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An Experiment on Diagnosis, Reliability Estimation, and Reliability Prognosis

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This paper introduces a small scale experiment, which simplifies validating the effectiveness of reliability approaches combining performance and reliability measures based on diagnosis, reliability estimation, and reliability prognosis. The experiment consists of a classical slot car setup. The vehicles motor power is mapped on an overloaded light-emitting diode (LED), which is operating in parallel to the direct current (DC) motor. The rule of the race is: If the LED fails during a race, the driver lost the game.

First of all LED reliability properties depending on constant power load are tested. For every power level an a posteriori mean time to failure (*MTTF*) value is calculated. The linear damage hypothesis approach is applied here estimating semiconductor reliability, namely an a priori estimated value of the time to failure. Based on this measure, it is easy to conduct a reliability prognosis. Prognosis can be conducted during the race. Comparing *MTTF*, estimated time to failure, and prognosis values, results are pretty interesting. However, individual vehicle prognoses are based on individual diagnosis data and mean value-based reliability statements. A driver should keep that in mind if he/she controls his/her vehicle based on prognosis values calculated during a race.

1. Introduction

The experiment consists of a classical slot car setup. The vehicles are powered by DC 18 V maximum voltage. Motor power is mapped on an (overloaded) LED, which is operating in parallel to the DC motor. The rule of the race is: If the LED fails during a race, the driver lost the game. The basic constraints are: If the car runs too fast, the LED may fail before the race is finished. If the car runs too slow, somebody else might win. The scenario supplies enough uncertainties justifying a detailed performance and reliability consideration. The objective is giving individual reliability data to drivers during the race so that they can change their ways of driving.

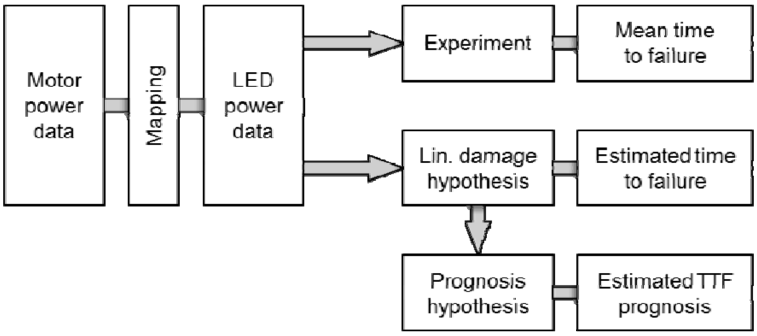

Figure 1: Overview on data flow, hypotheses, and results

The experiment shows a combination of diagnosis, reliability estimation, and reliability prognosis. Three different paths of reliability modelling are applied here, see Figure 1:

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(1) A sample of 36 LED is tested with power values equivalent to a race conducted. Result is an a posteriori (after the race) *MTTF* based on experimental data.
(2) A linear damage hypothesis (LDH) is applied based on diagnosis data. Result is an LDH estimated time to failure – not to be confused with the *MTTF* in this context.
(3) Finally, a prognosis is conducted based on LDH values.

2. Specifications and Diagnosis

The critical component applied in the experiment is a low-cost green LED of 5 mm diameter with a diffuse and wide angle lens without a series resistor. LED specifications are 25 mA DC forward current, 140 mA peak forward current at 1/10 duty cycle and 0.1 ms pulse width, and 2.0 V forward voltage (2.5 V maximum) at 20 mA. The LED is significantly overloaded during operation by forward voltages up to 6.5 V or forward currents up to 200 mA depending on changing internal resistance; however, the maximum power is limited to ca. 1100 mW. Avoiding regenerating effects after cooling-off, the minimum power was limited to 600 mW at the beginning and later stepped up to 700 mW. Overloading reduces the LED time to failure (*TTF*) to a feasible interval of 8.5 s minimum at 1100 mW and 90.5 s maximum at 600 mW.

2.1 Failure Criterion

It was difficult to find a reproducible failure criterion, e.g. luminous intensity could not be measured with adequate accuracy in a small-scale experimental environment. Finally, internal resistance R_{in} is measured at idle power with ca. 3 mW. Values of R_{in} show almost no ripple at this power stage compared to resistance variability when the LED under test is overloaded. If $R_{\mathrm{in}} < 1300\ \Omega$, the LED is defined as faulty. Failure causes are heating effects and in consequence irreversible physical destructions of the semiconductor.

2.2 Experimental Set-up

At this stage, the experimental setup consists of a PC connected via USB to a microcontroller unit (here a low-cost Olimexino board). The microcontroller is linked to an application unit, which carries a PWM D/A converter including an RC-circuit and finally an LED holder. A shot glass covers the LED under test shielding the component from convection air cooling and providing reproducible convective flow of heat. It is planned to conduct measuring aboard vehicle in later stages. Presently, the equipment is mounted alongside tracks.

2.3 Measuring and Diagnosis

Measuring and calculations are conducted by the microcontroller in 1 ms intervals. Operating time is recorded, voltage and current values are measured; power and energy are calculated on the basis of these characteristics. Power controlling according to a given set value is also carried out in 1 ms intervals. Moreover, LED internal resistance is calculated during idle sections as described below.

Operating duration is divided into intervals of 500 ms duration. Every interval consists of a power section (450 ms), where the LED is overloaded and an idle section (50 ms), where the LED is operated within its specification. The last 10 ms at the end of the idle section – denoted as diagnosis section – are used for calculating R_{in} based on the mean of 10 voltage and current values. Figure 2 illustrates sections, intervals, and voltage graph.

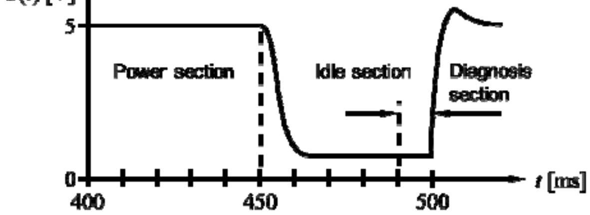

Figure 2: Illustration of power, idle, and diagnosis sections. Note that original voltage graphs show ripple in power sections.

The 40 ms wide “paranoia gap” between the end of the power section and the beginning of the diagnosis section assures steady state electrical properties. Actually, idle power is reached within in ca. 10 ms. It is assumed that thermal conditions do not change significantly during the 50 ms idle section. The microcontroller calculates following values and hands it over to the connected PC: (a) operating time at the end of the 500 ms interval, (b) mean voltage, mean current, and mean power related to the 450 ms power section, (c) cumulative energy input related to the complete 500 ms interval, (d) mean R_{in} related to the 10 ms diagnosis section.

3. Reliability Properties

Random samples of 36 pieces are taken for every test conducted from a set of 1000 LED, which is a compromise between feasibility and quality of results.

3.1 Experimental Mean Time to Failure

The time to failure (*TTF*) in this experimental context is the operating time when the LED internal resistance is not lower than 1300 Ω, see Section 2.1. The *TTF* is measured depending on the power applied to the LED under test. In total 11 power categories are defined starting from 600 mW in 50 mW steps and ending at 1100 mW, i.e. 396 LED have been melt. The arithmetic mean of every category gives a first hint at the mean time to failure (*MTTF*) – and when the test is finally finished.

The sample data of every power category are compared with reference probability functions. Applying the test of goodness of fit tool by Oberguggenberger and Andreas (2008), the following probability functions are considered: normal, Weibull, lognormal, Pareto, Gumbel, chi-squared, exponential, and gamma distribution. The Oberguggenberger–Andreas tool provides parameter(s) for every probability function and Kolmogorov–Smirnov and chi-squared test results, see Table 1.

Table 1: Results of goodness of fit at 800 mW, K–S means Kolmogorov–Smirnov, DoF means statistical degree of freedom

Probability function	Parameters	Param. value 1	Param. value 2	Param. value 3	K–S *p*-value	K–S statistics	Chi^2 *p*-value	Chi^2 statistics	Chi^2 DoF
Normal	μ, σ	24.01 s	1.85 s	—	0.22	0.17	**0.10**	6.34	3
Weibull	α, δ, λ	16.31	0 s	0.04 s^{-1}	0.01	0.28	0.01	8.91	2
Lognormal	μ, σ	3.18 s	0.08 s	—	**0.32**	0.15	**0.10**	6.25	3
Pareto	x_min, k	6.44 s	20.50	—	0.00	0.36	0.00	17.13	3
Gumbel	α, λ	23.11 s	0.59 s^{-1}	—	0.18	0.18	0.08	6.86	3
Chi-squared	N	25.00	—	—	0.00	0.31	0	51.46	4
Exponential	λ	0.04 s^{-1}	—	—	0	0.57	0	298.99	4
Gamma	α, λ	168.46	7.02 s^{-1}	—	**0.29**	0.16	**0.10**	6.26	3

The best fitting probability functions in the given case seem to be lognormal, gamma, and normal distributions, see bold typed values in Table 1. However, the first three digits of expected values of all three distributions are equal. At all power levels, the expected value(s) of best-fitting distributions equal the arithmetic value. Hence, the arithmetic value is a good estimation of the *MTTF*.

3.2 The MTTF curve

The *MTTF* values of every power level, see Figure 3, are used defining a curve – denoted as *MTTF* curve – similar to the well-known S-N curve, also known as Wöhler (1870) curve, graphically appearing as a (more or less straight) line in a full-log diagram.

Two simple approaches have been conducted composing *MTTF* curves. The first creates a full-log regression curve over all *MTTF* points given. The second interpolates all necessary points between each pair of neighboured *MTTF* points.

However, *MTTF* and S–N curves have to be distinguished clearly. The *MTTF* curve is not a sharp line separating functioning LED from faulty. Failure may occur left hand to the *MTTF* curve, and LED may still be functioning right hand to the curve. Assumed, the best fitting probability functions is a normal distribution, then the probability of a failure between $t = 0$ and the curve is 0.5. Probability for a failure in the interval $[0, MTTF - 1\cdot\sigma]$ or $[0, MTTF - 6\cdot\sigma]$ is ca. 0.159 or ca. 10^{-9}, respectively. This property is not typical for some mechanical engineering approaches.

3.3 Validation

An experimental step-stress test validates the *MTTF* curve. Tests are starting from two steps at different power level patterns and ending by six steps. The validation shows that the both lowest power levels (600 and 650 mW) defined prior to the step-stress test must be generally omitted in this experiment, due to the fact that they cause re-cooling effects in combination with the highest power levels.

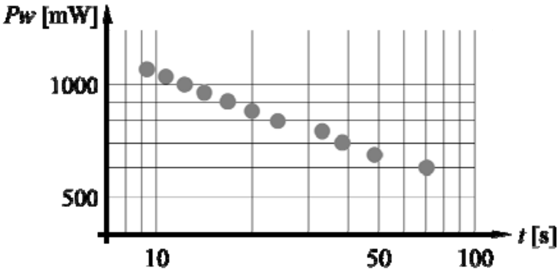

Figure 3: Estimated MTTF depending on power level values.

4. Reliability Race

First of all slot-car races are conducted, which is fun for students and lecturer. A minimalist race course of ca. 4.2 m has been composed of ca. two 1 m straight rails and two 180° curves with ca. 0.7 m diameter. Only one vehicle drives at a time at the outside bend. Following data are measured, calculated, and recorded for every 20 ms interval: (a) lap number, (b) operating time at the end of the 20 ms interval, see column 1 in Table 2, (c) mean voltage, mean current, and mean power related to the complete 20 ms interval, see columns 2 to 4 in Table 2, (d) cumulative energy input also related to the complete 20 ms interval. The fastest manual controlled race over 12 laps took 32.42 s. The data of this race have been used for further modelling.

4.1 Mapping

Motor power values $Pw_{\mathrm{motor}}(t)$ are calculated form motor voltage and current. Motor power range is [0 mW, 8270 mW], where the maximum value also depends on the race course topology. The motor power range is mapped linearly on the LED power level $Pw_{\mathrm{LED}}(\mathrm{t})$ = [700 mW, 1100 mW] by

$$Pw_{\mathrm{LED}}(t) = \frac{1100\,\mathrm{mW} - 700\,\mathrm{mW}}{8270\,\mathrm{mW}} \cdot Pw_{\mathrm{motor}}(t) + 700\,\mathrm{mW} \qquad (1)$$

4.2 A Posteriori Reliability Properties based on Experimental Data

Rounded mapped race data (Pw_{LED}), see column 5 in Table 2, are used for testing 36 LED. With that, *TTF* are recorded and hypotheses tests are conducted as described in Section 3.1. As expected, estimated values of the best fitting probability functions (lognormal, gamma, and hints on normal (K–S test) and Gumbel (chi-squared test) distribution) equal the arithmetic mean in the first three digits. With the above mentioned data from the 32.42 s winning race an *MTTF* of 23.21 s holds.

4.3 Reliability Estimation

The linear damage hypothesis (LDH), also known as Miner's Rule, has been invented by Palmgren (1924) and discussed later by Langer (1937) and Miner (1945). The approach is applied here in a similar way estimating semiconductor reliability in the given context. There is

$$D(t) = \sum_{0}^{t} \frac{m}{M(Pw)},$$

where *M*(*Pw*) is the *MTTF* assigned to the constant power level given, see column 6 in Table 2, applying an inverse function of the regression function as addressed in Section 3.2. Measure *n* is a constant parameter representing the 20 ms interval duration. The quotient *m*/*M*(*Pw*) is the so-called fraction of "reliability consumed" at each 20 ms interval. The LDH states that a failure occurs for $D(t) \approx 1$; however, this value is not understood as a sharp boundary between functioning and failure. It is just a hint with a character like mean values as applied in reliability engineering. Table 2 gives an extract of data collected, see especially columns 7 and 8.

As Table 2 includes a full 500 ms interval, it shows that the 50 ms LED idle section is calculated here as 40 ms at 3 mW power considering decrease of power within the first 10 ms, see graph in Figure 2.

The result is pretty interesting: Experimentally gained *MTTF* (23.21 s) and LDH estimated time to failure based on the overall regression approach (ca. 23.08 s) show a difference of just 130 ms (0.56 % of *MTTF*). LDH estimated time to failure based on interpolation yields in 22.84 s (not shown in Table 2) and a difference of 370 ms (1.59 % of *MTTF*). Presently, a second LED charge will be tested to validate these results.

Table 2: Columns 1 to 4 show motor data, 5 to 8 reliability estimation data, and 9 and 10 reliability prognosis data. Detailed descriptions are given in the related sections.

1	2	3	4	5	6	7	8	9	10
t [s]	*U* [V]	*I* [A]	*Pw* [W]	*Pw* [mW]	*M* [s]	*m/M*	*D*(*t*)	Mean m/M	Prog *MTTF*
0.00	0.01	0.000	0.00	3	—	0	0.0 %	0	—
0.02	1.06	0.069	0.07	700	39.3	509.0E-6	0.1 %	254.5E-6	78.6
0.04	4.23	0.412	1.74	780	27.5	726.5E-6	0.1 %	411.8E-6	48.6
0.06	6.87	0.611	4.20	900	17.2	1.2E-3	0.2 %	599.6E-6	33.4
0.08	7.71	0.467	3.60	870	19.2	1.0E-3	0.3 %	687.7E-6	29.1
0.10	7.93	0.657	5.21	950	14.4	1.4E-3	0.5 %	804.5E-6	24.9
0.12	8.27	0.699	5.78	980	13.0	1.5E-3	0.6 %	909.4E-6	22.0
0.14	8.30	0.686	5.69	980	13.0	1.5E-3	0.8 %	988.0E-6	20.2
0.16	8.34	0.687	5.73	980	13.0	1.5E-3	0.9 %	1.0E-3	19.1
0.18	8.62	0.699	6.03	990	12.6	1.6E-3	1.1 %	1.1E-3	18.1
0.20	8.85	0.722	6.39	1010	11.8	1.7E-3	1.3 %	1.2E-3	17.3
0.22	8.81	0.717	6.32	1010	11.8	1.7E-3	1.4 %	1.2E-3	16.6
0.24	8.90	0.679	6.04	990	12.6	1.6E-3	1.6 %	1.2E-3	16.2
0.26	9.08	0.696	6.32	1010	11.8	1.7E-3	1.8 %	1.3E-3	15.8
0.28	9.22	0.702	6.47	1010	11.8	1.7E-3	1.9 %	1.3E-3	15.4
0.30	9.10	0.677	6.16	1000	12.2	1.6E-3	2.1 %	1.3E-3	15.2
0.32	9.11	0.677	6.17	1000	12.2	1.6E-3	2.3 %	1.3E-3	15.0
0.34	9.30	0.693	6.44	1010	11.8	1.7E-3	2.4 %	1.4E-3	14.8
0.36	9.39	0.674	6.33	1010	11.8	1.7E-3	2.6 %	1.4E-3	14.6
0.38	9.27	0.629	5.83	980	13.0	1.5E-3	2.8 %	1.4E-3	14.5
0.40	9.33	0.634	5.92	990	12.6	1.6E-3	2.9 %	1.4E-3	14.4
0.42	9.55	0.630	6.02	990	12.6	1.6E-3	3.1 %	1.4E-3	14.3
0.44	9.57	0.608	5.82	980	13.0	1.5E-3	3.2 %	1.4E-3	14.2
0.46	9.35	0.587	5.49	970	13.4	1.5E-3	3.4 %	1.4E-3	14.2
0.48	9.42	0.589	5.55	3	∞	0	3.4 %	1.4E-3	14.8
0.50	9.55	0.595	5.68	3	∞	0	3.4 %	1.3E-3	15.4
0.52	9.24	0.535	4.94	940	14.9	1.3E-3	3.5 %	1.3E-3	15.3
0.54	8.57	0.457	3.92	890	17.8	1.1E-3	3.6 %	1.3E-3	15.4
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
23.06	9.37	0.532	4.98	940	14.9	1.3E-3	99.9 %	865.3E-6	23.1
23.08	9.40	0.540	5.08	950	14.4	1.4E-3	**100.0 %**	865.8E-6	23.1
23.10	9.60	0.543	5.21	950	14.4	1.4E-3	100.1 %	866.2E-6	23.1
23.12	9.60	0.527	5.06	940	14.9	1.3E-3	100.3 %	866.6E-6	23.1
23.14	9.37	0.509	4.77	930	15.4	1.3E-3	100.4 %	867.0E-6	23.1
23.16	9.43	0.523	4.93	940	14.9	1.3E-3	100.5 %	867.4E-6	23.1
23.18	9.67	0.536	5.18	950	14.4	1.4E-3	100.7 %	867.9E-6	23.0
23.20	9.60	0.509	4.89	940	14.9	1.3E-3	100.8 %	868.3E-6	23.0
23.22	9.41	0.496	4.67	930	15.4	1.3E-3	100.9 %	868.6E-6	23.0

4.4 A Priori Reliability Prognosis

With reliability data calculated as described above, reliability properties are extrapolated into the operating future. A very simple prognosis hypothesis is applied here stating, “The future is as god or bad as the mean of the past.”

If this hypothesis is applied to the mean LED power (column 5 in Table 2), the prognosis pretty soon ends up around 765 mW and assigns a rather optimistic time to failure prognosis of 29.4 s. LED power as prognosis input does not consider reliability properties described in Section 3. Hence, the given prognosis hypothesis is applied to the “reliability consumed” quotient $m/M(Pw)$, see Section 4.3. The i-th value of the 9th column in Table 2 represents the mean of the 7th column from the first to the i-th row, which is the mean of the past from the viewpoint of the i-th time interval extrapolated into the future. If a failure is expected for $D(n) \approx 1$ then

$$D(n) \approx 1 = n \cdot \left(\frac{1}{i} \cdot \sum_{0}^{i} \frac{m}{M(Pw)} \right) \quad \Rightarrow \quad n = \left(\frac{1}{i} \cdot \sum_{0}^{i} \frac{m}{M(Pw)} \right)^{-1} \tag{2}$$

holds, where *n* is number of intervals until *D* equals 1. With that the estimated value of the time to failure prognosis yields $E_i(TTF) = n \times 0.02$ s. Values of $E_i(TTF)$ are given in column 10 of Table 2. Figure 4 shows the graph of the estimated value of the time to failure prognosis. A pretty good prediction for an estimated value greater 22.0 s is given from ca. 6.08 s on (ca. 26 % of *MTTF*).

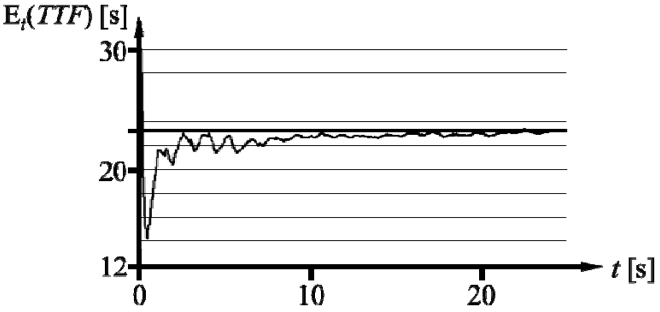

Figure 4: Estimated value of the time to failure prognosis depending on time operated. The bold horizontal line indicates the a posteriori (experimentally measured) MTTF of 23.21 s

5. Conclusions

The experiment combines performance and reliability measures based on diagnosis data and reliability estimation. Three different paths of calculating reliability measures are presented: testing components and calculating the mean time to failure (*MTTF*) based on experimental data; applying linear damage hypothesis (LDH) based on diagnosis data resulting in an estimated time to failure; and a prognosis based on LDH values. All measures have to be distinguished clearly in this context: *MTTF* is an a posterior measure, LDH estimated time to failure is an a priori measure, same holds for the prognosis result. All are mean values, i.e. individual vehicle prognoses are based on individual diagnosis data and mean value-based reliability statements. A driver should keep that in mind if he/she controls his/her vehicle based on prognosis values calculated during operation – during the race. The experiment is understood as one step towards a reliability-adaptive system as proposed e.g. in Rakowsky (2006).

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