

# Electro-Migration Model Parameters Sensitivity Analysis Based on the Monte Carlo Method

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With the narrowing of the size and the improvement of integration, Integrated Circuit devices' reliability is increasingly becoming the focus of attention. The Electro-Migration is the one of many reasons to cause circuit malfunction. When the Integrated Circuits is working, certain current goes through the metal interconnect inside, and the metal ion moves in the effect of the "electronic wind", making certain parts of the conductor produces empty or whisker and hill, which is Electro-Migration phenomena. The migration will cause the failure of the electrical connection, which can reduce the reliability of the structure of Integrated Circuit. The physical model established the relation between EM phenomena and current density, geometry size, material properties and the temperature distribution of the metal line, which can evaluate the electric wire lifetime under the impact of Electro-Migration. The sensitivity analysis of the electric parameters is very significant for improving the reliability of the IC.

Based on the classical EM failure physical model equation of Black, this paper analyzes this model by using the Monte Carlo method. Firstly, confirms the four parameters of distribution in the Black equations. Then, analyzes the Monte Carlo method about the value of sampling N, the value of  $r^*$  in the limit equations, and the finally number of digits effective sensitivity. And then, analysis the four parameters using the Matlab, mainly considers four parameters' change; Change three parameters and fix the remaining one.; Two parameters change, two parameters fixed; A parameter change, three parameters fixed. Eventually, get the sensitivity order of the parameters in Black equations. This paper is useful for the physical model parameter sensitivity analysis of reliability (Ogawa E. T, 2002) and life prediction of the product.

## 1. Introduction

For more than four decades, the semiconductor industry has distinguished itself by the rapid pace of improvement in its products, which is strictly follows the Moore's Law (that is, the number of components per chip doubles roughly every 24 months). The most significant trend is the decreasing of cost-per-function, which led to significant improvements in economic productivity and overall quality of life through proliferation of computers, communication, and other industrial and consumer electronics products.

Electro-Migration (EM) is the transport of material caused by the gradual movement of the ions in a conductor due to the momentum transfer between conducting electrons and diffusing metal atoms. Electro-Migration can always lead to serious accidents, which occurred in the pacemaker, F15 fighter radar, rocket engines, Patriot missiles, nuclear weapons according to statistics (Pierce D. G., 1997), which is also happened in the satellites (Tin Whisker Experiences, 2002). So it is very necessary to analyze the Electro-Migration. Reliability sensitivity is the rate to explain the effect of basic variable distribution parameters( $\theta_x$ ) on failure probability( $P_f$ ), that is  $\frac{\partial P_f}{\partial \theta_x}$ .

In this paper we introduce reliability sensitivity analysis method—Monte Carlo (Ye H, 2003) to the microelectronics failure model to get the important degree of sorting of parameters by analyzing the sensitivity of special parameter, which can be used in the analyze of failure physical model sensitivity of similar microelectronic devices.

## 2. Parameters of Monte Carlo

### 2.1 Parameter distribution of Electro-Migration

Select aluminum (Tu K N, 2003) as the material of the model, referring to the Electro-Migration Black equation (BLACK J R, 1969):

$$MTTF = \frac{A}{J^n} \exp\left(\frac{E_a}{kT}\right) \quad (1)$$

We can get the data of model's parameters from the JESD63 (JEDEC STANDARD 63)'s Table A.4A and Table A.4B, as listed in the Table 1 and Table 2. The data in Table 1 is about the different value of MTTF in different conditions.

Table 1: The data in the JESD63(J, T)

i	J <sub>i</sub>	T <sub>i</sub>	t50 (J <sub>i</sub> , T <sub>i</sub> )
1	2.0	458	2.446
2	2.0	413	7.067
3	2.0	433	4.227
4	2.53	413	4.680
5	2.53	433	2.916
6	2.53	373	14.582
7	1.67	413	13.503
8	1.67	433	6.355
9	1.67	458	4.053
10	2.53	458	1.375

Table 2: The data in the JESD63(E<sub>a</sub>, n)

Parameter	n	E <sub>a</sub>
Sample Estimate	2.322	0.407
Standard Deviation	0.336	0.034

From the two tables, we can easily get the probability distribution of each parameter. There are four parameters in the Black equation to be studied, the current density, J, the model parameter for current density, n, the activation energy, E<sub>a</sub>, and the temperature, T. Fitting parameters A and Boltzmann constant k are definite values. J and T are depends on the test condition, floating around a certain value following the normal distribution. The model parameter for current density, J, is the value of the exponent, n, to which J is raised and the model parameter for temperature is the activation energy, E<sub>a</sub>, of the Electro-Migration process. When J and T are fixed, the two parameters—E<sub>a</sub> and n also follow the normal distribution. Here are all the distributions of each parameter, J ~ N(2.10, 0.2482); T ~ N(415.5, 24.532); E<sub>a</sub> ~ N(0.407, 0.0342); n ~ N(2.322, 0.3362); A = 12.2306, k = 8.617\*10<sup>-5</sup>.

Suppose that each variable is independently distributed. And according to the multiplication rule for independent events, the overall distribution is the multiplication of the probability distribution of each variable.

### 2.2 The limit state equation of Monte Carlo

This equation works as a criterion, As the aim of this paper is to rank the sensitivity of each parameter we choose the mid-life as the r\* in the equation. That is g(x) = r(x) - r\*, and the r(x) = MTTF = 2.447h.

### 2.3 Cycle index—N

Cycle index N, the larger the more accurate result from Monte Carlo Method. In the test, we choose the value of N from 1000 to 500000 and get the Figure 1 and Figure 2.

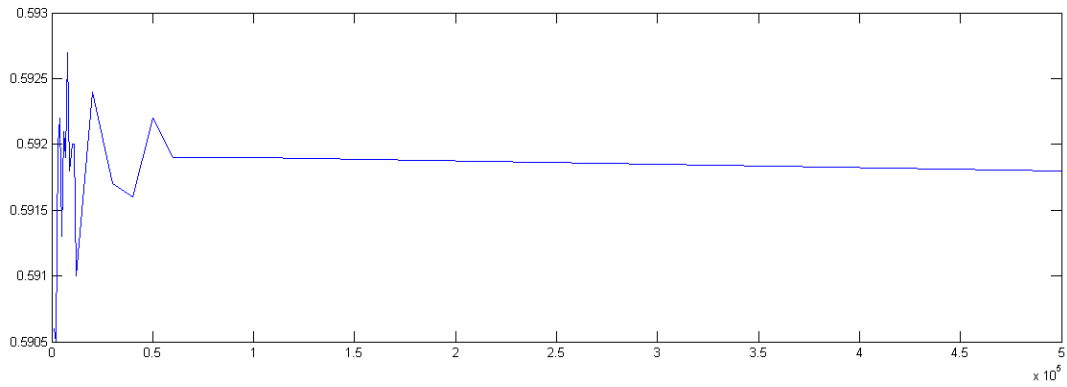


Figure 1 Mean of Pf

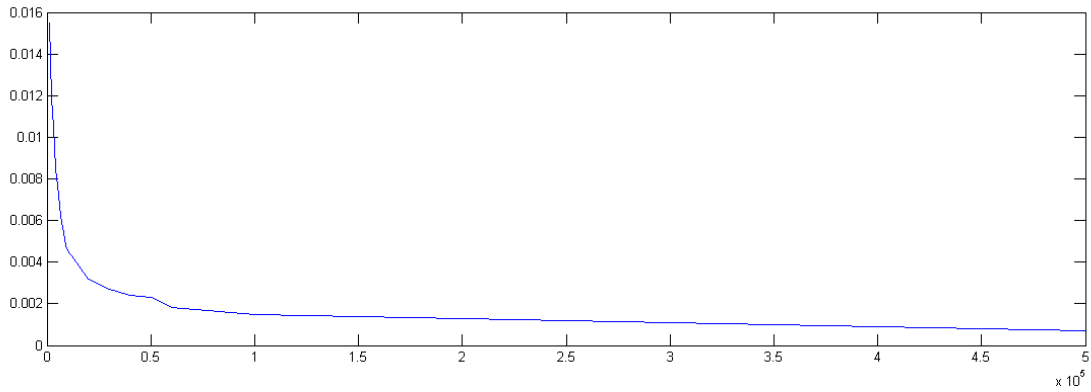


Figure 2 Variance of Pf

From the Figure 1 and the Figure 2, we can see each N can generate 100 values of Pf. The mean value of Pf gradually becomes stabilized and the variance is less than 0.01 when N=500000, which is acceptable. Finally, we choose Pf = 0.592, N = 500000.

#### 2.4 Discussion of Significant Digits

As all the parameters are following the normal distribution, the formula to accurate sensitivity becomes:

$$\frac{\partial \widehat{P}_f}{\partial \mu_{x_i}} = \frac{1}{N} \frac{1}{\delta_{x_i}} \sum_{k=1}^N \frac{x_{ki} - \mu_{x_i}}{\delta_{x_i}} I_F(x_k) \quad (2)$$

$$\frac{\partial \widehat{P}_f}{\partial \delta_{x_i}} = \frac{1}{N} \frac{1}{\delta_{x_i}} \sum_{k=1}^N I_F(x_k) \left[ \left( \frac{x_{ki} - \mu_{x_i}}{\delta_{x_i}} \right)^2 - 1 \right] \quad (3)$$

$I_F(x_k) = 1$  only if  $g(x_k) > 0$ , otherwise  $I_F(x_k) = 0$ . The final significant digits of the four parameters: J, there should be 3 significant figures; T, there should be 4 significant figures;  $E_a$ , there should be 2 significant figures; n, there should be 3 significant figures.

### 3. Calculation and analyze the sensitivity based on Monte Carlo

#### 3.1 Calculation of the sensitivity

As we have got the formula 2 and 3, we can get the value of the variance of mean of sensitivity and the coefficient of mean of sensitivity with the formula 4 and 5. And we calculate the sensitivity using the Monte Carlo method in Matlab.

$$\text{Var}\left(\frac{\partial P_f}{\partial \theta_{x_i}^k}\right) \approx \frac{1}{N-1} \left[ \frac{\partial P_f}{\partial \theta_{x_i}^k} - \left(\frac{\partial P_f}{\partial \theta_{x_i}^k}\right)^2 \right] \quad (4)$$

$$\text{Cov}\left(\frac{\partial P_f}{\partial \theta_{x_i}^k}\right) = \sqrt{\text{Var}\left(\frac{\partial P_f}{\partial \theta_{x_i}^k}\right)} \left| \frac{\partial P_f}{\partial \theta_{x_i}^k} \right| \quad (5)$$

(1) Preliminary calculations of the sensitivity

By using Monte Carlo method, we get the sensitivity as well as the coefficient of variation, which are shown in Table 3. As all the four parameters follow the normal distribution, each of them has two parameters to calculate:  $\mu$  and  $\delta$ . And the variance of sensitivity and coefficient of sensitivity reflect the data discrete degree.

Table 3: The raw data of the sensitivity

Parameter	T	J	E <sub>a</sub>	n
Mean of sensitivity	-9.81302902 1586238e-07	1.16678959 5770804	0.01853825 2121333	0.76392478 2709333
Variance of mean of sensitivity	1.96266041 8717530e-09	1.40575230 9296411e-05	0.00102255 1980363	8.04369759 6106890e-06
Coefficient of mean of sensitivity	45.14602302 9804844	0.00321337 8287964	1.72493968 0381844	0.00371259 2372700
Standard deviation of sensitivity	-3.28692874 7704330e-05	-0.30348586 3943906	-0.00758059 2388939	-0.21237588 4841834
Variance of Standard deviation of sensitivity	3.90390798 1888164e-09	3.36985417 7517731e-05	0.00205353 2988794	1.82295841 2096548e-05
Coefficient of Standard deviation of sensitivity	1.90090098 8181035	0.01912789 0952163	5.97788693 8735407	0.02010403 2570795

(2) Further calculation of the sensitivity

In order to get more knowledge of the Monte Carlo sampling methods and to get a more comprehensive analysis of the sensitivity of each parameter, we made further calculation of sensitivity, such as fix one parameter and change other three, two parameters fixed and the other two changed, three parameter fixed and the last one changed, from which we can make further operation and discussion about sensitivity.

**3.2 Analysis of the sensitivity**

(1) Preliminary analysis of the sensitivity

From Table 3 we get the final sensitivity of the four parameters in the Table 4 with the significant digits in consideration.

Table 4: The final sensitivity

Parameter	T	J	E <sub>a</sub>	n
Mean of sensitivity	-9.813e-07	1.17	0.019	0.764
Standard deviation of sensitivity	-3.287e-05	-0.303	-0.0076	-0.212

The positive and negative of the sensitivity indicate different kinds of impacts of the parameters on the probability. So we chose the absolute value of the sensitivity then rank it :  $J > n > E_a > T$ .

(2) Validation of the limit state equation

To prove the chosen limit state equation is correct, we use two different MTTF values 1 and 20 to calculate the sensitivity.. The result is in the Table 5 and 6.

Table 5: MTTF=1

Parameter(MTTF=1)	T	J	E <sub>a</sub>	n
Mean of sensitivity	-6.883e-06	0.123	-0.0020	0.0946
Standard deviation of sensitivity	-7.731e-06	0.170	-0.015	0.1359

Table 6: MTTF=20

Parameter	T	J	E <sub>a</sub>	n
Mean of sensitivity	-9.813e-07	1.17	0.019	0.764
Standard deviation of sensitivity	-3.287e-05	-0.303	-0.0076	-0.212

The value of the MTTF has no influence on the rank of the four parameters. The descending sort is that: J, n, E<sub>a</sub>, T.

(3) Further analysis of the sensitivity

The further analysis of the results is based on the further calculation of the sensitivity. First, set three parameter fixed and change remaining one. The result shows in Table 7

Table 7: Three fixed and one changed

Parameter (changed)	T	J	E <sub>a</sub>	n
Mean of sensitivity	4.687e-06	48	0.0034	1.07
Standard deviation of sensitivity	4.224e-05	1.598	1.2	-0.486

The rank of mean of sensitivity does not change but the rank of standard deviation of sensitivity changes and E<sub>a</sub> has the biggest SD. So if T, J and n can be ignored under certain conditions, there's more attention should be paid to E<sub>a</sub>, which gets a much bigger influence than before.

Table 8 to Table 11 shows the change of a specific parameter with different parameters fixed.

Table 8: The sensitivity of E<sub>a</sub>

Fixed parameters	Mean of sensitivity of E <sub>a</sub>	Standard deviation of sensitivity of E <sub>a</sub>
T, J, n	0.0034	1.2
T, n	-0.024	0.0091
T, J	-0.015	-0.059
J, n	0.0081	0.0026
T	0.024	-0.013
n	0.0031	0.052
J	0.015	-0.031
NONE	0.019	-0.0076

Table 9: The sensitivity of T

Fixed parameters	Mean of sensitivity of T (10 <sup>-6</sup> )	Standard deviation of sensitivity of T (10 <sup>-5</sup> )
E <sub>a</sub> , J, n	4.687	4.224
J, n	65.87	-5.489
E <sub>a</sub> , n	-25.97	-7.288
E <sub>a</sub> , J	-20.96	8.227
E <sub>a</sub>	42.04	-3.254
N	-5.364	-6.391
J	-28.66	5.281
NONE	-0.9813	3.287

Table 10: The sensitivity of J

Fixed parameters	Mean of sensitivity of J	Standard deviation of sensitivity of J
E <sub>a</sub> , n, T	1.48	-0.598
T, n	1.48	-0.599
E <sub>a</sub> , T	1.17	-0.305
E <sub>a</sub> , n	1.48	-0.594
E <sub>a</sub>	1.17	-0.299
n	1.49	-0.589
T	1.17	-0.306
NONE	1.17	-0.303

Table 11: The sensitivity of  $n$

Fixed parameters	Mean of sensitivity of $n$	Standard deviation of sensitivity of $n$
$E_a, J, T$	1.07	-0.486
$T, J$	1.07	-0.483
$E_a, T$	0.761	-0.210
$E_a, J$	1.07	-0.483
$E_a$	0.759	-0.209
$J$	1.07	-0.483
$T$	0.762	-0.212
NONE	0.764	-0.212

From Table 8, the mean of sensitivity of  $E_a$  changes a lot and if there are any laws. As it is still very important in the Black equation, the influence of  $E_a$  needs further research.

The Table 9 shows influence of  $T$  is always the smallest no matter which parameters fixed. So  $T$  has little effect on the probability of failure.

According to Table 10 and 11 the two parameters— $J$  and  $n$  have the similar regulation. They all only have 2 values, smaller one and bigger one. For  $J$ , if  $n$  is not fixed,  $J$  will take the smaller one and if the value of  $n$  is fixed, then  $J$  will take the bigger one. The state of other two parameters has little effect on  $J$ . And  $n$  also changes like this. That is, for  $J$  and  $n$ , when one parameter is fixed, the other one will take the maximum value.

#### 4. Conclusion

Based on the classical EM failure physical model equation of Black, this paper analyzes this model by using the Monte Carlo method. First, confirms the four parameters of distribution in the Black equations. Then, analyze the Monte Carlo method about the value of sampling  $N$ , the value of  $r^*$  in the limit equations, and the finally number of digits effective sensitivity, and at last gets the result.

For the four parameters  $E_a, T, J$ , and  $n$ , the sensitivity sort in descending order :  $J, n, E_a, T$ . Results of further analysis of the sensitivity show that the two parameters which have the most influence  $J$  and  $n$  varies in a regular. The changes from these two parameters are quite similar. They all only have 2 values and when one parameter is fixed, the other one will take the maximum value. In conclusion, the analysis of the sensitivity of the four parameters is meaningful for further research of the Electro-Migration and reliability assessment of devices.

This paper only analyze single failure mode and independent parameters, so the sort of sensitivity of multiple failure modes and related parameters is still unknown. Further research is needed.

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