

An Integrated Decision Model for Critical Component Spare Parts Ordering and Condition-based Replacement with Prognostic Information

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With advances in condition monitoring (CM) technologies, the joint decision-making paradigm for spare parts ordering as well as condition-based replacement with prognostic information has become a challenging but appealing issue in the systems health management field. To the best of our knowledge, few papers have focused on the joint decision for critical component spare parts ordering and replacement with prognostic information. In this paper we present an integrated decision model for jointly determining the condition-based replacement and critical component spare parts ordering decisions for a functioning component subject to condition monitoring. To do this, the degradation path of the component is modelled using a Wiener process, and the parameters are updated through the combination of the Bayesian method and the expectation maximization (EM) algorithm using real-time CM data. The probability density function (PDF) and cumulative density function (CDF) of the remaining useful life (RUL) are derived, which are then utilized to update the integrated decisions. The main advantage in our proposed decision-making model is that the prognostic information is fully utilized for joint decisions. An additional advantage lies in the updating mechanism of the RUL which enables the integrated decisions to be updated based on in-situ sensor data. The proposed integrated decision model is validated by a numerical example.

1. Introduction

The maintenance management strategy for engineering systems is one of the main elements in prognostics and systems health management (PHM) practice. During the past decades, various maintenance methodologies were proposed and applied by the researchers and practitioners, which can be roughly divided into corrective maintenance and preventive maintenance. Corrective maintenance is applied only after failures. Preventive maintenance can be further classified as time-based maintenance and condition-based maintenance. It is the maintenance that occurs before systems failure in order to maintain devices in a specified status. Among these studies, most maintenance related literature dealt with the maintenance issue under the following two assumptions (Horenbeek, 2012): 1) the amount of the available spare parts is infinite; 2) the spare parts can be acquired without any leading time. Unfortunately, these two assumptions cannot be always satisfied in practice and may lead to wrong conclusions.

In order to relax these assumptions in maintenance practice, the joint maintenance and inventory optimization issues have drawn more and more attention. The traditional joint decision methods are based on the time-based maintenance policies (Armstrong and Atkins, 1996). The failure time distributions used in these models are inferred from the statistical analysis of the failure time characteristics of component populations in an offline manner. However, the failure data is often hard to obtain, especially along with the increase of product reliability. On the other hand, this type of method does not consider the time-varying operational condition of specific individual components, and the failure distributions keep unchanged as for the integrated decision of an individual component. Fortunately, a large amount of sensory information can be available for the monitored components due to the recent advances in the sensory technology. If these CM data are properly collected and organized, they can be used to predict the health state of a functioning

device, and further, the integrated maintenance and inventory policies can be implemented in real time. For this reason, the condition-based order-replacement strategies have received considerable attention in recent years. For a single-unit system, Wang et al (2008) proposed a condition-based maintenance and spare parts provisioning approach for a single-unit system with gradual deterioration. However, they assumed that the degradation increments are nonnegative which limited its application scope. Elwany and Gebraeel (2008) also presented a sensory-driven prognostic model for single-unit system replacement and spare parts inventory, they modelled the degradation signal of a certain component as a continuous stochastic process, and the stochastic parameters were updated online based on the Bayesian paradigm. However, the diffusion coefficient σ and the prior distributions of the stochastic parameter λ in their degradation models have to be chosen by the history data of the other analogous units, and once these parameters are estimated, they are fixed without adjusting adaptively along with the dynamic observed CM data. Thus, the PDFs and CDFs which are then used in the joint decision models are not accurate enough. In this paper, we introduce a new condition-based order-replacement modelling method for a single-unit system with a room to store only one spare part. It is worth noting that the prognostic information is provided on the basis of a degradation path-dependent approach by employing the Wiener process model with deterministic and stochastic parameters. Further, all of the parameters in the degradation models are estimated only relying on the online CM information of a specific component without resorting to the history data of other components. Thus, more precise lifetime distributions can be obtained than the existing studies. Correspondingly, this leads to more reasonable replacement and inventory provisioning results. The reminder of this paper is organized as follows. Section 2 presents the degradation modelling framework and RUL estimation mechanism. In section 3, the condition-based order-replacement policy is given. A numerical example is provided in Section 4. Section 5 concludes this paper.

2. Degradation modelling and RUL estimation

Let $X(t)$ denotes the degradation at t , then a Wiener process-based degradation can be represented as

$$X(t) = \lambda t + \sigma B(t), \quad (1)$$

where λ is the drift coefficient, σ is the diffusion coefficient, and $B(t)$ is the standard Brownian motion (BM) with $B(t) \sim N(0, t)$ representing the stochastic dynamics of the degradation process. In this paper, we further assume $X(0) = 0$, and λ is stochastic with $\lambda \sim N(\mu_\lambda, \sigma_\lambda^2)$ that represents the unit-to-unit variability. These assumptions are widely used in degradation modelling literature (Si et al, 2013a).

For a specific functioning unit, we denote its RUL at t_k as L_k with its realization of l_k . The following equation can be easily deduced from Eq. (1)

$$Y(l_k) = x_k + \lambda l_k + \sigma W(l_k), \quad \text{for } l_k \geq 0, \quad (2)$$

where $Y(l_k) = X(l_k + t_k)$, x_k is the degradation at t_k , $W(l_k) = B(l_k + t_k) - B(t_k)$.

Based on the concept of FHT, the RUL at t_k can be defined as

$$L_k = \inf\{l_k : Y(l_k) \geq w\}, \quad (3)$$

where w represents the pre-set threshold of the degradation process.

From Eq. (3), the PDF and CDF of the RUL at t_k conditional on λ can be formulated respectively as follows,

$$f_{L_k|\lambda}(l_k | \lambda) = [(w - x_k) / (\sigma \sqrt{2\pi l_k^3})] \cdot \exp\{-(w - x_k - \lambda l_k)^2 / (2\sigma^2 l_k)\}, \quad (4)$$

$$F_{L_k|\lambda}(l_k | \lambda) = 1 - \Phi[(w - x_k - \lambda l_k) / (\sigma \sqrt{l_k})] + \exp\{[2\lambda(w - x_k)] / \sigma^2\} \cdot \Phi\{[-(w - x_k) - \lambda l_k] / (\sigma \sqrt{l_k})\}, \quad (5)$$

where $\Phi(\cdot)$ denotes the CDF of the standard normal distribution.

Considering the random effects of the stochastic parameter λ , the PDF and CDF of the RUL at t_k can be formulated by the law of total probability using $f_{L_k}(l_k) = \int f_{L_k|\lambda}(l_k | \lambda) p(\lambda) d\lambda$ and

$F_{L_k}(l_k) = \int F_{L_k|\lambda}(l_k | \lambda) p(\lambda) d\lambda$. From Eqs. (4) and (5), after some algebraic operations, the unconditional PDF and CDF of the RUL at t_k are shown as

$$f_{L_k}(l_k) = [(w - x_k) / \sqrt{2\pi l_k^3 (\sigma_k^2 + \sigma_{\lambda,k}^2 l_k)}] \cdot \exp\{-[(w - x_k - \mu_{\lambda,k} l_k)^2] / [2l_k (\sigma_k^2 + \sigma_{\lambda,k}^2 l_k)]\}, \quad (6)$$

$$F_{L_k}(l_k) = 1 - \Phi\left\{ \frac{(w - x_k - \mu_{\lambda,k} l_k) / \sqrt{\sigma_k^2 l_k + \sigma_{\lambda,k}^2 l_k^2}}{\sigma_k} + \frac{[2\mu_{\lambda,k} (w - x_k)] / \sigma_k^2 + [2\sigma_{\lambda,k}^2 (w - x_k)^2] / \sigma_k^4}{\sigma_k} \right\} \\ \cdot \Phi\left\{ -\frac{[2\sigma_{\lambda,k}^2 (w - x_k) l_k + \sigma_k^2 (\mu_{\lambda,k} l_k + w - x_k)] / (\sigma_k \sqrt{\sigma_k^2 l_k + \sigma_{\lambda,k}^2 l_k^2})}{\sigma_k} \right\}, \quad (7)$$

where σ_k is the updated value of the diffusion coefficient at t_k , $\mu_{\lambda,k}$ and $\sigma_{\lambda,k}^2$ are the mean and variance of stochastic parameter λ at t_k , respectively.

We have now obtained the analytical form of the PDF and CDF of the RUL at t_k , however, the parameters in these distributions should be estimated from observed data. In Gebraeel et al (2005) and Elwany and Gebraeel (2008), only the stochastic parameter λ is updated when a new CM data is available. In this paper, we adopt the approach proposed in Si et al (2013b), in which not only the stochastic parameter λ is updated but also the diffusion coefficient σ and the hyper-parameters in the prior distributions of the stochastic parameter are updated in real time.

(i) The updating of the stochastic parameter λ via a Bayesian mechanism

Given $\lambda \sim N(\mu_0, \sigma_0^2)$, the posterior distribution of the stochastic parameter λ at t_k is normal with the following parameters:

$$\mu_{\lambda,k} = (\mu_{0,k-1} \sigma_k^2 + x_k \sigma_{0,k-1}^2) / (t_k \sigma_{0,k-1}^2 + \sigma_{k-1}^2) \\ \sigma_{\lambda,k}^2 = \sigma_{k-1}^2 \sigma_{0,k-1}^2 / (t_k \sigma_{0,k-1}^2 + \sigma_{k-1}^2) \quad (8)$$

(ii) The updating of the diffusion coefficient σ and the hyper-parameters in the prior distributions of the stochastic parameter λ by the EM algorithm

From Eq. (8), the hyper-parameters in the prior distributions of the stochastic parameter λ , i.e. $\mu_{0,k-1}$ and $\sigma_{0,k-1}^2$, and the diffusion coefficient σ_{k-1}^2 should be known before employing the Bayesian rule at t_k . In this paper, these parameters are updated on the basis of $\mathbf{X}_{k-1} = \{x_1, x_2, \dots, x_{k-1}\}$ in an iterative manner, where \mathbf{X}_{k-1} represent all of the observations up to time t_{k-1} . We denote the prior values of the hyper-parameters and the diffusion coefficient before t_k as $\mathbf{Y}_{k-1} = [\mu_{0,k-1}, \sigma_{0,k-1}^2, \sigma_{k-1}^2]$. The EM algorithm is employed to estimate \mathbf{Y}_{k-1} due to the random effects and unobservability of the stochastic parameter λ . Based on all the observations \mathbf{X}_{k-1} till time t_{k-1} , the log-likelihood function can be written as

$$\ln f(\mathbf{X}_{k-1}, \lambda | \mathbf{Y}_{k-1}) = \ln f(\mathbf{X}_{k-1} | \lambda, \mathbf{Y}_{k-1}) + \ln f(\lambda | \mathbf{Y}_{k-1}) = -(k \ln 2\pi) / 2 - [\sum_{j=1}^{k-1} \ln(t_j - t_{j-1})] / 2 \\ - [(k-1) \ln \sigma_{k-1}^2] / 2 - \sum_{j=1}^{k-1} \{ [x_j - x_{j-1} - \lambda(t_j - t_{j-1})]^2 / [2\sigma_{k-1}^2 (t_j - t_{j-1})] \} - [\ln \sigma_{0,k-1}^2] / 2 - (\lambda - \mu_{0,k-1})^2 / (2\sigma_{0,k-1}^2) \quad (9)$$

In order to evaluate \mathbf{Y}_{k-1} in Eq. (9), the EM algorithm is utilized in the following two steps.

Step 1 E-step

Let $\hat{\mathbf{Y}}_{k-1}^{(i)} = [\hat{\mu}_{0,k-1}^{(i)}, \hat{\sigma}_{0,k-1}^{2(i)}, \hat{\sigma}_{k-1}^{2(i)}]$ denote the estimates in the i th step, the expectation of $\ln f(\mathbf{X}_{k-1}, \lambda | \mathbf{Y}_{k-1})$ can be formulated as follows,

$$E_{\lambda|\mathbf{X}_{k-1}^{(i)}} [\ln f(\mathbf{X}_{k-1}, \lambda | \mathbf{Y}_{k-1})] = -(k \ln 2\pi) / 2 - [\sum_{j=1}^{k-1} \ln(t_j - t_{j-1})] / 2 - [(k-1) \ln \sigma_{k-1}^2] / 2 \\ - \sum_{j=1}^{k-1} \{ [(x_j - x_{j-1})^2 - 2\mu_{\lambda,k-1} (t_j - t_{j-1})(x_j - x_{j-1}) + (t_j - t_{j-1})^2 (\mu_{\lambda,k-1}^2 + \sigma_{\lambda,k-1}^2)] / [2\sigma_{k-1}^2 (t_j - t_{j-1})] \}, \quad (10) \\ - [\ln \sigma_{0,k-1}^2] / 2 - (\mu_{\lambda,k-1}^2 + \sigma_{\lambda,k-1}^2 - 2\mu_{\lambda,k-1} \mu_{0,k-1} + \mu_{0,k-1}^2) / (2\sigma_{0,k-1}^2)$$

where $\mu_{\lambda,k-1}$ and $\sigma_{\lambda,k-1}^2$ are the estimates from Eq. (8) based on $\mathbf{X}_{k-1} = \{x_1, x_2, \dots, x_{k-1}\}$.

Step 2 M-step

Let $\frac{\partial E_{\lambda, \hat{\mathbf{Y}}_{k-1}^{(i)}}\{\cdot\}}{\partial \mathbf{Y}_{k-1}} = 0$ in Eq. (10), we can compute $\hat{\mathbf{Y}}_{k-1}^{(i+1)}$ as follows,

$$\begin{aligned} \hat{\mu}_{0,k-1}^{(i+1)} &= \mu_{\lambda,k-1} \\ \hat{\sigma}_{0,k-1}^{2(i+1)} &= \sigma_{\lambda,k-1}^2 \end{aligned} \quad (11)$$

$$\hat{\sigma}_{k-1}^{2(i+1)} = \frac{1}{k-1} \cdot \sum_{j=1}^{k-1} \frac{(x_j - x_{j-1})^2 - 2\mu_{\lambda,k-1}(t_j - t_{j-1})(x_j - x_{j-1}) + (t_j - t_{j-1})^2(\mu_{\lambda,k-1}^2 + \sigma_{\lambda,k-1}^2)}{t_j - t_{j-1}}. \quad (12)$$

Based on the iterative computations of the E-step and M-step, a series of $\hat{\mathbf{Y}}_{k-1}^{(i)}$ ($i=1,2,\dots$) can be obtained. The iterations are usually terminated when the difference between $\hat{\mathbf{Y}}_{k-1}^{(i+1)}$ and $\hat{\mathbf{Y}}_{k-1}^{(i)}$ is smaller than a pre-defined threshold. Thus, the optimal estimates $\hat{\mathbf{Y}}_{k-1}^* = \hat{\mathbf{Y}}_{k-1}^{(i)}$.

3. The condition-based replacement and spare provisioning policy

In this paper, we consider a single-unit critical component with a room to store only one spare part. Moreover, we assume the CM data can be obtained for free and the leading time is fixed. The condition-based replacement and spare provisioning method used in this paper is developed from the maintenance and spare parts inventory policies in (Elwany and Gebraeel, 2008). In this decision-making framework, the replacement time is confirmed firstly, and then the optimal ordering time is computed. According to the renewal theory, the long-run replacement cost and inventory cost per unit time can be formulated by Eqs. (13) and (14), respectively.

$$C_r^k(t_r^k) = [c_p + (c_f - c_p)\Pr(L_k < t_r^k - t_k)] / \{t_k + (t_r^k - t_k)[1 - \Pr(L_k < t_r^k - t_k)] + \int_{l_k=0}^{t_r^k - t_k} l_k f_{L_k}(l_k) dl_k\}, \quad (13)$$

where $\Pr(L_k < t - t_k) = \int_0^{t-t_k} f_{L_k}(l_k) dl_k$ and $f_{L_k}(l_k)$ can be obtained from Eq. (6), t_r^k is the decision variable representing the planned replacement time at t_k , c_p is the planned replacement cost, c_f is the failure replacement cost. After we obtain the optimal replacement time t_r^{k*} , the expected long-run inventory cost per unit time can be computed sequentially as follows

$$C_o^k(t_o^k) = \{k_s \int_{t_o^k}^{t_o^k + L} F_{L_k}(l_k) dl_k + k_h \int_{t_o^k}^{t_r^{k*}} [1 - F_{L_k}(l_k)] dl_k\} / \{t_k + \int_{t_o^k}^{t_o^k + L} F_{L_k}(l_k) dl_k + \int_0^{t_r^{k*}} [1 - F_{L_k}(l_k)] dl_k\}, \quad (14)$$

where $F_{L_k}(l_k)$ can be obtained from Eq. (7), t_o^k is the decision variable representing the inventory ordering time at time t_k , k_s is the shortage cost per unit time, k_h is the holding cost per unit time, and L is the fixed leading time elapsed from the moment of placing the order up till order arrival. Moreover, the optimal inventory ordering time t_o^{k*} at any CM point should satisfy the constraint $t_o^{k*} + L \leq t_r^{k*}$.

4. Numerical example

In order to show the adaptive nature of our presented degradation modelling method, we use a nonlinear process $X(t) = \lambda_s t^b + \sigma_s B(t)$ to generate the simulated degradation data. By the Euler discretization methodology, the simulated degradation process can be further transformed as $X((k+1)\Delta t) = X(k\Delta t) + \lambda_s b(k\Delta t)^{b-1} + \sigma_s Y \sqrt{\Delta t}$, where Δt is the discretization step and $Y \sim N(0,1)$. Here, the parameters are set as follows $\lambda_s = 0.1$, $b = 1.5$, $\sigma_s = 0.2$, and $\Delta t = 0.1$. One particular simulated degradation trajectory and the predicted one by our developed degradation modelling method are shown in Figure 1. After simulation, 120 suits of data are obtained with the final point $X(11.9) = 4.5520$. Given the failure threshold $w = 4.5600$, the FHT of this degradation path can be approximated as 12.0. Note that

the first data point $X(0)$ is set to be zero according to the model setting. The initial parameters are chosen arbitrarily as follows: $\mu_{\lambda} = 4, \sigma_{\lambda}^2 = 0.8, \sigma^2 = 0.5$. For comparison, we set the true parameters with the same values as that in the linear degradation model in (Elwany and Gebraeel, 2008). As shown in Figure 1, the projected path of our presented degradation modelling method tracks the actual degradation path well. This validates the predictability of our presented degradation modelling method.

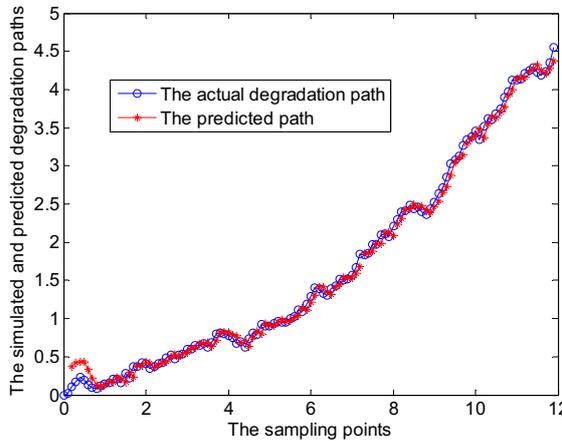


Figure 1: The simulated and projected degradation paths

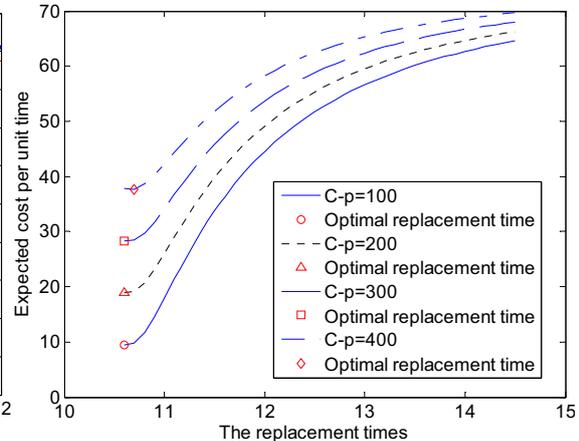


Figure 2: The expected replacement cost per unit time by Elwany's approach at $t_k = 10.5$

For simplicity, we just choose $t_k = 10.5$ as an example to show the feasibility and effectiveness of our proposed joint decision policy. Note that the other sampling points can be chosen for illustration too. Moreover, comparison works are done between our proposed method and the method used in (Elwany and Gebraeel, 2008). The related parameters are set as follows: $c_f = 1000$; $c_p = 100, 200, 300, 400$; $k_s = 3000$ / unit time; $k_h = 0.1$ / unit time, $L = 0.3$. It is worth pointing out that we fix the failure replacement cost $c_f = 1000$ and let the planned replacement cost c_p vary from 100 to 400 for a more comprehensive validation.

The average long-run replacement costs per unit time of the Elwany's approach and our proposed approach are illustrated in Figure 2 and Figure 3, respectively. From Figure 2, using the Elwany's approach, the replacement times are 10.6, 10.6, 10.6, and 10.7 when c_p varies from 100 to 400. It can be seen from Figure 3, the optimal replacement times are different when we choose varied planned replacement cost based on our proposed approach, however, the cost curves are evolving in a similar trend. Specifically, the corresponding replacement times are 11.5, 11.6, 11.7, and 11.9 when c_p varies from 100 to 400. As mentioned above, the FHT of this simulated degradation path is approximately to be 12.0 in this example. Compared with Elwany's approach, our approach gains more reasonable results since the replacement times obtained by Elwany's are too early. As mentioned previously, the prognostic information is typically important to the condition-based replacement and spare provisioning procedure. However, the diffusion coefficient σ and the prior distribution of the stochastic parameter λ rely on the history data heavily in Elwany's approach, and once they are estimated, they are fixed. This leads to an imprecise prognostic result for a critical component. If we adopt Elwany's approach in this numerical example, the spare part should be ordered immediately at $t_k = 10.5$, and it is too early according to the estimated FHT. If our approach is applied, the illustration of the long-run inventory cost per unit time is shown in Figure 4. Different lines mean that the evolutions of inventory cost are different along with the varying of the planned replacement cost c_p . However, the optimal ordering times are all at 11.0. As is pre-set in this simulation, the shortage cost per unit time is much higher than the holding cost per unit time. This is consistent with the situation in practice for these critical components.

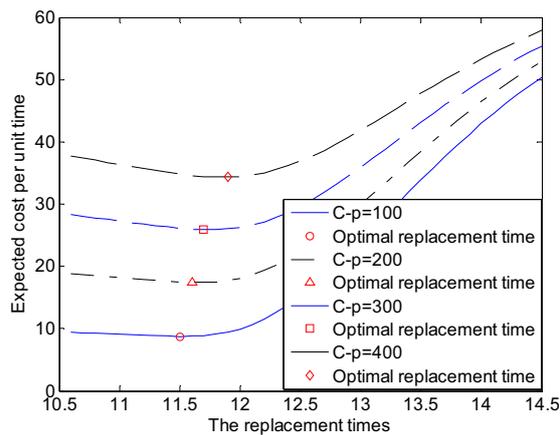


Figure 3: The expected replacement cost per unit time by our approach at $t_k = 10.5$

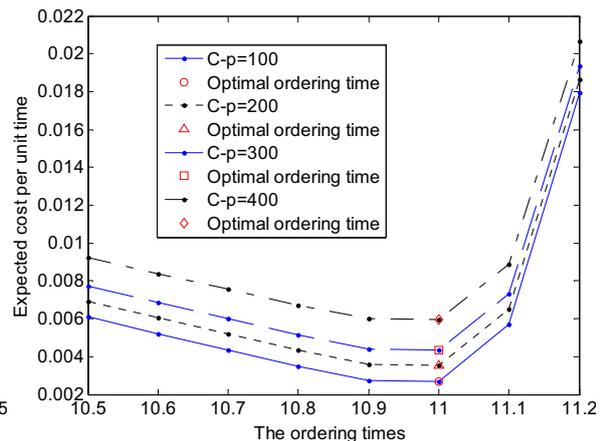


Figure 4: The expected inventory cost per unit time by our approach at $t_k = 10.5$

To sum up, the following conclusions can be drawn from this numerical example. Firstly, our presented condition-based replacement and spare provisioning approach can realize the optimization of replacement time and ordering time in a real time manner. Secondly, our presented joint decision approach gains more reasonable results than the existing approach in literature.

5. Conclusion

In this paper, we study the joint decision issue of condition-based replacement and inventory management for critical components. Firstly, a new degradation modelling method is introduced, which is independent of the information of the other components. Then, the prognostic information of one specific component is utilized to optimize the replacement time online, in turn, the optimal ordering time is confirmed sequentially. Compared with the joint decision approach in existing literature, the condition-based replacement and spare provisioning approach gains more reasonable results with more updated prognostic information.

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