is obtained using optimization model proposed in earlier section. The options available to satisfy thermal and electrical demands include combine PVT system and individual PV panel and thermal collector. The methodology is explained with the help of flowchart in Figure 1. The parameter values used for the calculation of thermal system are referred from Zondag et al. (2002). The reference cell efficiency is considered as 9.7 % at 25 °C where as battery charging and discharging efficiencies are assumed to be 86 %. The DOD of battery is assumed to be 50 %. The results obtained with both kind of electrical and thermal load dominant system are presented below.

### 3.1 Electrical load dominant system

The effect of change in energy ratio on sizing of PVT and separate PV and Thermal system is analyzed in this section. The solar insolation and ambient temperature data for March 15th for a rural location near Mumbai (19°17'N, 72°48'E) has been used for calculations (Krishna Priya et al., 2013). The electrical load pattern is adapted from Arun et al. (2008) and the thermal load pattern from Pillai and Banerjee (2007). The thermal load is 180 litres per day (LPD) at 60 °C. The Figure 2 shows sizing curve for the electrical load dominant system. It shows nature of ' $\alpha$ ' verses area ratio ' $a_r$ ' for solar devices. It indicates for ' $a_r$ ' more than 1, individual PV panel or solar thermal collector requires less space as compared to integrated PVT system. The choice of integrated PVT panel can be beneficial for ' $a_r$ ' less than 1. From the Figure 2, it can be seen that the value of ' $a_r$ ' is more than 1 for ' $\alpha$ ' nearer to 0 or 1. It means for the ' $\alpha$ ' approaches to 0 electrical energy requirement is dominant where as ' $\alpha$ ' closed to 1 thermal energy requirement is predominant. The value of ' $\alpha$ ' closed to zero suggests PV panel require less space as compared to PVT panel where as the ' $\alpha$ ' closed to 1 indicate solar thermal collector require less space over integrated PVT panel. It is due to the fact that the cogeneration of electrical energy and thermal energy decide the area of PVT system. It indicates, if one kind of the load is not present then that energy generated is not being used, which results in increase in area as compare to side by side PV and Thermal system. It is observed for electrical dominant system, for ' $\alpha$ ' in the range of 0.1 to 0.93, integrated PVT panel has ' $a_r$ ' less than 1. The area required to satisfy both thermal and electrical demands is less for PVT system during this ' $\alpha$ ' range. It is due to the combined efficiency of integrated PVT panel is more than the individual PV panel and solar thermal collector (Zondag et al., 2003). It can be seen that the point 'x' in the Figure 2 corresponds to ' $\alpha$ ' of 0.67 and ' $a_r$ ' of 0.6 represent optimal value of  $\alpha$  to fetch maximum benefit of space reduction. The point 'x' indicates balance kind of thermal and energy requirement for PVT panel. The appropriate value of ' $\alpha$ ' can result in less space requirement in building integrated solar applications. It also helps to use of solar energy optimally to satisfy both thermal and electrical demands.

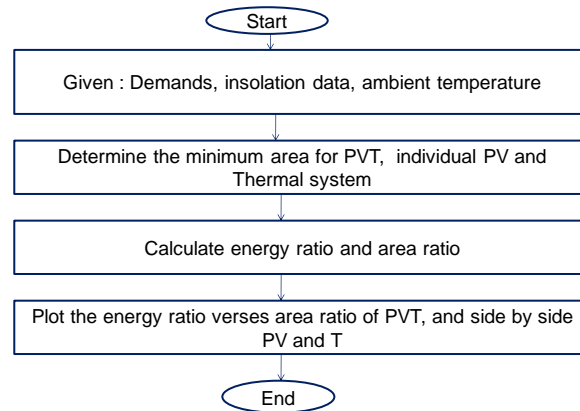


Figure 1: Methodology for sizing of solar devices

### 3.2. Thermal load dominant system

To analyze effect of ' $\alpha$ ' on sizing of PVT and individual PV and Thermal system, the electrical load pattern is adapted from Arun et al. (2008) and the thermal load pattern from Kulkarni et al. (2009). The domestic thermal load is considered as 4,500 LPD at 60 °C. For this case, monthly mean values of hourly solar radiation (Mani, 1981) on April 15 are used. The sizing curve for the thermal load dominant system is shown in Figure 3. It shows usefulness of ' $\alpha$ ' to decide option available to serve thermal and electrical demands. The area requires to supply both the demands is less for a PVT system for ' $\alpha$ ' in the range of 0.08 to 0.95. The ' $\alpha$ ' lying outside the range (0.08 - 0.95), suggests less area require for individual PV and thermal system to supply electrical and thermal demands respectively. The point 'y' in Figure 3 corresponds to ' $\alpha$ ' of 0.82 and ' $a_r$ ' of 0.55 indicates the optimal balance of thermal and electrical demand for the thermal load dominant system. It gives idea of balanced thermal and electrical load for PVT system to serve at overall optimum. Such balanced load can results into significant reduced area for PVT system over side by side PV and thermal system in limited space application.

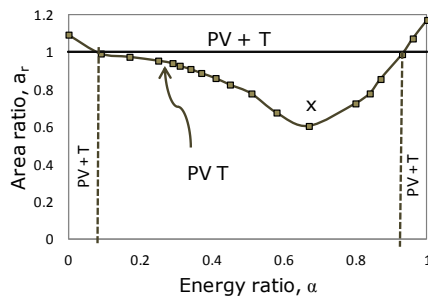


Figure 2: Sizing curve for electrical load dominant system

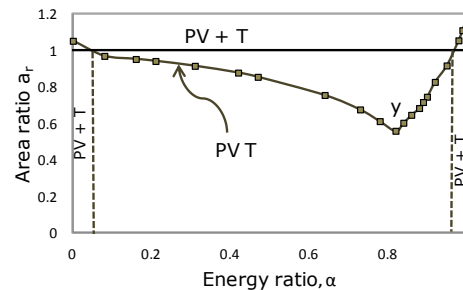


Figure 3: Sizing curve for thermal load dominant system

The ' $\alpha$ ' infers the proportion of both kind of load to be supplied by PVT system and individual PV and thermal system. The thermal energy is predominant after the point 'y' i.e. above ' $\alpha$ ' of 0.82, hence the curve is having positive slope. The higher the area required to meet the prominent thermal load for both PVT and solar thermal collector result in increase in value of ' $a_r$ '.

## 4. Conclusions

A mathematical model, based on MINLP formulation, is presented to select appropriate solar device to meet simultaneously thermal and electrical energy demands. A simple methodology is proposed to take decision about appropriate solar device in view of limited space availability. The concept of energy ratio and area ratio is proposed to decide for sizing either of integrated PVT and side by side PV and thermal system. The energy ratio infers significant reduction in area for PVT system over individual PV and Thermal system for balance kind of thermal and electrical demands. It is seen that there exists a range of energy ratio (0.1 to 0.93 for two examples considered in this paper), within which it is optimal to use PVT system. The future direction of research is to evaluate the effect of energy ratio for other configuration of PVT over side by side PV and

Thermal system. The model formulated is so for unity solar fraction. The further study is directed towards the development of model for variation of solar fractions. The effect of variation in packing factor of PVT on energy ratio is required to be carried out.

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