CHEMICAL ENGINEERING TRANSACTIONS

VOL. 35, 2013

A publication of AIDIC
The Italian Association of Chemical Engineering
www.aidic.it/cet

Guest Editors: Petar Varbanov, Jiří Klemeš, Panos Seferlis, Athanasios I. Papadopoulos, Spyros Voutetakis

ISBN 978-88-95608-26-6; **ISSN** 1974-9791

DOI: 10.3303/CET1335077

Application of H_2 and H_∞ Approaches to the Robust Controller Design for a Heat Exchanger

Anna Vasičkaninová*, Monika Bakošová

Institute of Information Engineering, Automation, and Mathematics, Faculty of Chemical and Food Technology, Slovak University of Technology in Bratislava, Radlinského 9, 812 37 Bratislava, Slovakia
anna.vasickaninova@stuba.sk

A shell-and-tube heat exchanger and possibilities to use the robust controllers for its control are studied, tested and compared by simulations in this paper. Simulation results obtained using designed controllers were measured calculating integral performance index IAE. The control objective is to keep the output temperature of the heated stream at a reference value. The controlled output is the measured output temperature of the heated stream - kerosene and the control input is the volumetric flow rate of the heating stream - water. The use of the robust controllers can lead to smaller consumption of the heating medium. For controller design the heat exchanger was identified in the form of the 3^{rd} order plus time delay system. As several step responses were measured, interval values of the gain, the time constant and the time delay were obtained. Simulations of control were done in the Matlab/Simulink environment. The simulation results confirmed that designed robust controllers represent the possibilities for successful control of heat exchangers. Comparison with classical PID control demonstrates the superiority of the proposed control especially in the case, when the controlled process is affected by disturbances.

1. Introduction

H_2 and H_∞ control theories have been active areas of research for the years and have been successfully introduced to many engineering applications. H_2-optimization finds a controller which minimizes the H_2 norm of the closed-loop transfer function and internally stabilizes the system. The polynomial solution of the standard H_2 problem is proposed in Meinsma (2000), which is based on factorizations over polynomials and stable matrices. Kwakernaak (2000) derived an alternative solution in which operations with polynomial matrices replace those with rational matrices. The closed-loop transfer function to be minimized is located between the external signal and the control error signal (Kučera, 2008).

There exist various solutions also of the standard H_∞ problem. While the H_2 norm of a signal is the mean energy with respect to the frequency, the H_∞ norm is the maximum energy with respect to the frequency. If there are uncertainties in the system model, some quantity combining the H_2 norm and the H_∞ norm can be a desirable measure of a system's robust performance. Thus the mixed H_2/H_∞ performance criterion provides an interesting measure for the controller evaluation. The theoretic motivation for the mixed H_2/H_∞ control problem has been discussed in Kwakernaak (2002). The same method is used for convex parameterization of fixed-order H_∞ controllers in Yang et al. (2007). Many problems in systems and control are susceptible of convex reformulation via e.g. LMIs (Scherer, 2006).

Most processes are nonlinear, and their control is a difficult and important problem. Heat exchangers represent nonlinear processes (Janna, 2009) and control of them is a complex process due to the non-linear behaviour and complexity caused by many phenomena such as leakage, friction, temperature-dependent flow properties, contact resistance, unknown fluid properties, etc. (Serth, 2007). Many factors enter into the design of heat exchangers, including thermal analysis, weight, size, structural strength, pressure drop and cost. Owing to the wide utilization of heat exchangers in industrial processes, their cost minimization is an important target for both, designers and users (Pan et al., 2011). Cost evaluation is obviously an optimization process dependent upon the other design parameters. The method which is

Please cite this article as: Vasickaninova A., Bakosova M., 2013, Application of h2 and h∞ approaches applied to the robust controller design for a heat exchanger, Chemical Engineering Transactions, 35, 463-468 DOI:10.3303/CET1335077

capable of utilizing the maximum allowable stream pressure drops is described in Panjeshahi et al. (2010). The approach can result in minimum surface area requirements. Economics plays a key role in the design and selection of heat exchanger equipment. The weight and size of heat exchangers are significant parameters in the overall application and thus may still be considered as economic variables (Holman, 2009). A particular application will dictate the rules that one must follow to obtain the best design considering size, weight, economic criteria, etc. They all must be considered in practice (Holman, 2009).

In the presented paper, the tasks of the set point tracking and the disturbance rejection in H_2 and H_∞ control techniques are investigated. The presented experimental results show applicability of mentioned approaches to safer control of a nonlinear process. The control responses obtained by the H_2 controller have smaller overshoots. On the other side, the use of the H_∞ approach leads to smaller consumption of the heating medium.

2. Process description

Consider a co-current tubular heat exchanger (Vasičkaninová et al., 2011), where kerosene is heated by hot water through a copper tube. The controlled variable is the outlet kerosene temperature T_{1out}. Among the input variables, the water flow rate $q_3(t)$ is selected as the control variable. The tubes are described by a linear coordinate z, which measures the distance of a generic section from the inlet. The fluids move in a plug velocity profile and the kerosene, tube and water temperatures $T_1(z,t)$, $T_2(z,t)$ and $T_3(z,t)$ are functions of the axial coordinate z and the time t. The kerosene, water and tube material densities ρ_i as well as the specific heat capacities C_{Pi}, i = 1, 2, 3, are assumed to be constant. The simplified nonlinear dynamic mathematical model of the heat exchanger is described by three partial differential equations (Vasičkaninová et al., 2011). Parameters and steady-state inputs of the heat exchanger are enumerated in (Vasičkaninová et al., 2012). For the identification, the step changes ±15 %, ±30 %, ±50 % of the inlet mass flow-rate of heating water were generated at the time t = 0. Step responses of the outlet temperature are shown in Figure 1, where step responses on the input changes ±15 % are represented by the solid lines, on the input changes ±30% by the dashed lines, on the input changes ±50 % by the dotted lines.

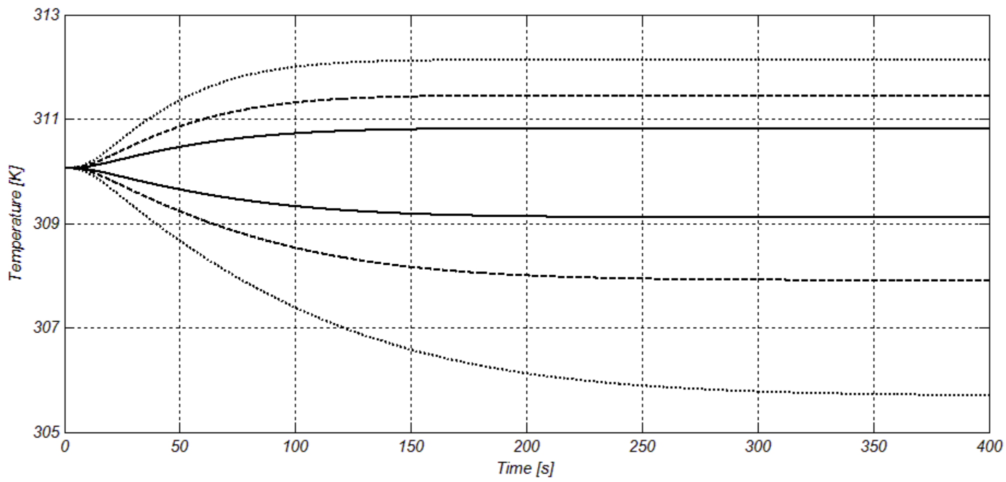

Figure 1: Step responses of the outlet temperature on the step changes of the control input

According to these step changes, the heat exchanger is a time-delay nonlinear system with asymmetric dynamics. The model was identified using the Strejc method from the step responses in the form of the n^{th} order plus time delay transfer function in Eq.(1).

$$G = \frac{K}{(\tau s + 1)^n} e^{-Ds} \tag{1}$$

Because the heat exchanger can be represented also as a system with interval parametric uncertainty, for various step responses were obtained intervals for values of the gain K, the time constant τ and the time delay D (Table 1). The system order n = 3. The mean values of the parameters are considered to be nominal.

Table 1: Identification of the process dynamics

τ_{min}	τ_{max}	τ_{mean}	K_{min}	K_{max}	K_{mean}	D_{min}	D_{max}	D_{mean}
15	26	19.33	3.734×10^4	7.8407×10^4	5.4136×10^4	0.24	2.00	0.91

3. Control of the heat exchanger

PID controllers described by the transfer function in Eq(2).

$$C = k_p\left(1+\frac{1}{t_i s}+t_d s\right) \quad (2)$$

In the transfer function k_p is the proportional gain, t_i the integral time, t_d the derivative time and these parameters were tuned using the Cohen-Coon method (Bequette, 2003). The controller parameters were designed for the model with the mean (nominal) values of the identified parameters. The PID controller parameters obtained using the Cohen-Coon formulas are $k_p = 1.7\times10^{-4}$, $t_i = 35.12$ s, $t_d = 5.44$ s.

3.1 H_2 control

Consider the plant model with the feedback as shown in Figure 2. The signal w is the external input, z is the error signal, which ideally should be zero, u is the control input, and y the observed output. The block G is the generalized plant, and C the controller.

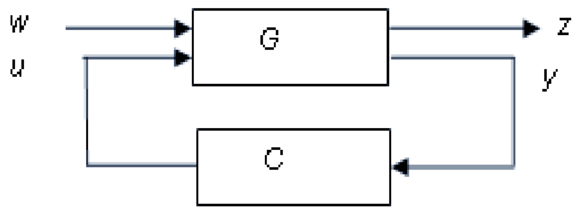

Figure 2: The standard system configuration

The standard H_2 problem is finding a controller which minimizes H_2 norm in Eq(3) of the closed-loop transfer function and internally stabilizes the system.

$$\|H\|_{H_2} = \sqrt{tr\int_0^\infty h^T(t)h(t)dt} = \sqrt{\frac{1}{2\pi}tr\int_{-\infty}^{\infty} H(j\omega)H^T(-j\omega)d\omega} \quad (3)$$

Assume G has following realization

$$\begin{aligned} \dot{x}(t) &= Ax(t)+B_1w(t)+B_2u(t) \\ z(t) &= C_1x(t)+D_{12}u(t) \\ y(t) &= C_2x(t)+D_{21}w(t)+D_{22}u(t) \end{aligned} \quad (4)$$

It is assumed further that: (A, B_2) is stabilizable, (A, C_2) is detectable, $D^T_{12}D_{12}$ and $D_{21}D^T_{21}$ are non-singular. The problem is essentially an LQG problem, hence may be solved using solutions of two Riccati equations (observer & state-feedback). The controller C may be seen as an observer interconnected with a state-feedback law.

Consider a more flexible configuration as shown in Figure 3 (Bosgra et al., 2000).

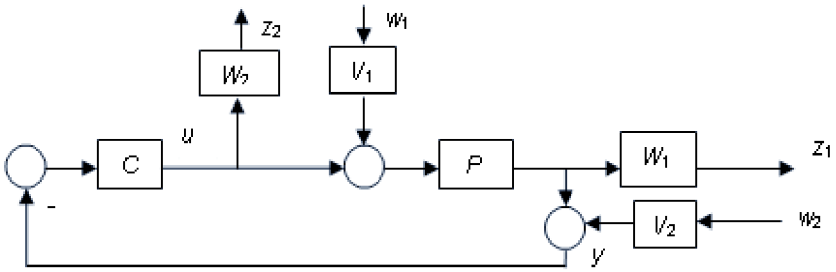

Figure 3: Generalized configuration

Here, P is the transfer function of the controlled system, C is the compensator to be designed, w comprises the external inputs, including perturbations, measurement noise and reference inputs, z is the control error signal, y is the measured output, u is the control input, V_1, V_2 are (frequency dependent) shaping filters, W_1,W_2 are (frequency dependent) weights. Choose V_1, V_2, W_1, W_2 so that H is a sensible cost function.
The loop gain has a direct effect on the important closed-loop transfer functions which determine the norm, such as the sensitivity S and the complementary sensitivity T. The sensitivity and the complementary sensitivity functions are given by Eq.(5).

$$\begin{aligned} S &= (I+PC)^{-1} \\ T &= (I+PC)^{-1}PC \end{aligned} \quad (5)$$

U is the input sensitivity matrix given in Eq.(6).

$$U = C(I+PC)^{-1} \quad (6)$$

For the system in Figure 2

$$H = \begin{bmatrix} W_1SPV_1 & W_1TV_2 \\ W_2T'V_1 & W_2UV_2 \end{bmatrix} \quad (7)$$

Then the sum of squares of norms of all entries of H is

$$\|H\|_2^2 = \|W_1SPV_1\|_2^2 + \|W_1TV_2\|_2^2 + \|W_2T'V_1\|_2^2 + \|W_2UV_2\|_2^2 \quad (8)$$

The H_2-controller can be found in the form

$$C(s) = \frac{1995s^3 + 309.7s^2 + 16.02s + 0.2763}{s^4 + 28.93s^3 + 418.6s^2 + 3546s + 35.41} \quad (9)$$

The closed-loop poles are given by [-0.010 -0.0518 -0.0517 -0.0517 -4.2501±10.2292i -10.2164±4.2187i]

3.2 H_∞ Control

H_∞-optimization resembles H_2-optimization, where the criterion is the 2-norm. Because the 2- and ∞-norms have different properties, the results naturally are not quite the same. An important aspect of $H\infty$-optimization is that it allows including robustness constraints explicitly in the criterion (Bosgra et al., 2000).
In the H_∞–controller design, the H_∞-norm of the mapping from w to z can be minimized. The inputs w are typically reference or disturbance signals, whereas the outputs z can be the control error or the controller output.
Using H_∞ theory

$$\min_{\text{stabilizing controller}} \left\| \begin{matrix} W_1SV \\ -W_2TV \end{matrix} \right\|_\infty \quad (10)$$

Controller design is equivalent to the choice of W_1, W_2,V. Essentially the same is

$$\min_{\text{stabilizing controller}} \left\| \begin{matrix} W_1SV \\ W_2UV \end{matrix} \right\|_\infty \quad (11)$$

where $U = C(I+PC)^{-1}$ is the input sensitivity matrix.
The criterion in Eq(6) is reduced to the square root of the scalar quantity for the SISO mixed sensitivity problem:

$$\sup_{\omega \in R} \left(|W_1(j\omega)S(j\omega)V(j\omega)|^2 + |W_2(j\omega)U(j\omega)V(j\omega)|^2 \right) < \gamma^2 \quad (12)$$

Consider the block diagram of Figure 4. In this diagram, the external signal w generates the disturbance v after passing through a shaping filter with the transfer matrix V. The control error z has two components, z_1 and z_2. Here, z_1 is the control system output after passing through a weighting filter with the transfer matrix W_1, z_2 is the plant input u after passing through a weighting filter with the transfer matrix W_2.

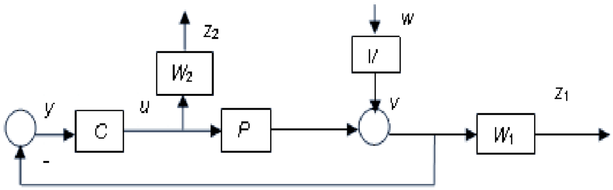

Figure 4: The mixed sensitivity problem.

It is easy to show that for the closed-loop system described by Eq.(13)

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} W_1SV \\ -W_2UV \end{bmatrix} w = Hw \tag{13}$$

minimization of the ∞-norm of the closed-loop transfer matrix H leads to minimization of

$$\left\| \begin{matrix} W_1SV \\ W_2UV \end{matrix} \right\|_\infty \tag{14}$$

The H_∞-controller can be found in the form

$$C(s) = \frac{349.3s^3 + 40.3s^2 + 1.55s + 0.02.}{s^4 + 16.9s^3 + 136.9s^2 + 645.3s + 6.44} \tag{15}$$

The closed-loop poles are given by [-0.010 -0.0385 -6.0988±2.4524i -2.3564±5.5367i -0.0384±0.0001i], γ=0.01.
Simulation results obtained using designed H_2 and H_∞ controllers are shown in Figure 5, where reference is represented by the dashed-dotted line, PID control is represented by the solid line, H_2 control is represented by the dotted line and H_∞ control is represented by the dashed line. The figure presents the simulation results of the control of the heat exchanger in the task of set point tracking and in the task of disturbance rejection. The set point changes from 313.15 K to 312.15 K at 400 s and then to 313.65 K at 800 s. Disturbances were represented by water temperature changes from 348.15 K to 344.15 K at 200 s, from 344.15 K to 351.15 K at 600 s and to 346.15 K at 1000 s.

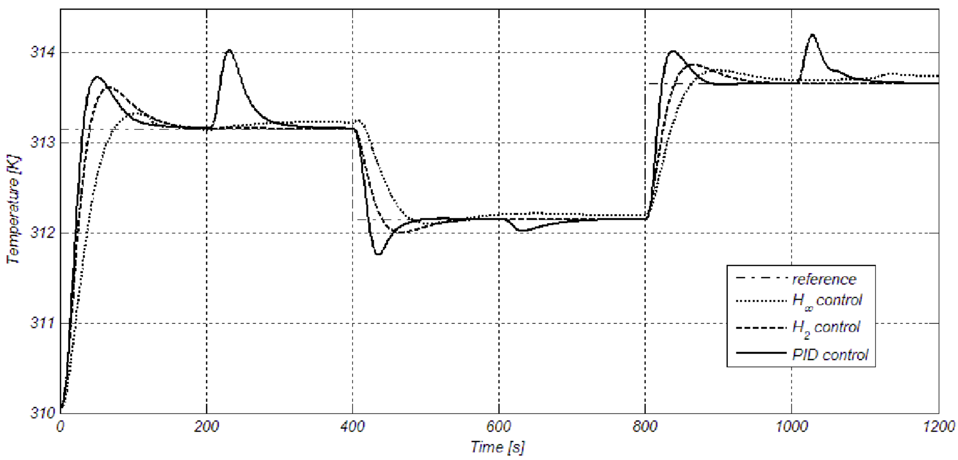

Figure 5: Comparison of the outlet kerosene temperature control

The control response obtained by the H_∞ controller has smaller overshoots, but longer settling times.
The energy consumption is measured by the total amount of hot water consumed during the control process. Smaller energy consumption is assured using the H_∞ controller. The simulation results were compared also using IAE (integral absolute value of error) criteria (Ogunnaike and Ray, 1994). The IAE values and the consumption of the heating medium are given in Table 2.

Table 2: Values of IAE and hot water consumption V

controller	IAE	V [m^3]
PID Cohen-Coon	210	0.1436
H_2	186	0.1426
H_∞	247	0.1415

4. Conclusions

The aim of the described work was to apply H_2 and H_∞ optimization techniques to the control of the nonlinear heat exchanger. Simulation results obtained using designed controllers were measured calculating integral performance index IAE and consumption of the heating medium. The control response obtained by the H_2 controller had smaller overshoots and also smaller IAE value. The use of the H_∞ controller led to smaller consumption of the heating medium. The simulation results confirm that designed robust controllers represent the possibilities for successful control of heat exchangers. Comparison with classical PID control demonstrates the superiority of the proposed control especially in the case, when the controlled process is affected by disturbances.

Acknowledgments

The authors gratefully acknowledge the contribution of the Scientific Grant Agency of the Slovak Republic under the grant 1/0973/12.

References

Bequette W.B., 2003, Process Control: Modeling, Design, and Simulation, Prentice-Hall of India, New Delhi, India.

Bosgra O.H., Kwakernaak H., Meinsma G., 2000, Design methods for control systems, DISC lecture notes 2000-2001 <web.boun.edu.tr/eskinat/ME687.html> accessed 18.03.2013.

Holman J.P., 2009, Heat Transfer, McGraw-Hill, New York, USA.

Janna W.S., 2009, Engineering Heat Transfer, 3rd Edition, The University of Memphis, Tennessee, USA.

Kučera V., 2008, A comparison of approaches to solving the H_2 control problem. Kybernetika, 44, 328-335.

Kwakernaak H., 2000, H_2 optimization – Theory and applications to robust control design, in: Kučera V. and Šebek M. (eds), Robust Control Design, 3rd IFAC Symposium, Prague, Czech Republic, 437-448, Pergamon, Oxford, UK.

Kwakernaak H., 2002, Mixed sensitivity design, In: 15th IFAC World Congress, Barcelona, Spain.

Meinsma G., 2000, On the standard H_2 problem, in: Kučera V. and Šebek M. (eds), Robust Control Design, A Proceedings Volume from the 3rd IFAC Symposium, Prague, Czech Republic, 681-686, Pergamon, Oxford.

Ogunnaike B.A., Ray W.H., 1994, Process Dynamics, Modelling, and Control. Oxford University Press, New York, USA.

Pan M., Bulatov I., Smith R.. Kim J.K., 2011, Improving energy recovery in heat exchanger network with intensified tube-side heat transfer, Chemical Engineering Transactions, 25, 375-380

Panjeshahi M.H, Joda F., Tahouni N., 2010, Pressure drop optimization in multi -stream heat exchanger using genetic algorithms, Chemical Engineering Transactions, 21, 247-252

Serth R.W., 2007, Process Heat Transfer Principles and Applications, Academic Press, Burlington, USA.

Scherer C.W., 2006, LMI relaxations in robust control, European Journal of Control,12, 3-29

Vasičkaninová A., Bakošová M., Mészáros A., Klemeš J., 2011, Neural network predictive control of a heat exchanger, Applied Thermal Engineering, 31, 2094-2100

Vasičkaninová A., Bakošová M., 2012, Robust control of heat exchangers, Chemical Engineering Transactions, 29, 1363-1368

Yang F., Gani M., Henrion D., 2007, Fixed-order robust H_∞ controller design with regional pole assignment, IEEE Transactions on Automatic Control, 52, 1959-1963