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Combining Multi-Parametric Programming and NMPC for the Efficient Operation of a PEM Fuel Cell

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This work presents an integrated advanced control framework for a small-scale automated Polymer Electrolyte Membrane (PEM) Fuel Cell system. At the core of the nonlinear model predictive control (NMPC) formulation a nonlinear programming (NLP) problem is solved utilising a dynamic model which is discretized based on a direct transcription method. Prior to the online solution of the NLP problem a pre-processing search space reduction (SSR) algorithm is applied which is guided by the offline solution of a multi-parametric Quadratic Programming (mpQP) problem. This synergy augments the typical NMPC approach and aims at improving both the computational requirements of the multivariable nonlinear controller and the quality of the control action. The proposed synergetic framework is deployed to the automation system of the unit and its online response is explored by a comparative experimental case study that reveals its applicability and efficiency with respect to the fulfilment of multiple desired objectives under physical and operational constraints.

1. Introduction

Polymer Electrolyte Membrane Fuel cells (PEMFCs) are a versatile and efficient electricity generation technology that can be applied in a wide range of industries, from autonomous back-up power units and portable consumer electronic devices to vehicles and primary energy systems. Driven by their emerging potential it is important to ensure their optimum behaviour. Thus, a control strategy able to manage the interactions between the various devices and components of an integrated PEMFC system must be employed, considering the long-term reliability and safety of the system. Such strategy can be developed with the aid of advanced model predictive control (MPC) methods which can effectively handle multivariable problems under physical and operating constraints (Menon and Marechal, 2012). More specifically in this work, a novel combination between two well established MPC methods will be presented and subsequently the resulting controller will be deployed to a PEM fuel cell unit in order to verify its ability to concurrently achieve the desired operating objectives.

2. Proposed synergetic NMPC framework

This work presents the combination of two MPC-based strategies in a unified control framework that takes advantage of their individual features and characteristics. The first methodology is an online Nonlinear Model Predictive control (NMPC) strategy that handles dynamic nonlinearities of the process under consideration. The second methodology is an explicit or multi-parametric Model Predictive Control

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(mpMPC) strategy, which provides the optimal control action in real-time, as the solution is computed offline and is implemented online by simple look-up functions (Bemporad et al., 2002). The unified framework is based on an NMPC formulation where an online finite-time constrained optimization problem is solved over a prediction horizon (T_p), using the current state of the process as the initial state. The optimization yields an optimal control sequence ($u_k..u_{k+Nc}$) over a control horizon (T_c), which is partitioned into N_c intervals and only the first control action (u_k) for the current time is applied to the system. We consider the following formulation of the NMPC problem (Mayne et al., 2000):

$$\min_u J = \sum_{j=1}^{Np} (\hat{y}_{k+j} - y_{sp,k+j})^T QR(\hat{y}_{k+j} - y_{sp,k+j}) + \sum_{l=0}^{Nc-1} \Delta u_{k+l}^T R1 \Delta u_{k+l} \tag{1}$$

s.t.: $\dot{x} = f_d(x,u), \quad y = g(x,u)$

$\hat{y}_{k+j} = y_{pred,k+j} + e_k$, $e_k = (y_{pred} - y_{meas})_k$

where *u,y,x* are the manipulated, the controlled and the state variables, $y_{pred}, y_{meas}, y_{sp}$ are the predicted, the measured variables and the desired set-points and *QR, R1* are the output tracking and the input move weighing matrices. The minimization of functional *J* (eq. 1) is also subject to constraints of *u, x* and *y*.

2.1 Simultaneous dynamic optimization

The finite-time constrained optimization problem of the NMPC formulation (1) is solved by a direct simultaneous method using an augmented Lagrangian solver (Murtagh and Saunders, 1998). The nonlinear dynamic model of the process is discretized based on an orthogonal collocation on finite elements method, whereas the variable profiles are approximated with a family of Lagrange polynomials on finite elements (NE). Within each finite element these profiles are further discretized around collocation points (N_{cop}), determined as the shifted roots of Legendre polynomials:

$$x(t) \approx \sum_{j=0}^{Ncop} x^{i,j}\Omega_j(t), \quad \Omega_j(t) = \prod_{k=0,k\neq j}^{Ncop} \frac{(t - t_{i,k})}{(t_{i,j} - t_{i,k})} \quad , i = 1..NE, \ j = 0..N_{cop}, \ t \in [t_i, t_{i+1}] \tag{2}$$

where t is the scalar independent dimension defined in the fixed domain $[0, t_f]$, $x^{i,j}$ is the value of the state vector at collocation point *j* of the *i*th finite element and Ω is the basis function. Respectively the algebraic variables ($z^{i,j}$) and input variables (u^i) are approximated. After this discretization the optimization problem is expressed as a large-scale but sparse NLP problem (Biegler et al., 2002):

$$\min_{x^{i,j}, z^{i,j}, u^i} \sum_{i=1}^{NE} \sum_{j=1}^{Ncop} w_{i,j}\phi\left(x^{i,j}, z^{i,j}, u^i\right) \quad , i = 1..NE, \ j = 1..N_{cop} \tag{3}$$

$$\text{s.t.}: \ \sum_{k=0}^{Ncop} \dot{\Omega}_k(\tau_{i,j})x^{i,k} = h_i f_d\left(u^i, x^{i,j}, z^{i,j}\right), \quad 0 = f_a\left(u^i, x^{i,j}, z^{i,j}\right) \tag{3a}$$

$$x^{1,0} = x_0, \quad x(t_f) = \sum_{j=0}^{Ncop} x^{NE,j}\Omega_j(1) \tag{3b}$$

$$x^{i,0} = \sum_{j=0}^{Ncop} x^{i-1,j}\Omega_j(1), \ \ \mathrm{i} = 2..NE \tag{3c}$$

where ϕ is the objective function and f_d, f_a are the differential and the algebraic equations, h_i is the length of each element. To enforce zero-order continuity of the state variables at the element boundaries, connecting equations are used (3c). Also all variables are constrained within upper and lower boundaries.

2.2 Multi-parametric quadratic programming problem (mpQP) formulation

The second method that is used for the synergetic framework is the explicit or multi-parametric MPC (mpMPC) method. In mpMPC the optimization problem is solved offline with multi-parametric quadratic programming (mpQP) techniques in order to obtain the objective function and the control actions as functions of the measured state/outputs (parameters of the process) and the regions in the state/output space where these parameters are valid i.e. as a complete map of the parameters (Bemporad et al., 2002). From the solution of the mpQP problem a piecewise affine (PWA) function is derived that describes a polyhedral partition of the feasible variable space. The mpMPC approach involves the use of a discrete-time constrained linear time-invariant system:

$$x_{t+1} = Ax_t + Bu_t + Dw_t \;\;,\;\; y_t = Cx_t \tag{4}$$

where *u,y,x,w* are the manipulated, the controlled, the state, and the disturbance variables and A, B, C, D are fixed matrices. Considering the linear model (4), the MPC problem (1) can be recast as a multi-parametric Quadratic Programming (mpQP) problem which can be solved with standard multi-parametric programming techniques and involves a systematic exploration of the parameter space (ϑ). The resulting map consists of a set of convex non-overlapping polyhedra, critical regions (*CR)*, which are bounded by a unique set of active constraints while the corresponding control law is piecewise linear in the form:

$$A_{CR,i}\vartheta \le b_{CR,i} \Rightarrow u = C_{CR,i}\vartheta + d_{CR,i} \quad i = 1,2,\ldots N_{CR} \tag{5}$$

where N_{CR} is the number of critical regions, $A_{CR}, b_{CR}, C_{CR}, d_{CR}$ are constants defining each region CR, i and the derived optimal control action within.

2.3 Search Space Reduction approach

A major challenge for the online applicability of the NMPC controller is the computational burden caused by the optimization problem which is solved at every sampling interval. A number of very promising works appear in the literature aiming towards this direction such as the use of a warm-start homotopy path method (Ferrau et al., 2008) or advanced preprocessing (Zavala and Biegler, 2009). Our primary objective focuses on the same issue using a warm-start initialization procedure for the NLP solver and a newly proposed Search Space Reduction (SSR) algorithm of the feasible space. The warm-start initialization takes advantage of the information gained from the previous iteration which can significantly decrease the number of iterations towards the optimum point. This warm-start initialization is complemented by an SSR algorithm that defines the region in a variable's feasible space that includes the optimum solution for the given objective function. The center of this region is determined by the suggested solution (u_{mp}) provided by a piecewise affine (PWA) approximation of the system's feasible space. This PWA function is derived offline by solving an mpQP problem using the same concepts of the mpMPC method. In that context, prior to the solution of the NLP problem the upper and lower bounds ($bu_{act,low}, bu_{act,up}$) of the selected manipulated variables are modified based on the suggested solution from a PWA function and a deviation term (e_{bu}) (Ziogou et al., 2013b):

$$e_{bu} = \frac{bu_{f,up} - bu_{f,low}}{by_{f,up} - by_{f,low}} e_{y,\max} \tag{6}$$

where $bu_{f,up}, bu_{f,low}$ are the feasible upper and lower bounds of variable u and $by_{f,up}, by_{f,low}$ are the respective bounds for variable y . The term $e_{y,\max}$ is the maximum model mismatch between the linearized and the nonlinear model and it is determined by an offline simulation study that involves the whole operating range of y . Subsequently the modification of the search space is reduced based on eq. (7):

$$\begin{aligned}
bu_{act,low} &= \begin{cases} u_{mp} - e_{bu} & ,(u_{mp} - e_{bu}) \ge bu_{f,low} \\ bu_{f,low} & ,(u_{mp} - e_{bu}) < bu_{f,low} \end{cases} \\
bu_{act,up} &= \begin{cases} u_{mp} + e_{bu} & ,(u_{mp} + e_{bu}) \le bu_{f,up} \\ bu_{f,up} & ,(u_{mp} + e_{bu}) > bu_{f,up} \end{cases}
\end{aligned} \tag{7}$$

where $bu_{act,low}, bu_{act,up}$ are the active bounds for u . Apart from the bounds modification the rest of the NLP problem formulation remains the same. Based on the above analysis the explicit solution can direct the warm-start procedure for the solution of the NLP problem and thus improve its performance. The proposed method (explicit NMPC, exNMPC) will be subsequently developed and deployed for a fuel cell system.

3. Control of a PEM Fuel Cell unit

The performance of a PEMFC is affected by the operating conditions and the interaction of its subsystems as various phenomena are evolving during its operation. Therefore, it is necessary to control every subsystem in order to protect the long-term performance of the overall system and to address the numerous challenges that arise as the PEMFC operates. A suitable approach for this task is the aforementioned NMPC framework that can handle constraints, multiple objectives and consider the process dynamics. In this work, a small-scale fully automated fuel cell unit is used which was designed and constructed at CPERI/CERTH (Ziogou et al., 2010). The PEMFC unit has four distinct, yet interacting, subsystems which are related to the power production, the gas supply, the temperature and the water management, equipped with a number of sensors and actuators that constitute the unit's Input/Output (I/O) field. A Supervisory Control and Data Acquisition (SCADA) system is developed to acquire the I/O signals and monitor the behaviour of the unit, whereas the control is performed by the aforementioned NMPC framework. At the core of the NMPC framework there is a nonlinear dynamic model which is experimentally validated against data from the PEMFC unit and consists of a set of first principles equations combined with semi-empirical equations (Ziogou et al., 2013a). This model takes into account main variables and parameters, such as, the partial pressures of all gases, the FC current and the operating temperature. The structure of the model is based on a five volume approach, considering the mass dynamics in the gas flow channels, the gas diffusion layers (GDL) and the membrane, an energy balance and a set of electrochemical algebraic equations for the voltage calculation.

3.1 Operation and Control objectives

As stated earlier it is of vital importance to control the various subsystems responsible for maintaining a stable operating environment while ensuring an economically attractive operation. Overall the objective is to effectively address the issue of power generation in an optimum manner. In this context the optimality is defined by the three elements:

- Operation at a safe region regardless of the load fluctuations
- Minimization of the fuel consumption
- Maintenance of stable temperature conditions ensuring proper gas humidification

The power demand (P_{SP}) is achieved by a proper current (I) determination which is applied to the FC by the converter (DC load) connected to the system. The safe operation is maintained by manipulating the air and hydrogen flows ($\dot{m}_{air}, \dot{m}_{H2}$) in order to achieve certain set points of the excess ratios (λ_{O2} , λ_{H2}) and avoid starvation. Furthermore, another significant factor that should be considered is the proper maintenance of the operating temperature ($T_{fc,SP}$) which is achieved by two mutually exclusive subsystems, one for the heat-up (x_{ht}) and another for the cooling (x_{cl}).

3.2 Problem formulation

In order to explore the behaviour of the PEMFC unit two controllers are developed. The first one uses a typical NMPC formulation as described at Section 2.1 and the second one uses the combined method using the mpQP problem of Section 2.2 and the SSR algorithm of Section 2.3. In both cases the optimization method remains the same. Initially the nonlinear dynamic model of the PEMFC needs to be descretized. More specifically is has nine differential equations and one algebraic, discretized at 10 finite elements (NE) having 4 collocation points (N_{cop}) each, resulting to 441 variables and 381 equations. The Jacobian matrix is computed analytically and has 3375 non-zero elements (density: 2.009 %). The NLP solver uses this model and solves the optimization problem (1) having a prediction (T_p) and control horizon (T_c) of 5 s and 500 ms respectively. The next step involves the development of the newly proposed method (exNMPC). Three of the manipulated variables (I, x_{ht}, x_{cl}) are selected to have varying bounds, that mainly affect two of the controlled variables (P_{SP}, T_{fc}) while the bounds of the other two variables ($\dot{m}_{air}, \dot{m}_{H2}$) are fixed at their feasible points. Based on the proposed SSR algorithm a PWA function for each controlled variable is required. Thus, two linear discrete state space models are derived, one for each control objective, by a model identification procedure. The first linear model (SS_P) approximates the behaviour of power and has one input variable (I), one output variable (P) and two states ($x_{P,1}, x_{P,2}$). The second linear model (SS_{Tfc}) approximates the temperature behaviour of the FC and has two input variables (x_{ht}, x_{cl}), one output (T_{fc}), one disturbance (T_{amb}) and two states ($x_{Tfc,1}, x_{Tfc,2}$). For each linear model (SS_P, SS_{Tfc}) an mpQP problem is formulated in order to derive a PWA function. The first involves one decision variable (I) and five parameters $\vartheta_P = [x_{P,1}\ x_{P,2}\ I\ P\ P_{sp}]$ and the second involves two variables (x_{ht}, x_{cl}) and six parameters $\vartheta_{Tfc} = [x_{Tfc,1}\ x_{Tfc,2}\ T_{amb}\ T_{fc}\ T_{fc,sp}]$. From their individual solution a set of critical regions are derived ($N_{CR,P} = 57$ and $N_{CR,Tfc} = 23$) which are explored during the SSR algorithm in order to determine the active bounds at every sampling interval.

4. Comparative experimental case study

The behaviour of the PEMFC unit is presented though a comparative experimental case study that explores the response of the two control methods (NMPC, exNMPC). The experiment involves a few step changes at the power demand (2.8 W, 3.4 W, 2.4W, 3 W) and the operating temperature (60 °C, 52 °C, 63 °C), while the oxygen and hydrogen excess ratio set-points are determined by a feedforward scheme (Ziogou et al., 2013) aiming at the minimization of the fuel and air supply. The performance of the underlying controllers is assessed based on the error from the desired set-point, qualitative response characteristics (settling time, rise time, overshoot) and most importantly the computational requirements.

4.1 Tracking of the desired set-points

Figure 1a illustrates the tracking of the power profile where we observe that the fuel cell exhibits similar behaviour regardless of the control configuration.

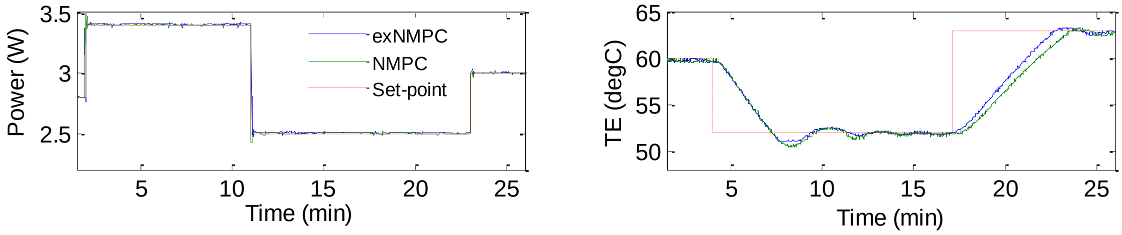

Figure 1: a) Produced power, b) fuel cell temperature using NMPC and exNMPC

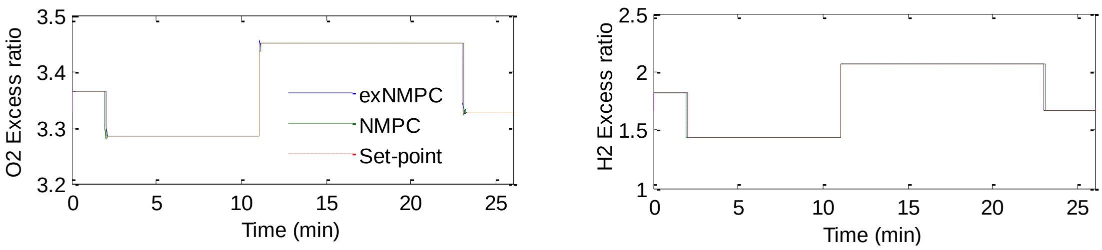

Figure 2: a) O_2 Excess ratio, b) H_2 Excess ratio using NMPC and exNMPC

The maximum power error for the exNMPC at steady state is 9.5 mW with an average error of 4 mW and for the NMPC is 12.0 mW and 3 mW. Figure 1b illustrates the modification of the temperature where both controllers respond similarly. In the case of the oxygen and hydrogen excess ratio, both profiles are accurately followed which is illustrated at Figures 2a and 2b and at the metrics of Table 1.

Table 1: Performance metrics related to set-point tracking (NMPC, exNMPC)

	$\lambda_{O2,SP}$ error	$\lambda_{H2,SP}$ error	$T_{fc,SP}$ overshoot	T_{fc} Settling time	T_{fc} Rise time
NMPC	4.38×10^{-4}	3.81×10^{-4}	0.6 °C	4 min	6.3 min
exNMPC	4.21×10^{-4}	2.42×10^{-4}	0.7 °C	5.5 min	5.4 min

4.2 Computational Requirements

Although a controller might achieve its objectives, the necessary time for the computation of the optimal values of the manipulated variables should also be considered. From the previous analysis we concluded that the exNMPC and NMPC exhibit similar behaviour in terms of set-point tracking. However, there is a significant difference between those two schemes related to the computational requirements. In fact this is the main contribution of the exNMPC approach, the reduction of the optimization time compared to the NMPC method. Figure 3b illustrates that the exNMPC can solve the NLP problem faster than the NMPC scheme (Figure 3a) with a maximum decrease of 70 %. These experimental results verify that a significant improvement is achieved regarding the computational demands of the controller by the implementation of the proposed synergetic approach. The maximum optimization time for the NMPC is 496 ms with an average of 98 ms, whereas for the exNMPC is 144 ms and 58 ms.

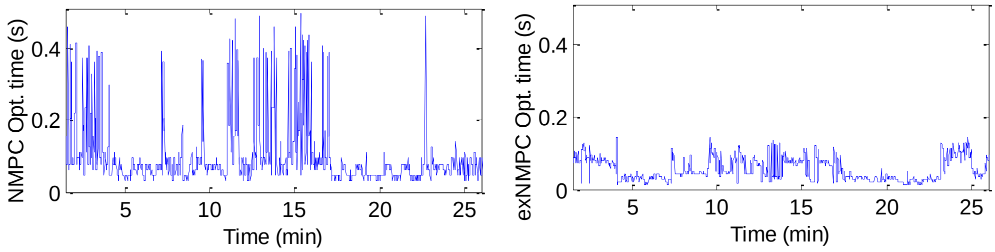

Figure 3: a) Optimization time of NMPC and b) Optimization time of exNMPC

Overall this case study reveals the benefits that arise from the deployment of the newly proposed exNMPC method. Apparently the combination of the NMPC with the PWA approximation shows interesting results for the underlying control problem. Furthermore, the fact that a nonlinear model of the fuel cell is used ensures that the exNMPC controller has the same performance with the NMPC controller.

5. Conclusions

This work presented a new approach that empowers the performance of a typical NMPC formulation. In order to reduce the computational requirements of the optimization problem which is solved at every sampling interval, a pre-processing methodology is designed. The newly proposed method relies on a PWA approximation of the variable's feasible space, derived offline by the solution of an mpQP method. A warm-start initialization is complemented by an SSR technique, relying on the PWA function that sets the basis for the improved behaviour of the optimizer. Overall this work reveals the potential of the synergy between the mpQP and the NMPC. The importance of this synergy is exemplified through a challenging multivariable nonlinear control problem with measured and unmeasured variables that involves concurrently four operation objectives for a PEM fuel cell system. Two controllers, using the NMPC and exNMPC methods, were designed, deployed online to the industrial automation system and their behaviour was evaluated at the PEMFC unit. From the results of the experimental case study it is observed that the response of the exNMPC controller is significantly enhanced without compromising the quality of the obtained solution.

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