# Minimization of Off-Grade Production in Multisite Multiproduct Plants by Solving Multiple Traveling Salesman Problem

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Continuous multiproduct plants allow the production of several products (product grades). During grade transitions off-spec products are produced. The economic losses and the environmental impact of these transitions are sequence dependent, so the amount of off-grade products can be minimized by scheduling the sequence of the production of different products. Applying parallel production sites ($m$) increases the flexibility of multiproduct plants. Since market demands are changing, the production cycles of these sites should be re-scheduled in certain intervals. Therefore, our task is to design $m$ production cycles that contains all required products by minimizing the total length of grade transitions. Most production scheduling problems such as the one considered in this paper are NP-hard. Our goal is to solve realistic problem instances in no more than a couple of minutes. We show that this problem can be considered as a multiple traveling salesmen problem (mTSP), where the distances between the products are based on the time or costs of the grade transitions. The resulted mTSP has been solved by multi-chromosome based genetic algorithm. The proposed algorithm was implemented in MATLAB and is available at the website of the authors (www.abonyilab.com). For demonstration purposes, we present an illustrative example. The results show that multiproduct multisite scheduling problems can be effectively handled as mTSPs, and the proposed problem-specific representation based genetic algorithm can be used in wide range of optimization problems.

## 1. Introduction

Thanks to the increasing need for flexible processing facilities to produce more than one product, the planning problem of multiproduct plants is becoming more and more important. The planning of process systems involves the procedures and processes of allocating the available resources and equipment over a period of time to perform a series of tasks required to manufacture one or more products. Typical example for such problem is the optimization of the transformation of biomass to energy. Biomass is usually locally available, which defines it as a distributed resource, and requires extensive infrastructure networks for harvesting, transportation, storage, and processing. The design and management of biomass supply chains should account for the local conditions and constrains, such as the existing infrastructure, geographical features of the studied region and the competition among several consumers. Biomass are wood and forestry residues, energy crops, various kinds of straw, as well as biowaste from food production, wood processing and use. Primary biomass resources are distributed over the area in a region and often available in remote locations. Building the infrastructure to transfer biomass energy over longer distances would tend to increase its cost. On the other hand, biomass offers the potential to reduce the environmental impact of energy supply and potentially saving costs for reacting to natural disasters in the future. An important factor to be considered is the security of energy supply, which has significant importance – considered from strategic viewpoint (Nagy, 2009) as well as efficiency (Klemeš and Lam,

2009). Energy generation from domestic sources helps reducing the dependence on foreign imports of crude oil and natural gas. It increases the economic stability and can improve significantly the foreign trade balances of the respective regions and countries. The relatively low energy density (energy per unit volume) of most raw biomass feedstocks tends to increase the cost, emissions and complexity of supply chain. Therefore, the developments of complete procedure for regional energy supply optimisation become important task. (Čuček et al., 2010)

The goal of this paper is the development production planning and scheduling algorithm for parallel (multi-site) continuous processes in the presence of sequence-dependent switchover times. A similar problem is already studied in (Kopanos et al., 2011). Based on the classic formulation of travelling salesman problem (TSP), a mixed-integer liner programming (MILP) is developed, whose objective function considers the profit, inventory deviations from the desired trajectories and price changes simultaneously (Liu et al., 2012). TSP-based model for medium-term planning of a single-stage plant with a single continuous processing unit producing several products with sequence-dependent changeovers has been already studied in (Liu et al., 2008). TSP is a widely applied technique for scheduling of parts in a flowshop, lot streaming and scheduling problems, and optimization of robot movements in automated production cells (Bagchi et al., 2006). The problem becomes more complex when a multisite production system is needed to be optimized. Such production–distribution network is made up of several production sites distributing to different markets. The planning and scheduling model has to include spatial scales that go from a single production unit within a site to a geographically distributed network (Terrazas-Moreno and Grossmann, 2011).

In this paper we propose multiple-Traveling Salesman Problem based representation for the optimization of multiproduct and multisite production systems, where the distances between the products are based on the time or costs of the grade transitions. The resulted mTSP has been solved by multi-chromosome based genetic algorithm. The chromosome representation and especially the applied operators make our modified genetic algorithm especially effective in the optimization of mTSP problems (Király and Abonyi, 2011). Furthermore, taking additional constraints into consideration, like the maximum number of salesmen or the maximal time a salesperson can travel makes it capable to solve complex structures and in production planning to prevent a single site from overloading.

The proposed algorithm was implemented in MATLAB and available at the website of the authors (ABONYILAB, 2014). For demonstration purposes, we present an illustrative example from the literature, which is from a real world polymer processing plant and discussed by Liu et al. (2008). In this simple example we present the optimization of the production of 10 products by 3 sites, using our genetic algorithm based mTSP solver.

## 2. Problem formulation

As we discussed in the introduction, production scheduling problem usually handled by integer programming methods, like MIP, MILP. For mathematical formulation we can't find any better approaches than these articles present, therefore we follow the conventions of these formulations. The problem can be represented by a distribution network as it can be seen in Figure 1. In this little example, we have a factory with 3 sites which produce 7 different products ($m=3, n=7,$ see later), e.g. site 2 produces product 3 and 7.
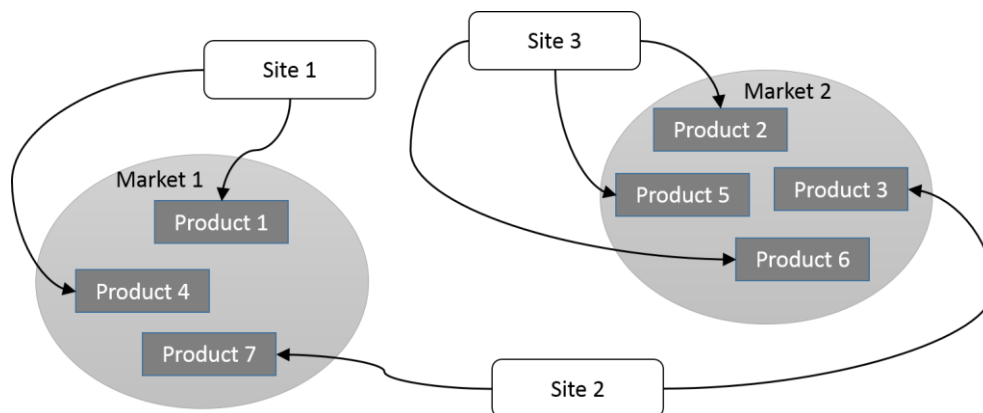


*Figure 1: Layout of the multi-site production planning problem as a distribution network for m=3, n=7*

As the figure demonstrates, there are several (*m*) parallel production sites in the plant which produce several (*n*) products to fulfil the market's demand. Within a site during the transition of production from one product to another, many waste or off-grade products are producing. Our task is to minimize the amount of these waste items. The problem can be defined by simple MIP formulation as follows.

$$\min \sum_m \sum_{i,j \in J_m} C_{ij} \, , \ \forall i, j \in I, m \geq 1 \tag{1}$$

where *I* is the set of items, and $J_m$ is the mth production site. $C_{ij}$ represents the changeover cost from product *i* to *j*. Furthermore, to prevent the sites from overloading, we need to define additional constraints about the minimum number of product types:

$$|J_m| > r \, , \ \forall m \geq 1 \tag{2}$$

### 2.1 mTSP-based formulation

As we mentioned in the previous sections, the minimization of waste items in parallel multiple production sites plants can be handled as a multiple traveling salesmen problem or mTSP. With the following reconciliation we can handle and solve the problem as a route planning task and use an mTSP solver as we will see in the next section:

- grade transition times ←→ traveling distances
- products ←→ locations
- production sites ←→ salesmen

In this ways, the problem is transformed into an mTSP, thus, the following formulation can be used. Let us define the following binary variable $x_{ijk}$:

$$x_{ijk} = \begin{cases} 1 & \text{if } (i, j) \text{ is used on the tour of the } k^{th} \text{ salesman} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Using this binary variable, our objective function can be expressed as follows in Eq(4-9):

$$\min \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij}^{t} \cdot \sum_{k=1}^{m} x_{ijk} \tag{4}$$

so that

$$\sum_{j=1}^{n} \sum_{k=1}^{m} x_{1jk} = m \tag{5}$$

$$\sum_{i=1}^{n} \sum_{k=1}^{m} x_{i1k} = m \tag{6}$$

$$\sum_{i=0}^{n} \sum_{k=1}^{m} x_{ijk} = 1, \ i = 2, \ldots, n \tag{7}$$

$$\sum_{j=0}^{n} \sum_{k=1}^{m} x_{ijk} = 1, \ j = 2, \ldots, n \tag{8}$$

$$\sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij}^{t} \cdot x_{ijk} \geq r \, , \ k = 1, \ldots, m \tag{9}$$

## 3. The proposed GA-based method to solve mTSP problems

There are many articles discussing how to solve TSP and mTSP problems by the help of genetic algorithms (GAs). In this paper we use our previously published method, which is a multi-chromosome representation based GA (Király and Abonyi, 2011). The concept of this approach is presented in the following figure, together with the corresponding route system. In the bottom part of the figure, the correspondence between the parallel multi-production site plant and the route system is demonstrated. As it can be seen, the whole problem is decoded into a single individual, where different chromosomes represent the different salesmen or sites. Note that first location or product is not presented here, since it has a special attribute. In route systems, it is the depot from where all trucks depart and arrives back, and in production scheduling it represents the idle state of the site.
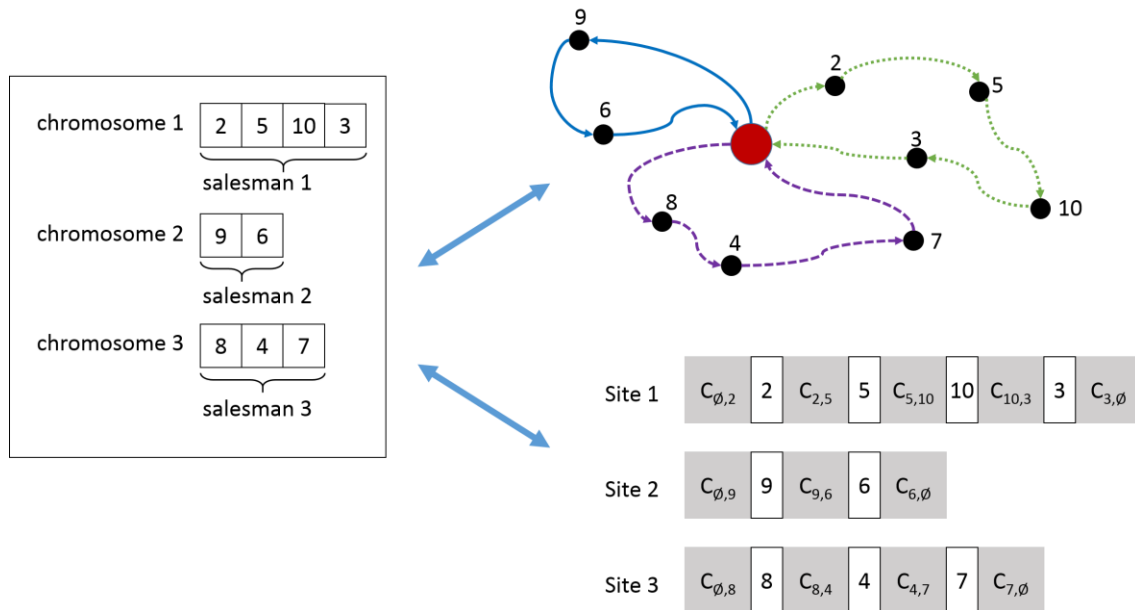


*Figure 2: Concept of the multi-chromosome representation based GA for production site scheduling. On the left side, a single individual is shown, containing 3 chromosomes, which corresponds to the 3 salesmen in the upper right corner depicting a route plan, or to the 3 production sites of the production scheduling in the bottom right corner*

As all GA, our algorithm starts from a random population, where each product is produced by a random site, in a random order. The algorithm uses the evolutionary strategy to select, recombine and mutate the individuals with best fitness value to produce the new population. The fitness function is defined in Eq() and Eq(), as the sum of all changeover times in the production, therefore the smallest the fitness the better the individual. After predefined iteration steps, the procedure stops and the best individual will represent the optimal solution of the problem.

Our method uses the special chromosome representation discussed above, and therefore requires special operators to recombine the individuals inside a population. These special and complex operators ensure the strong convergence of our GA as well as the aforementioned constraints. We present one complex operator hire, other ones can be found in (Király and Abonyi, 2011). The operator depicted in Figure 4 is a complex one, which combines two mutation operations applying them sequentially. The firs one is the so-called "slide" which moves the last gene (location, product) from each chromosome (route, production site) to the beginning of another one. The second operator, "swap" choses two random sequences of genes from two chromosomes and transposes them. Of course, these randomly chosen sequences can be empty, thus, the operator will be realized as the insertion of the nonempty sequence to a randomly chosen place in the other chromosome.
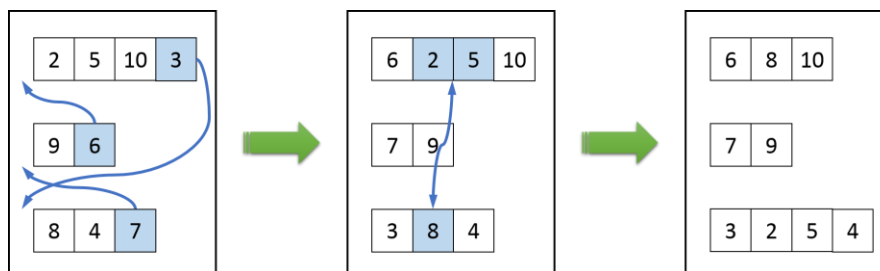
*Figure 3: Complex genetic operator used by the algorithm for mutation. From left to right: applying a "slide" and a "swap" sequentially*

## 4. Results and discussion

In this paper we present a representative example to illustrate our GA based mTSP solver to schedule multiproduct multisite problems. The example derived from a real world polymer processing plant is the same which is discussed by Liu et al. (2008) as an extension of the instance presented in (Chen et al., 2008). Here, 10 types of products (A-J) are manufactured, and we use a multi-site plant to satisfy customer demands. Table 1 shows the changeover times, and Ø represents the initial state of a site.

*Table 1: Representative example: Product grade transition times (min)*

| From/To | Ø | A | B | C | D | E | F | G | H | I | J |
|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Ø | | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| A | 20 | | 45 | 45 | 45 | 60 | 80 | 30 | 25 | 70 | 55 |
| B | 20 | 55 | | 55 | 40 | 60 | 80 | 80 | 30 | 30 | 55 |
| C | 20 | 60 | 100 | | 100 | 75 | 60 | 80 | 80 | 75 | 75 |
| D | 20 | 60 | 100 | 30 | | 45 | 45 | 45 | 60 | 80 | 100 |
| E | 20 | 60 | 60 | 55 | 30 | | 35 | 30 | 35 | 60 | 90 |
| F | 20 | 75 | 75 | 60 | 100 | 75 | | 100 | 75 | 100 | 60 |
| G | 20 | 80 | 100 | 30 | 60 | 100 | 85 | | 60 | 100 | 65 |
| H | 20 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | | 60 | 60 |
| I | 20 | 80 | 80 | 30 | 30 | 60 | 70 | 55 | 85 | | 100 |
| J | 20 | 100 | 100 | 60 | 80 | 80 | 30 | 45 | 100 | 100 | |

Using the multi-chromosome representation, we used our genetic algorithm to construct optimal production cycles with minimal overall grade-transition cost. We wanted to produce a balanced production plan, therefore the number of plants was chosen to 3 as well as the minimum tour length of each salesman. The number of individuals in the population was 80 and the GA took 2,000 iterations in less than 10 s. The result is demonstrated by Figure 4.
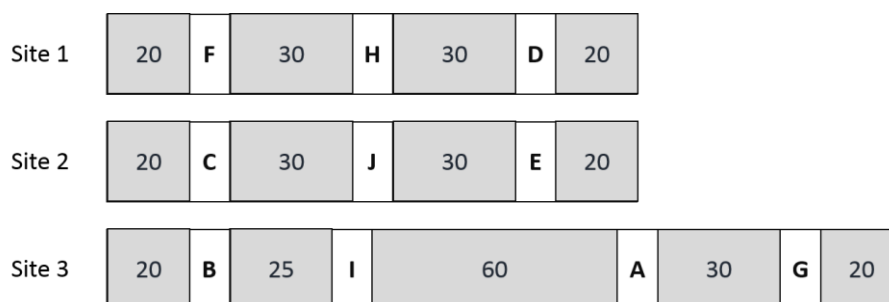


*Figure 4: Optimal production schedule of the representative example as an mTSP problem, using 3 salesmen. The overall changeover cost is 355 min*

The results show that multiproduct multisite scheduling problems can be effectively handled as mTSPs, and the proposed problem-specific representation based genetic algorithm can be used in wide range of optimization problems

## 5. Conclusions

Multiproduct multisite scheduling problems can be effectively handled as mTSPs. We proposed a problem-specific representation based genetic algorithm that can be used in wide range of optimization problems. The proposed algorithm was implemented in MATLAB and available at the website of the authors (www.abonyilab.com). For demonstration purposes, we presented an illustrative example from the literature, which is from a real world polymer processing plant and discussed by Liu et al. (2008). In this simple example we presented the optimization of the production of 10 products by 3 sites, using our genetic algorithm based mTSP solver. Further research will focus on the extension of more general vehicle rooting problems and more detailed application study on the optimization of a complex bioenergy production supply chain, where the sequence of the processing of different waste materials is scheduled.

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