

# Energy Evaluation of an Absorption Heat Transformer Operating with Water-Carrol Solution by Means of Mathematical Programming

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The objective of this work is an energy evaluation based on the coefficient of performance (COP) of an experimental absorption heat transformer (AHT) operating with Water–Carrol solution. The COP is a measure of the thermal efficiency of the system. An optimization tool was used, as it is mathematical programming. The mathematical model was obtained by means of mass and energy balances for each stage of thermodynamic cycle and global balances of AHT. Design parameters (i.e. effectiveness of the economizer), correlations of thermodynamic properties of water and Water-Carrol solution (i.e. specific enthalpy (H), equilibrium temperatures and concentrations) have been used. The optimal values of decision variables to maximize the COP (objective function) are determined using a mathematical programming formulation. The model was programmed in the General Algebraic Modelling System (GAMS) software. Energy performance curves were developed in order to show and establish the best behaviour of the COP in function of the decision variables.

## 1. Introduction

Increasingly is required of alternative systems of energy production from others sources than fossil fuels. The indiscriminate use of fossil fuels, affect to our planet by producing greenhouse gases, mainly carbon dioxide (CO<sub>2</sub>), which involves environmental changes, global warming, and climate and habitability of our world. The technology implementation of absorption heat transformers (AHT) helps to mitigate CO<sub>2</sub> emissions, because we can supply in whole or in part the energy required to perform a process, which otherwise would consume 100 % of non-renewable energy. Another significant advantage of this technology is that it generates savings because energy consumption is reduced by recycling waste energy. The AHT utilizes a thermodynamic cycle with a working pair (Water-Carrol in our case) to collect low-grade waste heat (at low temperature) and generates high-grade useful heat (at high temperature) which can be supplied to another process. For example, the heat recovery by an AHT can be used for a water purification process (Meza et al., 2014); to recover waste heat produced in heavy oil production (Zhang et al., 2014); to build a novel cogeneration system (Huicochea et al., 2013); and to desalinate water (Sekar and Saravanan, 2011). Ibarra-Bahena et al. (2014) carried out a thermodynamic evaluation of an AHT operating with Water-Carrol mixture obtaining a maximum value for the COP of 0.35. Zebbar et al. (2012) worked on thermodynamic optimization of an AHT using the so called structural analysis to find out the optimal operating parameters. Colorado et al. (2011) proposed a methodology to calculate the optimal operating conditions for an AHT by means of an artificial neural network inverse. In last two cases, the AHT operated with Water-LiBr mixture.

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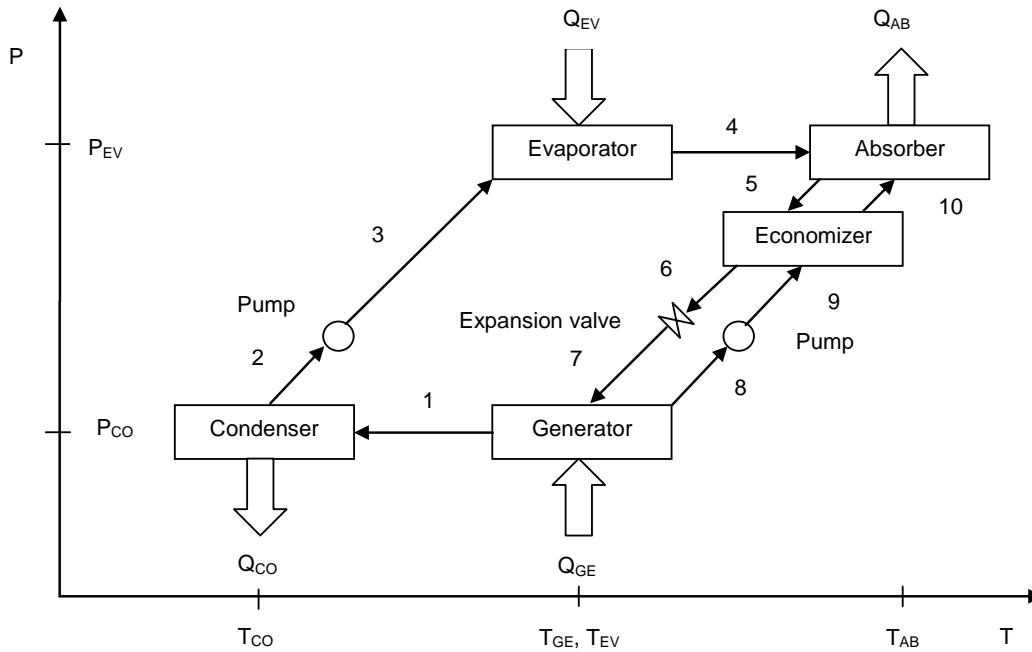


Figure 1: Absorption heat transformer cycle in the pressure-temperature process variables

## 2. Thermodynamic cycle

An AHT has five main heat exchanger devices: evaporator, condenser, generator, absorber and an economizer (see Figure 1). A constant quantity of waste heat ( $Q_{GE}$ ) is supplied to generator (with solution concentration  $X_{GE}$ ) to vaporize the working fluid (water) from a solution of an absorbent (Carrol) in the working fluid at temperature ( $T_{GE}$ ). The vaporized working fluid is condensed in the condenser at temperature ( $T_{CO}$ ) and low pressure ( $P_{CO}$ ), rejecting the heat ( $Q_{CO}$ ) to a cooling medium. The working fluid in the condenser is pumped to the evaporator at high pressure ( $P_{EV}$ ), where it evaporates by extracting heat ( $Q_{EV}$ ) from the waste heat source at temperature ( $T_{EV}$ ). The vapour of working fluid is then absorbed by absorbent at high temperature ( $T_{AB}$ ) in the absorber (with solution concentration  $X_{AB}$ ) and the heat ( $Q_{AB}$ ) is delivered to a heat sink as useful heat. Finally, the diluted solution from the absorber is throttled and returned to the generator through the economizer, where it exchanges heat to preheat the concentrated solution pumped from the generator to the absorber before repeating the cycle again.

## 3. Mathematical model

For the development of the mathematical model the following assumptions were made: (a) there is thermodynamic equilibrium throughout the entire system; (b) the analysis is made under steady-state conditions; (c) a rectifier is unnecessary since the absorbent does not evaporate in the temperature range under consideration; (d) the solution is saturated leaving the generator and the absorber, and the working fluid is saturated leaving the condenser and evaporator; (e) heat losses and pressure drops in the tubing and the components are considered negligible; (f) the work done by the pumps is considered negligible; (g) the work done by the pumps is isentropic; (g) the flow through the valves is isenthalpic; (h) the temperature in the evaporator is equal to temperature in the generator; (i) the heat load in the evaporator  $Q_{EV}$ , and the effectiveness of the economizer ( $EF_{HX}$ ) are known. Taking account the above assumptions, and applying the first law of thermodynamics and the principles of mass and species conservation the following mathematical model can be obtained.

### 3.1 Nomenclature

Symbols:  $Q$  (heat power, kW);  $H$  (specific enthalpy,  $\text{kJ kg}^{-1}$ );  $M$  (mass flow,  $\text{kg s}^{-1}$ );  $T$  (temperature,  $^{\circ}\text{C}$ ),  $X$  (salt solution concentration, % w/w).

Subscripts: AB (absorber), CO (condenser), EV (evaporator), GE (generator), HX (economizer), V (saturated steam), S (solution in equilibrium), L (saturated liquid), HXT (stream entering the exchanger from the generator but at absorber temperature).

Equations are referred to the system of Figure 1.

### 3.2 Generator

$$Q_{GE} = M_1 H_1 + M_8 H_8 - M_5 H_6 \quad (1)$$

$$M_8 = M_5 - M_1 \quad (2)$$

$$H_1 = H_V(T_{GE}) \quad (3)$$

$$H_8 = H_S(X_{GE}, T_{GE}) \quad (4)$$

$$H_6 = \frac{M_8}{M_5} (H_8 - H_{10}) + H_5 \quad (5)$$

### 3.3 Evaporator

$$Q_{EV} = M_1 (H_4 - H_2) \quad (6)$$

$$H_4 = H_V(T_{EV}) \quad (7)$$

$$H_2 = H_L(T_{CO}) \quad (8)$$

### 3.4 Condenser

$$Q_{CO} = M_1 (H_2 - H_1) \quad (9)$$

### 3.5 Absorber

$$Q_{AB} = M_1 H_4 + M_8 H_{10} - M_5 H_5 \quad (10)$$

$$T_{AB} = T_S(X_{AB}, T_{EV}) \quad (11)$$

$$H_5 = H_S(X_{AB}, T_{AB}) \quad (12)$$

$$H_{HXT} = H_S(X_{GE}, T_{AB}) \quad (13)$$

$$H_{10} = H_8 - EF_{HX} (H_8 - H_{HXT}) \quad (14)$$

### 3.6 Coefficient of performance (COP)

$$COP = \frac{Q_{AB}}{Q_{EV} + Q_{GE}} \quad (15)$$

### 3.7 Flow ratio (FR)

The flow ratio (FR) is an important design and optimizing parameter. The FR is related to the size of all AHT components and the power of pumps; a high FR value implies high power of the pumps and a higher size of AHT. It can be defined as the ratio of the mass flow rate of solution coming from the absorber to the generator ( $M_8$ ) by the mass flow rate of the working fluid ( $M_1$ ).

$$FR = \frac{M_8}{M_1} \quad (16)$$

It can be rewritten in terms of solution concentration as follows:

$$FR = \frac{X_{GE}}{X_{GE} + X_{AB}} \quad (17)$$

### 3.8 Effectiveness of the economizer (EF<sub>HX</sub>)

Another important design parameter is the effectiveness of the economizer (EF<sub>HX</sub>). It is defined as the ratio of the actual heat transfer rate by the maximum possible transfer heat exchange rate (Wang et al., 2007). An acceptable value of the effectiveness of the economizer is 0.7, as was evidenced by the work of Ibarra-Bahena et al. (2013) in a study of a plate heat exchanger (PHE). In the present work, the value of the effectiveness of the economizer was assumed equal to 0.7 (EF<sub>HX</sub>=0.7).

### 3.9 Correlations of thermodynamics properties of water and Water-Carrol solution

Eq(8), (3) and (7) were calculated by means of following equations reported by Torres (1997), for specific enthalpy of water and specific vaporization enthalpy:

$$H_L = -0.3349 + 4.2298T - 9.1418 \times 10^{-4}T^2 + 5.4512 \times 10^{-6}T^3 \quad (18)$$

$$H_v = 2501.9 + 1.7464T + 1.3868 \times 10^{-3}T^2 - 1.4289 \times 10^{-5}T^3 \quad (19)$$

Eq(4), (11), (12) and (13) were calculated by means of correlations reported by Reimann and Biermann (1984), for properties of Water-Carrol solution.

## 4. Statement of the optimization problem

### 4.1 Equations, variables and input parameters of the model

There are 17 equations and 22 variables in this mathematical model. The heat supplied to the evaporator (Q<sub>EV</sub>= 1 kW) and effectiveness of the economizer (EF<sub>HX</sub>=0.7) are input parameters of the model.

### 4.2 Degrees of freedom analysis

This analysis provides the number of decision variables one can change to obtain the optimum design, and is fundamental in optimization (Diwekar, 2008). This analysis is shown in Table 1.

### 4.3 The objective function and decision variables

The selected objective function to be optimized is the COP (maximization), because it is a measure of the thermal efficiency of the system. Based on the degrees of freedom, the five decision variables of interest are as follows: FR (flow ratio), X<sub>GE</sub> (solution concentration in the generator), T<sub>CO</sub> (temperature in the condenser), T<sub>EV</sub> (temperature in the evaporator) and T<sub>GE</sub> (temperature in the generator).

### 4.4 Constraints

The constraints of the optimization problem are the feasible operating ranges of the decision variables in the AHT.

Table 1: Degrees of freedom analysis

Number of variables	Number of equations	Degrees of freedom
22	17	5

Table 2: Constraints of the decision variables

Decision variable	Constraint	Units
FR	13 ≤ FR ≤ 60	dimensionless
X <sub>GE</sub>	55 ≤ X <sub>GE</sub> ≤ 75	% w/w
T <sub>CO</sub>	20 ≤ T <sub>CO</sub> ≤ 30	°C
T <sub>EV</sub>	70 ≤ T <sub>EV</sub> ≤ 95	°C
T <sub>GE</sub>	70 ≤ T <sub>GE</sub> ≤ 95	°C
T <sub>EV</sub> , T <sub>GE</sub>	T <sub>EV</sub> = T <sub>GE</sub>	°C

Table 3: Results of GAMS software, optimal values for the decision variables to maximize the COP

Decision variable	Constraint	Optimal value	Units
FR	$13 \leq FR \leq 60$	13	dimensionless
$X_{GE}$	$55 \leq X_{GE} \leq 75$	68.8	% w/w
$T_{CO}$	$20 \leq T_{CO} \leq 30$	30	°C
$T_{EV}$	$70 \leq T_{EV} \leq 95$	70	°C
$T_{GE}$	$70 \leq T_{GE} \leq 95$	70	°C

The maximized value of the COP is 0.470 (dimensionless)

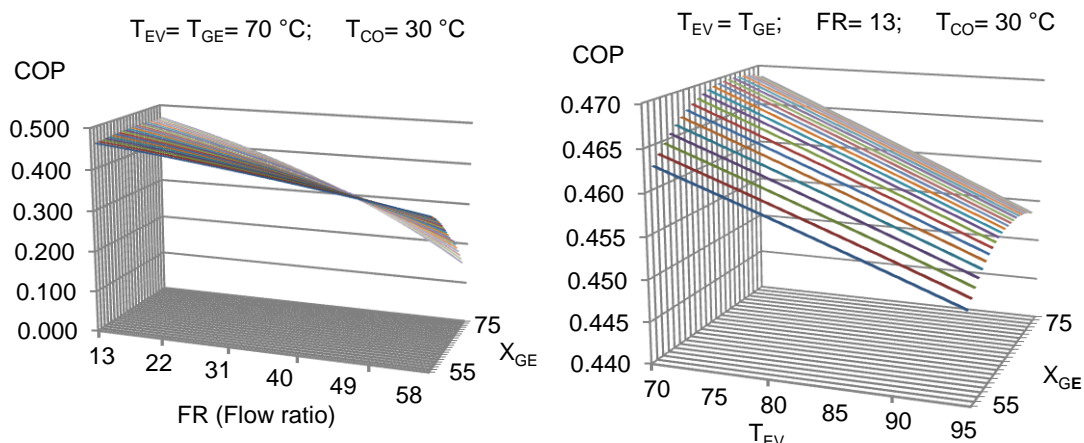
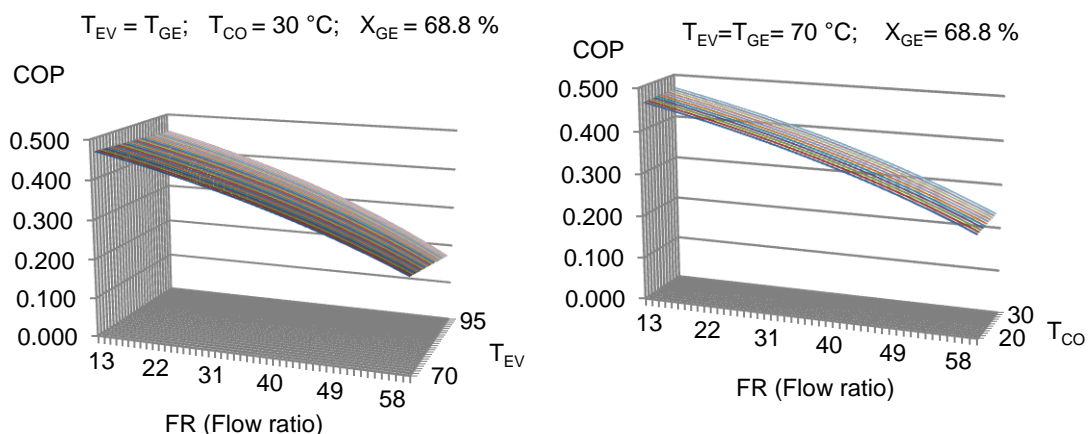
#### 4.5 Numerical optimization

The software used was *General Algebraic Modelling System* (GAMS), that it is optimization software. Because correlations of thermodynamics properties of water and Water-Carrol solution are nonlinear, the type of optimization problem is nonlinear programming (NLP).

### 5. Results and discussion

In Table 3 are shown the calculated optimal values by means of GAMS software.

The optimal value for FR is the lower limit and also for  $T_{EV}$  and  $T_{GE}$ . In the case of  $T_{CO}$ , the optimal value is the upper limit. With respect to  $X_{GE}$ , its optimal value is close to the upper limit (75 %) and it has a value of 68.8 %. Each of the energy performance curves (Figures 2, 3 and 4) show the best behaviour of the COP in function of specific decision variables, all other variables remain constant with their optimal values and are identified. Also, in all performance curves, the optimized value of the COP (0.470) is reached.

Figure 2: Energy performance curves, COP in function of FR and  $X_{GE}$  (left);  $T_{EV}$  and  $X_{GE}$  (right)Figure 3: Energy performance curves; COP in function of FR and  $T_{EV}$  (left); FR and  $T_{CO}$  (right)

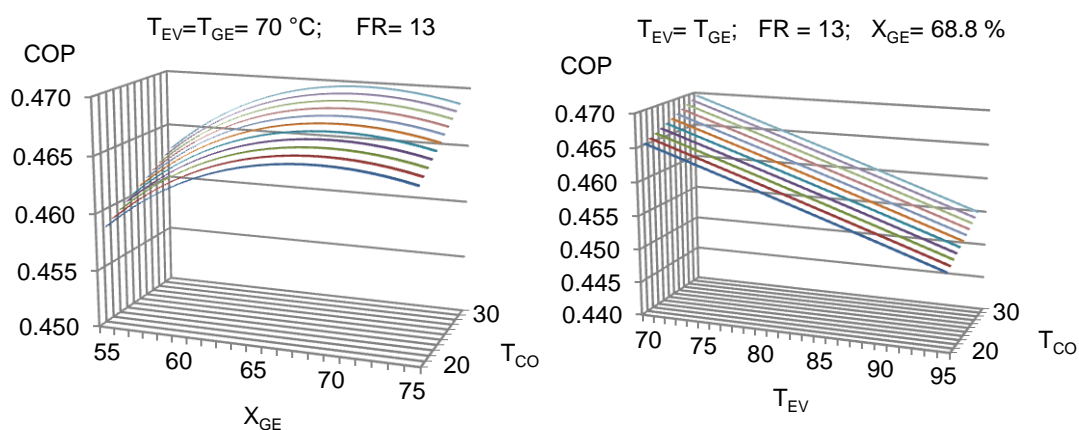


Figure 4: Energy performance curves, COP in function of  $X_{GE}$  and  $T_{CO}$  (left);  $T_{EV}$  and  $T_{CO}$  (right)

## 6. Conclusions

Mathematical programming was applied to AHT in order to obtain its behaviour of the COP in function of the decision variables (FR,  $X_{GE}$ ,  $T_{CO}$ ,  $T_{EV}$  and  $T_{GE}$ ). The maximum COP of the system is reached (0.47), and thereby the maximum thermodynamic efficiency. The energy performance curves show the best energy behaviour of the system model in function of the specific decision variables with their optimal values. In these curves, we can observe that the maximum COP and the optimal value of each decision variable are reached (FR = 13,  $X_{GE}$  = 68.8 %,  $T_{CO}$  = 30 °C,  $T_{EV}$  =  $T_{GE}$  = 70 °C) confirming the results obtained by means of GAMS software. This paper aims to contribute to focus our efforts for maximum utilization of waste energy and thereby obtain energy savings and minimize environmental pollution.

## References

- Colorado D., Hernández J. A., Rivera W., Martínez H., Juárez D., 2011, Optimal operation conditions for a single-stage heat transformer by means of an artificial neural network inverse, *Applied Energy*, 88, 4, 1281-1290.
- Huicochea A., Romero R. J., Rivera W., Gutierrez-Urueta G., Siqueiros J., Pilatowsky I., 2013, A novel cogeneration system: A proton exchange membrane fuel cell coupled to a heat transformer, *Appl. Therm. Eng.*, 50, 2, 1530-1535.
- Ibarra-Bahena J., Romero R. J., Velazquez-Avelar L., Valdez-Morales C. V., Galindo-Luna Y. R., 2014, Experimental thermodynamic evaluation for a single stage heat transformer prototype build with commercial PHEs, *Appl. Therm. Eng.*, DOI: 10.1016/j.applthermaleng.2014.05.018.
- Ibarra-Bahena J., Romero R. J., Velazquez-Avelar L., Valdez-Morales C. V., Galindo-Luna Y. R., 2013, Evaluation of the thermodynamic effectiveness of a plate heat exchanger integrated into an experimental single stage heat transformer operating with Water/Carrol mixture, *Exp. Therm. Fluid Sci.*, 51, 257-263.
- Meza M., Márquez-Nolasco A., Huicochea A., Juárez-Romero D., Siqueiros J., 2014, Experimental study of an absorption heat transformer with heat recycling to the generator, *Exp. Therm. Fluid Sci.*, 53, 171-178.
- Reimann R., Biermann W., 1984, Development of a single family absorption chiller for use in solar heating and cooling system, Phase III Final Report, Prepared for the U. S. Department of Energy under contract EG – 77 – C – 03 – 1587, Carrier Corporation.
- Sekar S., Saravanan R., 2011, Experimental studies on absorption heat transformer coupled distillation system, *Desalination*, 274, 292-301.
- Torres M., 1997, Gas-liquid contactors for multi-stage absorption heat pumps, PhD thesis, L'institut National Polytechnique de Lorraine, Vandoeuvre-lès-Nancy, France.
- Wang L., Sundén B., Manglik, 2007, Plate Heat Exchangers: Design, applications and performance, WIT Press, Billerica, USA.
- Zebbar D., Kherris S., Zebbar S., Mostefa K., 2012, Thermodynamic optimization of an absorption heat transformer, *International Journal of Refrigeration*, 35, 5, 1393-1401.